NOTE

SOLVING RENJU

János Wágner[1] *István Virág*[2]

Budapest, Hungary

ABSTRACT

Renju is a very old two-player board game with subtle tactics and strategies. It comes from Japan and is a professional variant of Go-Moku. In many countries, such as China, Estonia, Japan, Latvia, Russia, and Sweden, Renju is played on a high level of performance. Compared to Renju, the game of Go-Moku, mainly played in China and Japan, has rather simple rules. Go-Moku was solved by Allis in 1992. He proved that it is a won game for the player to move first. In this note we claim that Renju is a first-player win too. We describe our research approach and how we arrived at this game-theoretical value.

1. INTRODUCTION

In 1992 Allis solved the game of Go-Moku by means of the computer program VICTORIA (Allis, Van den Herik, and Huntjes, 1993; Allis, 1994). The main techniques applied were proof-number search (Allis, Van der Meulen, and Van den Herik, 1994) and threat-space search (Allis, Van den Herik, and Huntjes, 1995). Since the advantage for the first player was well-known (Sakata and Ikawa, 1981), many variants of Go-Moku were developed so as to reduce the advantage of the first player, attempting to create a balanced game. Already around 1900 Japanese professional players improved the simple rules of Go-Moku by introducing that the first player (Black) is prohibited from making a *double three*, a *double four*, and an *overline*. After that the Renju board was diminished from 19×19 (Go board size) to 15×15 when it was discovered that the larger board size increases Black's advantage. These improvements were to restrict Black's moves so as to offset his initial advantage. In 1936 – when *the Japanese Federation of Renju* was founded – the rules of Renju were fully accomplished. Professional players believed that the new rules would result in a more evenly-balanced game.

When the Renju players became stronger they found that the above-mentioned advanced new rules still gave great advantage to Black. Some Japanese players even showed a black victory in two frequently-played opening patterns (cf. Sakata and Ikawa, 1981), but their solution was incomplete because some good white moves were not analysed and later on even mistakes were found in it. In the Sakata's book – Figures 31 and 32 facing page 77 – japanese authors suggested an strong 15th move (**A**, see Figure 1). After move **A**, if the White's reply was move **B** (16th move) the Black victory will be become very complicated, but not fully analysed by pro players. White's defence move **C** (16th move) is not mentioned in this book, after this White's response we were not able to find the Black victory in some weeks. So move A was rejected (and considered to be not good move) and we chose another 15th move (**D**), which turn out a success. Another problem was, that there are 35 distinct second white moves possible, but only two main variations (adjacent to Black's first move) were exhaustively analysed in that book. In Japan professional Renju players continued to study Renju in detail. Sakata and Ikawa (1981) published their analysis in 32 pages. We have exploited the analysis for our work and found other mistakes and lacunae (see also Allis, 1994). Nevertheless, the Japanese prediction is now confirmed, extended and corrected by our computer program which established Renju's game-theoretical value. Our project was to carry out a complete proof about the game of Renju by a computer program and to create a database of the solved game tree.

In 1988, *the Renju International Federation* was created. At the same time, to equalise the chances of the players at the official tournaments new opening-rules regulations were adopted. The official Renju rules and the new opening regulations are described in Section 2. However, this article focuses on establishing the game-theoretical value of Renju without the new opening-rules regulations. The game we deal with is called *free*

[1] 1193 Budapest Deák F. u. 1 VIII/32. E-mail: wagjan@mail.datanet.hu.

[2] Department of Inorganic and Analytical Chemistry, Eötvös Loránd University, PO. Box 32, Budapest 112, H-1518, Hungary. E-mail: virag@para.chem.elte.hu.

Renju, it is a quite aggressive game in which almost every black move is a threat. The remainder of this note is structured as follows. After the description of the rules in Section 2, Section 3 provides a list of terms and gives their definitions. In Section 4 we explain our experimental approach and provide empirical evidence for our claim. In Section 5 some technical details on a checking program are given and in Section 6 results are presented. Section 7 contains our conclusions.

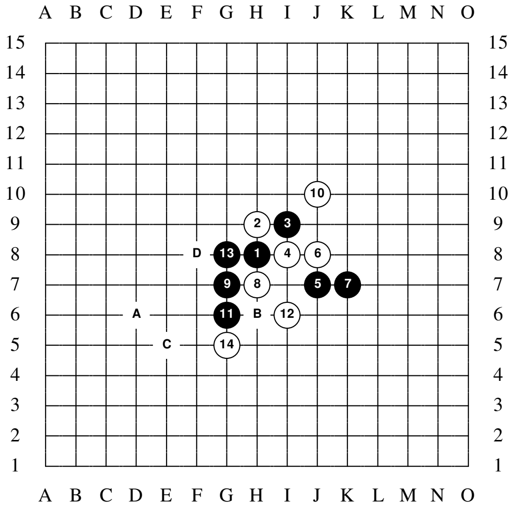

Figure 1: An instance, that human players may have some incompleteness their analysis.

2. THE RULES OF RENJU

The game of Renju is a professional variant of Go-Moku. The rules of Renju, although more complex than those of Go-Moku, are still rather simple (Sakata and Ikawa, 1981). The game is played on a board with 15×15 points of intersections. Vertical lines are lettered from 'a' to 'o', and the horizontal lines are numbered from 1 to 15. The left bottom position on the board is a1. Two players, Black and White, place alternately a stone of their own colour on an empty intersection. Black starts the game and must place a black stone on the centre intersection (h8). The first player completing an unbroken line of five stones (vertically, horizontally, or diagonally) wins the game. In Renju Black has some restrictions with respect to Go-Moku, e.g., some moves are considered to be "prohibited" for Black. If Black makes a prohibited move either accidentally or by being forced to, White wins the game. The prohibited moves for Black are: *overline*, *double-four*, *double-three* (for definitions, see Section 3). White does not have prohibited moves, so White may make, e.g., an *overline* and wins the game. If none of the players succeeds in completing a five-in-a-row, and Black did not make a prohibited move and the board is full of stones, the game is considered to be drawn.

The above-mentioned rules are known as the "*free* Renju rules". In an attempt to make the game more evenly-balanced for a fair competition in official tournaments, the following eight opening-rules restrictions have been imposed on Black (i.e., the professional Renju Rules).

a. Players start the game as tentative Black and tentative White. Tentative Black plays the first move on the centre intersection.
b. Tentative White makes the second move diagonally, horizontally or vertically to the first move and adjacent to the first move (i.e., two different openings).
c. Tentative Black plays the third move on an empty place within a zone of 5×5 intersections in the centre of the board (26 opening patterns).
d. Tentative White has the right to change the colour of the stones (swap option).
e. The player, who now has the white stones, plays the fourth move wherever (s)he wants.
f. Black has to offer the opponent two possible asymmetrical fifth moves.
g. White chooses one of the fifth moves which will be more advantageous to him/herself and tells Black to make the moves (s)he prefers.
h. There are no restrictions on the sixth and later moves.

3. TERMS AND DEFINITIONS

Below we provide a list of the terms together with their definitions.

Overline: Six or more stones of the same colour in an unbroken row, either horizontally, vertically or diagonally.
Five: Exact five stones of the same colour arranged in an unbroken row, either horizontally, vertically or diagonally.
Straight four: Four stones of the same colour in an unbroken row (horizontally, vertically or diagonally) with both ends open. A *straight four* ensures a win.
Four: Four stones of one colour in a row, which in one move can become a *five*.
Three: Three stones of the same colour in an unbroken row, or with one-intersection gap between the stones that can become a *straight four* on the next move.
Double-Four: A single move, which builds simultaneously more than one *four* at one time.
Double-Three: A single move, which builds simultaneously more than one *three* at one time.
Four-three: A *four* and a *three* produced simultaneously with a single move. It is the usual way for Black to create a winning formation.
VCF: This acronym means: *Victory by Consecutive Fours*. A win that results from making *fours* one after another.
VCT: This acronym means: *Victory by Consecutive Threats*. A win that results from making threats (*four* or *three*) one after another.

Japanese terms:
Fukumi: A move which threatens to win by *VCF*. Who played the *fukumi* move has a *VCF* if the opponent does not defend against this move.
Yobi: A positional offensive move, which threatens to win by *VCT*. A *yobi* move is not a direct forced method of attack.

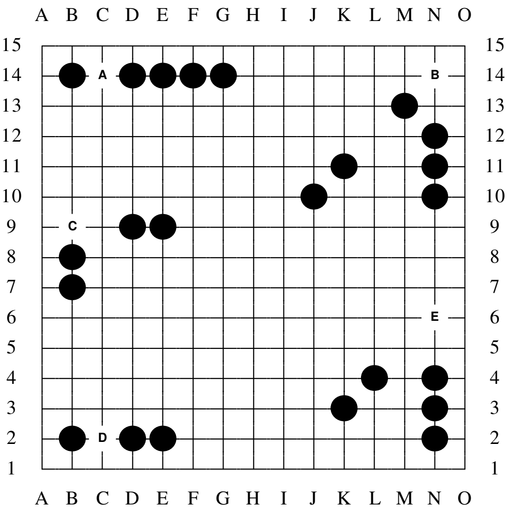

Figure 2: Five definitions illustrated.

In Figure 2 we illustrate five definitions given above.

Playing A leads to an *overline*.
Playing B leads to a *double-four*.
Playing C leads to a *double-three*.
Playing D leads to a *straight four*.
Playing E leads to a *four-three*.

4. THE PROGRAM AND THE EXPERIMENTAL APPROACH

The solving program was written in Borland Pascal language. The program's goal was to establish the game-theoretical value. The program generated a winning tree for Black and stored the tree in a database. The search process used transposition tables to avoid searching symmetrical positions. Moreover, the solving program used the threat-sequence search as suggested by Allis *et al.* (1993). There was only a very limited possibility to play no-threat moves, e.g., the first black moves after a bad opining by White (such as moves far away from the board centre). If a white defending move in such a variation also was a threat then Black countered it and did not lose a tempo in the end. The program operated an iterative-deepening search based on threat sequences up to 17 plies. The speed of the threat-sequence search was 400 nodes/second on a Pentium 200 MHz machine. The deepest variations of the iterative-deepening search (17 plies) took about 15 minutes, then a *VCT* line was found.

It was not possible to speed up the threat-sequence search (cf. Allis, 1994) since by the overline, double-four and double-three rule the order of moves became more complex. In the solving program we had implemented an automatic database builder, but all openings, *fukumi*, *yobi*, and positional moves were inputted by the hand of a human expert. In the past, the human expert assistance has been used in game solving (O. Patashnik, 1980). In total, human players produced approximately 5000 positional moves. The threat-sequence search module built all black threat sequences up to a given depth and investigated whether the position is a win for Black against every possible defence by White. After a positional black move, another module examined how Black can win if White passes. If a winning line exists then this module stores the moves of these sequences and in the next step the solving program will use these moves (usually about 60 to 70 moves, resulting in some 200 lines) as white answers. All moves of the game tree were stored in the database with the exception of the last 10 plies, since 10 or fewer plies of winning sequences can be found quickly by the threat-sequence search engine. Generating the whole game tree took about 3000 hours on a Pentium 200 MHz PC.

5. THE CHECKING PROCEDURE

After the generation of the whole game tree, the second stage of the project was checking the database. To making sure of generated database, there is necessary to develop a “checking” program, which plays backwards all possible white moves and then communicates with the solving program on the forward white moves (meanwhile inspecting the database). When a not-analysed white move was encountered, the solving program first took the black answer most frequently-played in that stage of the game tree. This approach gave many typical first omissions. In the checking loop, we stored white wins, black prohibited answers, and non-proved positions. The checking file contained approximately 2000 previously missing positions, which were corrected in the next proof run. The proof runs and the correction of positions were repeated several times even when the checking log did no longer contain any lacking positions. The programming of the “checking” program was performed by the second author in Borland Pascal programming language. This part of the solving took about 6000 CPU hours on a Pentium 200 MHz machine.

6. SOME RESULTS

Table 1 shows the statistics of the database of the solved game tree. The first column contains the opening pattern depending on White’s second move. Numbers 10, 11, 20, 21, 22 mean relative x and y coordinates according to the centre intersection of the board (h8). So 10 means positions H9, 11 means I9 and so on. Numbers 3x means 3 differences in one direction and any difference in the other direction, which leads to non-symmetric patterns (30, 31, 32, 33). The average depth was calculated by the following equation:

$$\text{Average Depth} = \frac{\sum_{D=1}^{MaxD} D * Nodes_D}{\sum_{D=1}^{MaxD} Nodes_D}$$

where D is the depth (Black moves) given a winning line and MaxD is the maximal depth. (Note that final 5 moves (Black moves) not stored.)

White’s 2nd move	Number of nodes	Average depth
10	169228	9.070
11	69682	7.107
20	73121	7.058
21	30236	6.019
22	27953	6.131
3x	88315	5.454
4x	85879	5.092
5x	92389	4.950
6x	87354	4.752
7x	87996	4.660
Total: 813674		

Table 1: Database results of the solved Renju game tree.

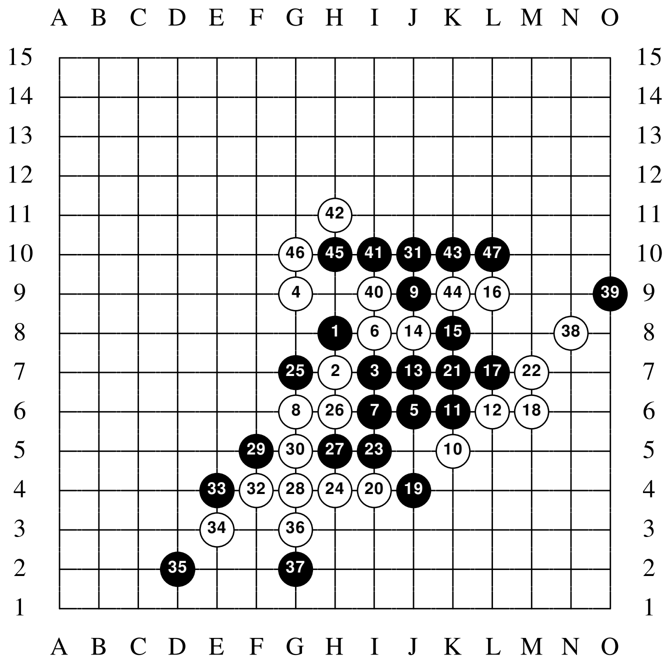

Figure 3: Our shortest solution (in best White protection): a win in 47 plies.

We admit that the method of solving the game as performed by us does not necessarily leads to producing an optimal solution. In fact we only established the game-theoretical value (our goal). Hence, it is possible that our shortest (database) winning lines is not optimal since we missed a stronger black move somewhere. Whatever the case, Figure 3 contains our solution of the best line in solving Renju, where White is on the best defensive. According to the database it takes 47 plies. Table 1. points to the fact that White best moves are 10 and 11 (according to japanese professional Renju players). But we were led to the conclusion, if White's second move is far from the centre point, Black victory is not unambigously easy. After White's second move 20, 21 and 22 there were a lot of hard line for Black. In all winning lines, distance of the black moves from remainder stones are not bigger than 2 intersections. It means that all played Black moves are very close to other stones.

7. CONCLUSIONS

We verified and expanded the statement of Japanese Renju experts that the game of *free* Renju is a theoretical win for the first player to move. The game was solved after 9000 hours of using a Pentium 200 MHz. The size of the overall database is 7 Megabyte. After the final database creation, the database was checked once more and accepted as correct by a "checking" program written by an independent programmer. In the last checking run, no missing variations were found by the "checking" program. In conclusion, *free* Renju should be considered a solved game. The result is that the game is a first-player win.

In Figure 3 we provided forced winning line. However it may be that it is not the shortest one. Finally, we have good reason to believe that after building a good opening book, say of 5000 moves, there is no middle game, provided that an appropriate endgame-analysing routine is available.

ACKNOWLEDGEMENT

We would like to acknowledge the help by Leonid Gluchovsky for sharing his Renju knowledge with us. He proposed many positional moves in difficult positions.

8. REFERENCES

Allis, L.V. (1994). *Searching for Solutions in Games and Artificial Intelligence*. Ph.D. Thesis, University of Limburg. Maastricht, The Netherlands. ISBN 90-9007488-0.

Allis, L.V., Herik, H.J. van den, and Huntjens, M.P.H. (1993). Go-Moku and Threat-Space Search. *Report CS 93-02*, Department of Computer Science, Faculty of General Sciences, University of Limburg. Maastricht, The Netherlands. ISSN 0922-8721.

Allis, L.V., Herik, H.J. van den, and Huntjens, M.P.H. (1996). Go-Moku Solved by New Search Techniques. *Computational Intelligence: An International Journal*, Vol. 12, No. 1, pp. 7-24. Special Issue on Games. Blackwell Publishers. ISSN 0824-7935.

Allis, L.V., Meulen, M. van der, and Herik, H.J. van den (1994). Proof-Number Search. *Artificial Intelligence*, Vol. 66, No. 1, pp. 91-124. ISSN 0004-3702.

Sakata, G. and Ikawa, W. (1981). *Five-In-A-Row, Renju.* The Ishi Press, Inc. Tokyo, Japan.

O. Patashnik, *Qubic: 4x4x4 Tic-Tac-Toe*, Mathematics Magazine 53, (1980), 202-216