FÜGGVÉNYEK

1. (a) Legyen $f(x) = 2x - 3$. Mivel egyenlő $f(4x)$? $[f(4x) = 8x - 3]$

   (b) Legyen $f(x) = 2x - 5$. Mivel egyenlő $f\left(\frac{x}{3}\right)$? $\left[f\left(\frac{x}{3}\right) = \frac{2}{3}x - 5\right]$

   (c) Legyen $f(x) = 2x + 7$. Mivel egyenlő $f\left(\frac{2}{x}\right)$? $\left[f\left(\frac{2}{x}\right) = \frac{4}{x} + 7\right]$

   (d) Legyen $f(x) = x^2 + 3x - 2$. Mivel egyenlő $f(5x)$? $[f(5x) = 25x^2 + 15x - 2]$

   (e) B Legyen $f(x) = x^2 + 3x - 2$. Mivel egyenlő $f\left(\frac{x}{2}\right)$? $\left[f\left(\frac{x}{2}\right) = \frac{1}{4}x^2 + \frac{3}{2}x - 2\right]$

   (f) B Legyen $f(x) = x^2 + 3x - 2$. Mivel egyenlő $f\left(\frac{1}{x}\right)$? $\left[f\left(\frac{1}{x}\right) = \frac{1}{x^2} + \frac{3}{x} - 2\right]$

2. Határozza meg a következő összetett függvényeket!
   $[g \circ f = g(f(x)); f \circ g = f(g(x)); f \circ f = f(f(x))]$

   (a) B $f(x) = \cos x + x^2; g(x) = \sqrt{x}; f(g(x)) =?; g(f(x)) =?$
   $\left[f(g(x)) = \cos(\sqrt{x}) + (\sqrt{x})^2 = \cos(\sqrt{x}) + x; g(f(x)) = \sqrt{\cos x + x^2}\right]$

   (b) B $f(x) = \sin x; g(x) = x^2; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \sin(x^2) = \sin x^2; g(f(x)) = (\sin x)^2 = \sin^2 x]$

   (c) B $f(x) = \sqrt{x+3}; g(x) = \sqrt{x} + 3; f(g(x)) =?; g(f(x)) =?$
   $\left[f(g(x)) = \sqrt{(\sqrt{x}+3)+3} = \sqrt{\sqrt{x}+6}; g(f(x)) = \sqrt{\sqrt{x+3}} + 3 = \sqrt[4]{x+3} + 3\right]$

   (d) B $f(x) = \ln x + 4x^5; g(x) = \mathrm{e}^x; f(g(x)) =?; g(f(x)) =?$
   $\left[f(g(x)) = \ln(e^x) + 4(\mathrm{e}^x)^5 = x + 4\mathrm{e}^{5x}; g(f(x)) = \mathrm{e}^{\ln x + 4x^5}\right]$

   (e) B $f(x) = x^2 - 3x; g(x) = \sqrt{5-2x}; f(g(x)) =?; g(f(x)) =?; f(f(x)) =?$
   $[f(g(x)) = \left(\sqrt{5-2x}\right)^2 - 3\sqrt{5-2x} = 5 - 2x - 3\sqrt{5-2x};$
   $g(f(x)) = \sqrt{5 - 2(x^2-3x)} = \sqrt{5 - 2x^2 + 6x};$
   $f(f(x)) = (x^2-3x)^2 - 3(x^2-3x) = x^4 - 6x^3 + 9x^2 - 3x^2 + 9x = x^4 - 6x^3 + 6x^2 + 9x]$

   (f) B $f(x) = 1 - x + x^2; g(x) = \mathrm{e}^x; f(g(x)) =?; g(f(x)) =?; f(f(x)) =?$
   $[f(g(x)) = 1 - \mathrm{e}^x + (\mathrm{e}^x)^2 = 1 - \mathrm{e}^x + \mathrm{e}^{2x}; g(f(x)) = \mathrm{e}^{1-x+x^2};$
   $f(fx)) = 1 - (1 - x + x^2) + (1 - x + x^2)^2 = x^4 - 2x^3 + 2x^2 - x + 1]$

   (g) B $f(x) = \cos(7-x); g(x) = x^4 - 3x + 2; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \cos(7 - (x^4 - 3x + 2)) = \cos(-x^4 + 3x + 5);$
   $g(f(x)) = (\cos(7-x))^4 - 3\cos(7-x) + 2 = \cos^4(7-x) - 3\cos(7-x) + 2]$

   (h) B $f(x) = \sqrt[3]{2-3x}; g(x) = 4x - x^3; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \sqrt[3]{2 - 3(4x - x^3)} = \sqrt[3]{2 - 12x + 3x^3};$
   $g(f(x)) = 4\sqrt[3]{2-3x} - (\sqrt[3]{2-3x})^3 = 4\sqrt[3]{2-3x} - 2 + 3x]$

   (i) B $f(x) = \sqrt[5]{4-x}; g(x) = x^5 - 3^x; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \sqrt[5]{4 - (x^5 - 3^x)} = \sqrt[5]{4 - x^5 + 3^x};$
   $g(f(x)) = (\sqrt[5]{4-x})^5 - 3^{\sqrt[5]{4-x}} = 4 - x - 3^{\sqrt[5]{4-x}}]$

(j) B $f(x) = (x-2)^2; g(x) = 2 - x^2; f(g(x)) =?; g(f(x)) =?$
$[f(g(x)) = [(2-x^2)-2]^2 = x^4;$
$g(f(x)) = 2 - [(x-2)^2]^2 = -x^4 + 8x^3 - 24x^2 + 32x - 14]$

3. Határozza meg a hiányzó függvényeket!

(a) $f(g(x)) = \sin(x+4); f(x) = \sin x; g(x) =?$ $[g(x) = x + 4]$

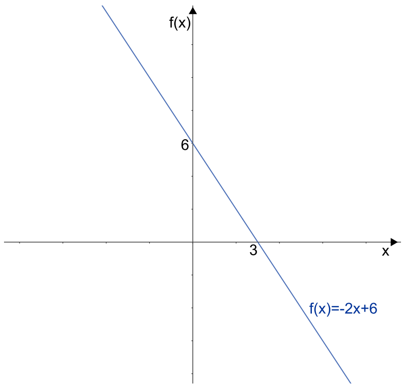
(b) B $f(g(x)) = \cos^4 x + 3\cos x; g(x) = \cos x; f(x) =?$ $[f(x) = x^4 + 3x]$

(c) B $g(f(x)) = x - \mathrm{e}^{\sqrt{x}}; g(x) = x^2 - \mathrm{e}^x; f(x) =?$ $[f(x) = \sqrt{x}]$

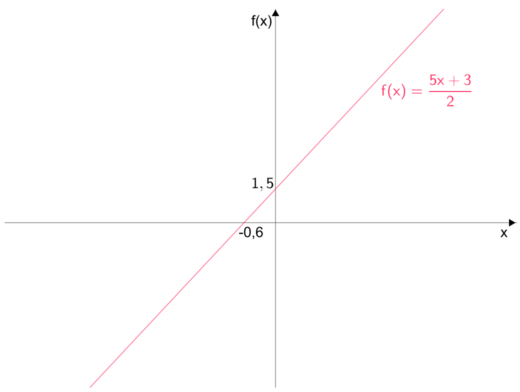
(d) B $g(f(x)) = \dfrac{x}{1+x^4}; f(x) = x^2; g(x) =?$ $\left[g(x) = \frac{\sqrt{x}}{1+x^2}\right]$

4. Ábrázolja az alábbi $f : R \to R$ függvényeket, majd az ábra alapján határozza meg a függvények értelmezési tartományát és értékkészletét!

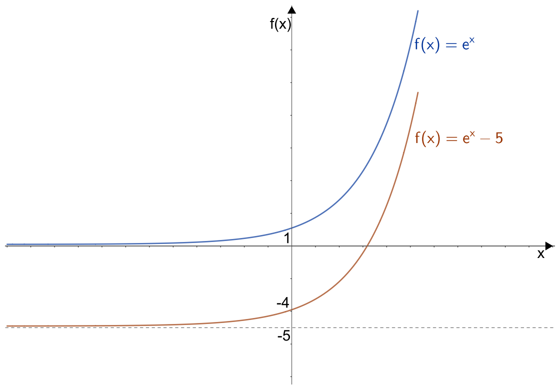
(a) $f(x) = -2x + 6$ $[D_f = R; R_f = R\ ]$



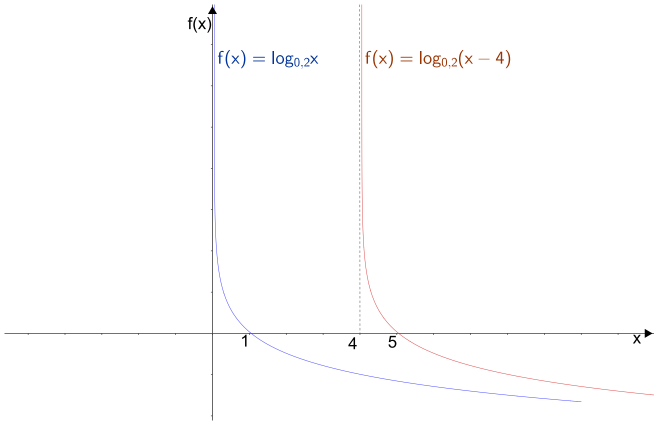
(b) $f(x) = \dfrac{5x+3}{2}$ $\left[f(x) = \frac{5x+3}{2} = \frac{5}{2}x + \frac{3}{2}, D_f = R; R_f = R\ \right]$



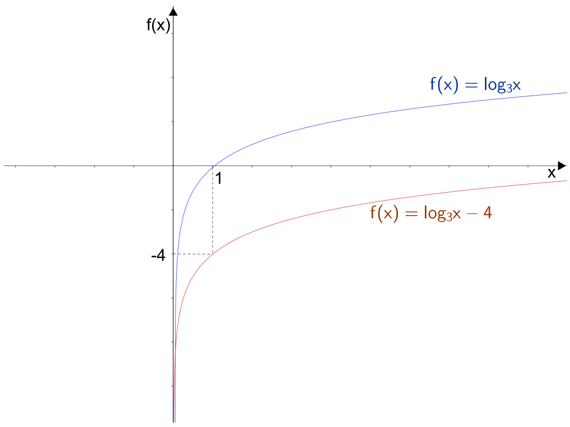
(c) $f(x) = \mathrm{e}^x - 5$ $[D_f = R; R_f =]-5;\infty[\ ]$



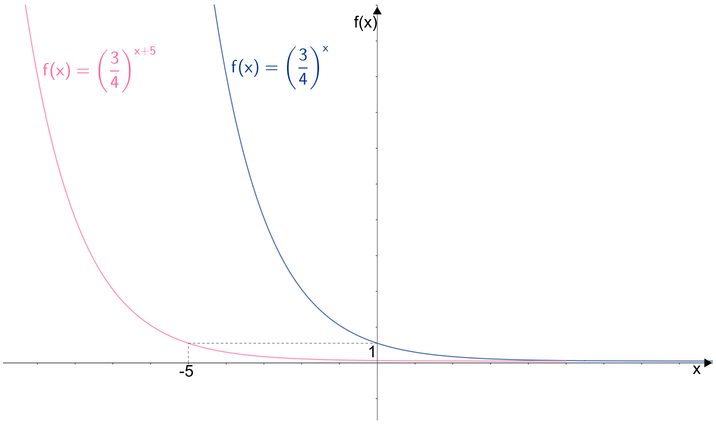
(d) $f(x) = \log_{0,2}(x-4)$ $[D_f =]4;\infty[; R_f = R\ ]$



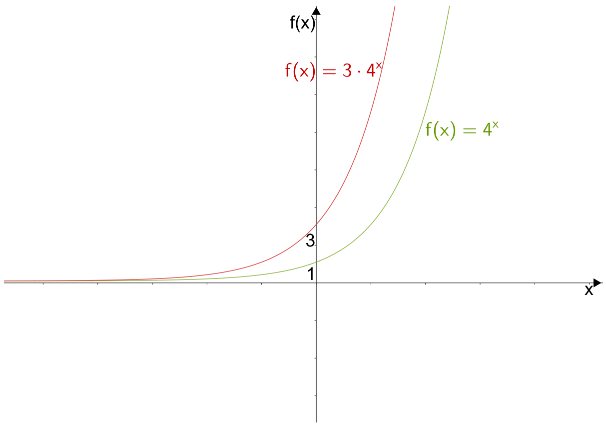
(e) $f(x) = \log_3 x - 4$ $[D_f =]0;\infty[; R_f = R\ ]$



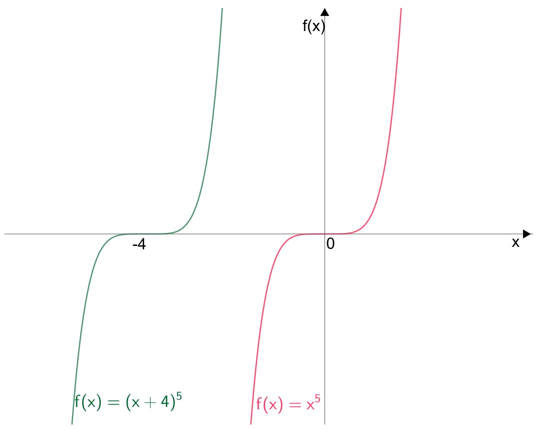
(f) $f(x) = \left(\frac{3}{4}\right)^{x+5}$ $[D_f = R; R_f =]0; \infty[\ ]$



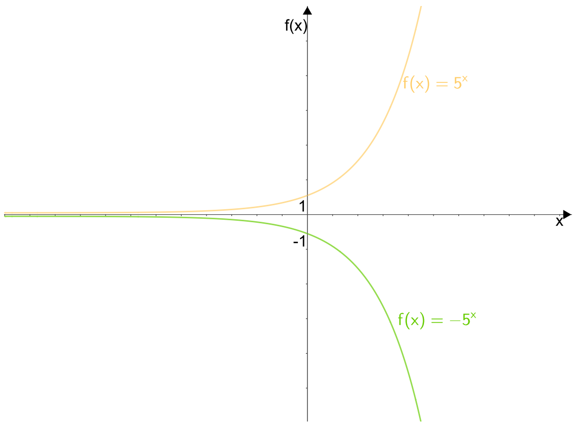
(g) $f(x) = 3 \cdot 4^x$ $[D_f = R; R_f =]0; \infty[\ ]$



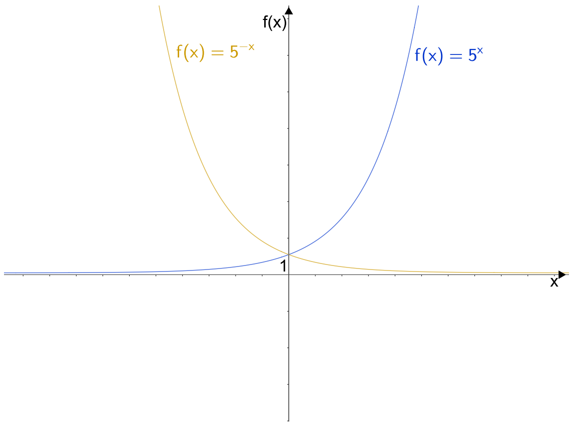
(h) $f(x) = (x + 4)^5$ $[D_f = R; R_f = R\ ]$



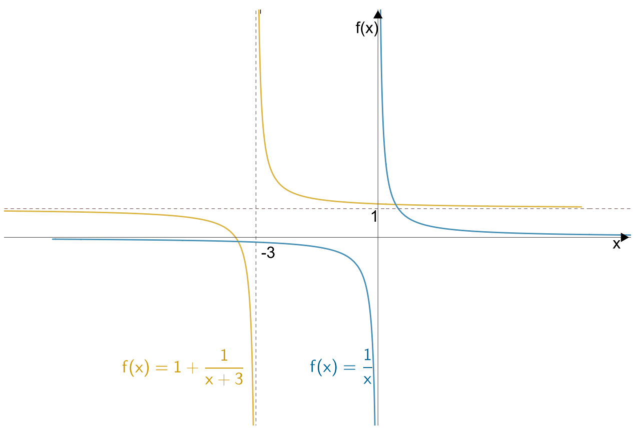
(i) $f(x) = -5^x$ $[D_f = R; R_f =]-\infty; 0[\ ]$



(j) $f(x) = 5^{-x}$ $[D_f = R; R_f =]0; \infty[\ ]$

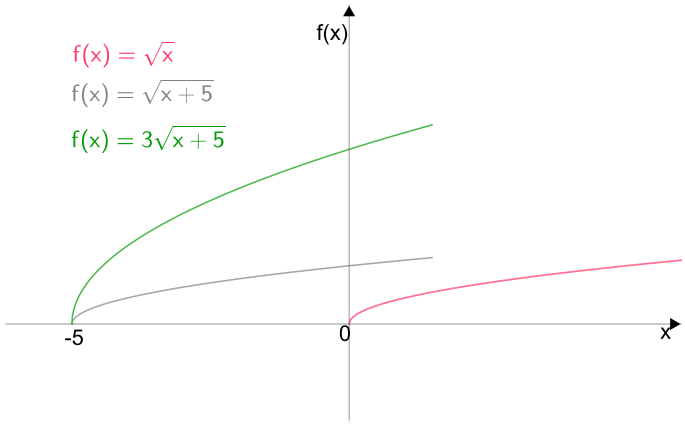


(k) $f(x) = 1 + \dfrac{1}{x+3}$ $[D_f = R \setminus \{-3\}; R_f = R \setminus \{1\}\ ]$

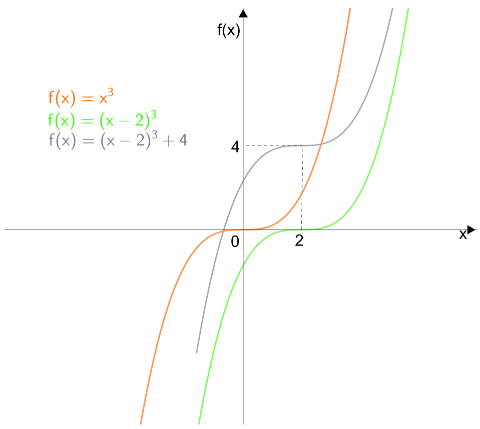


5. Ábrázolja az alábbi $f : R \to R$ függvényeket, majd az ábra alapján határozza meg a függvények értelmezési tartományát és értékkészletét!

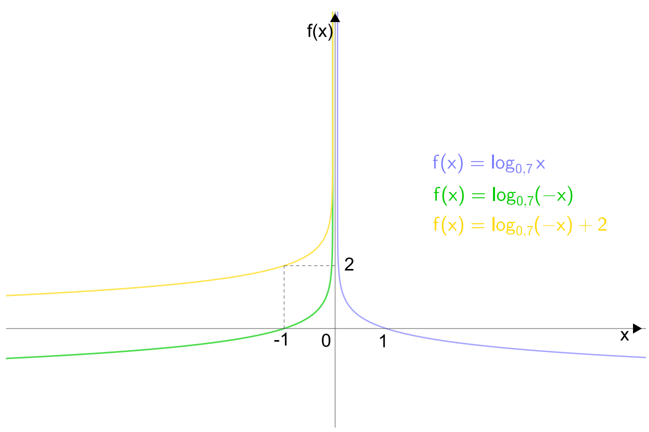
(a) B $f(x) = 3\sqrt{x+5}$ $[D_f = [-5, \infty[; R_f = [0; \infty[\ ]$



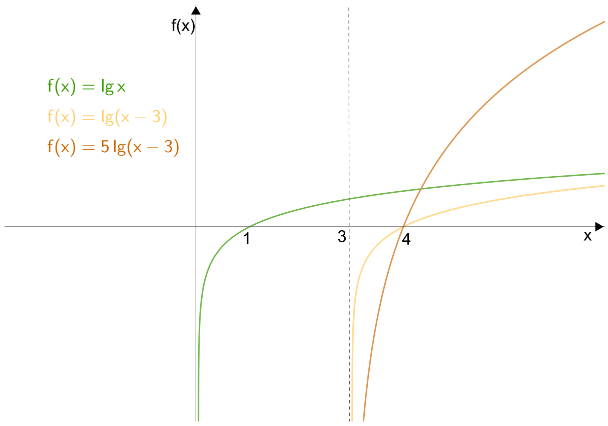
(b) B $f(x) = (x-2)^3 + 4$ $[D_f = R; R_f = R\ ]$



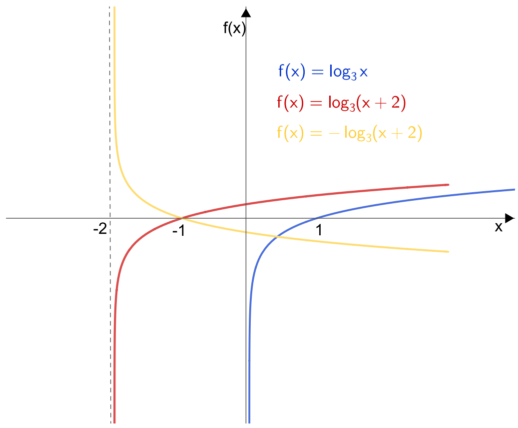
(c) B $f(x) = \log_{0,7}(-x) + 2$ $[D_f =]-\infty, 0[; R_f = R\ ]$



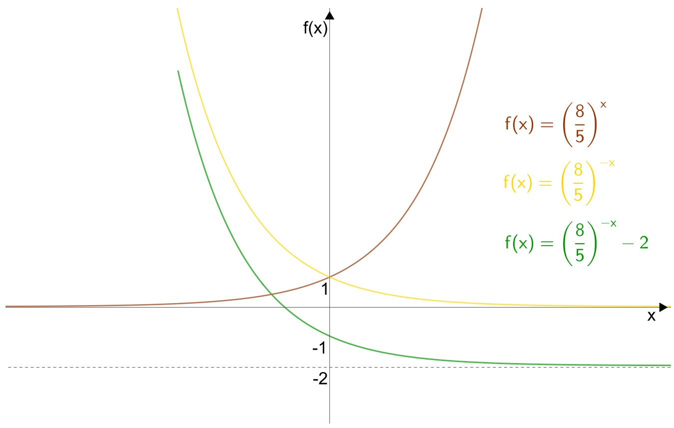
(d) B $f(x) = 5\lg(x-3)$ $[D_f =]3, \infty[; R_f = R\ ]$



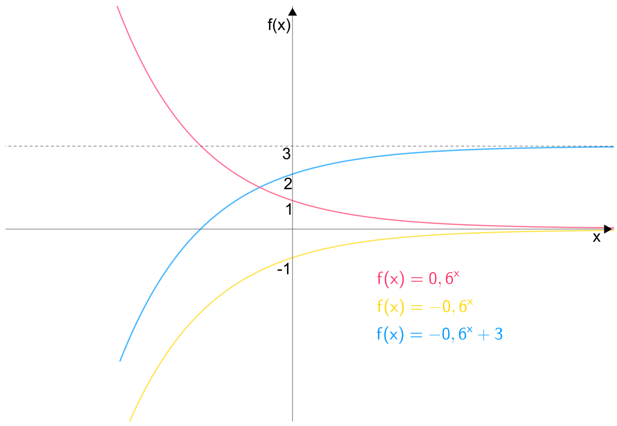
(e) B $f(x) = -\log_3(x+2)$ $[D_f =] -2, \infty[; R_f = R\ ]$



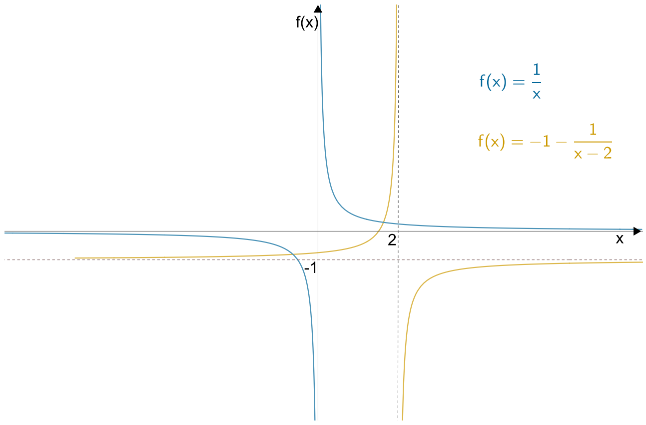
(f) B $f(x) = \left(\frac{8}{5}\right)^{-x} - 2$ $[D_f = R; R_f =] -2, \infty[\ ]$



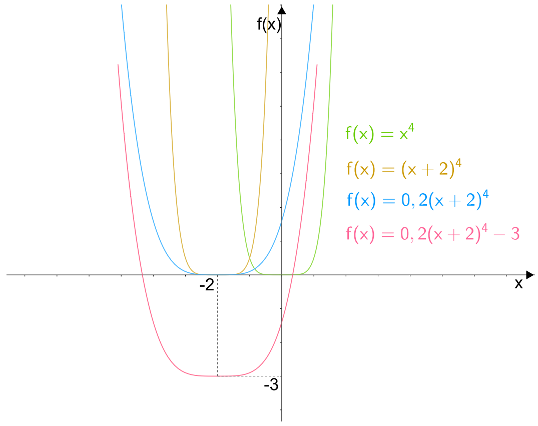
(g) B $f(x) = -0,6^x + 3$ $[D_f = R; R_f =]-\infty, 3[\ ]$



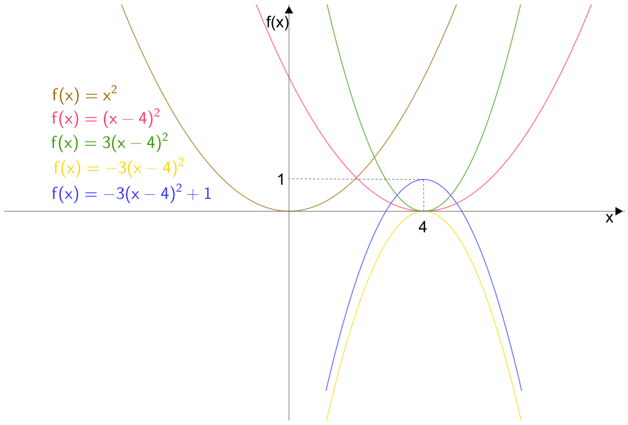
(h) B $f(x) = -1 - \dfrac{1}{x-2}$ $[D_f = R \setminus \{2\}; R_f = R \setminus \{-1\}\ ]$



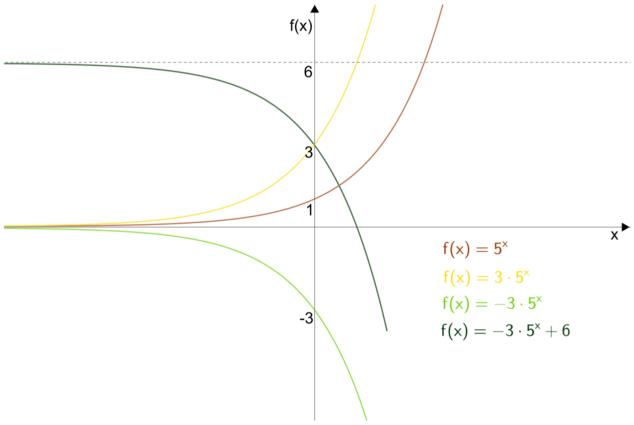
(i) V $f(x) = 0,2(x+2)^4 - 3$ $[D_f = R; R_f = [-3; \infty[\ ]$



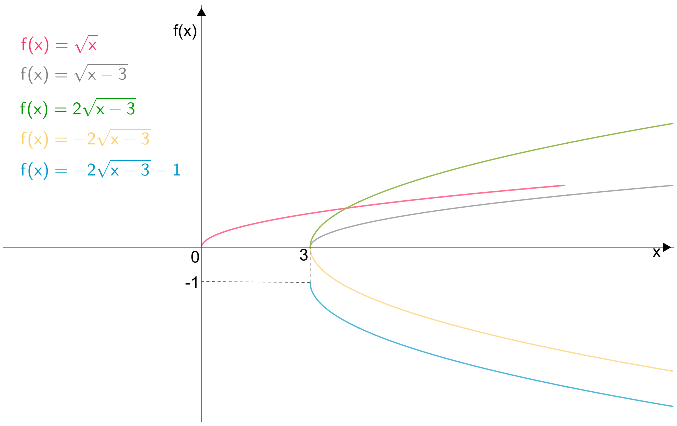
(j) V $f(x) = -3(x-4)^2 + 1$ $[D_f = R; R_f =]-\infty; 1]\ ]$



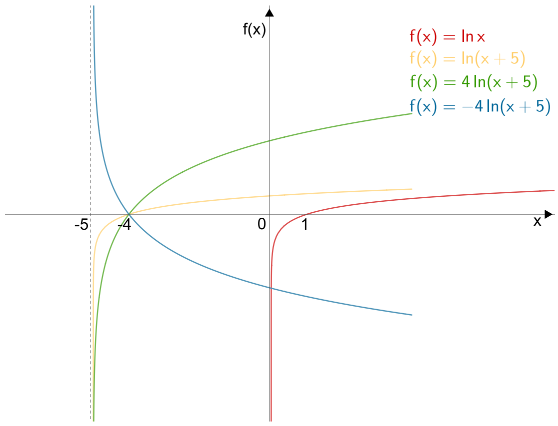
(k) V $f(x) = -3 \cdot 5^x + 6$ $[D_f = R; R_f =]-\infty; 6[\ ]$



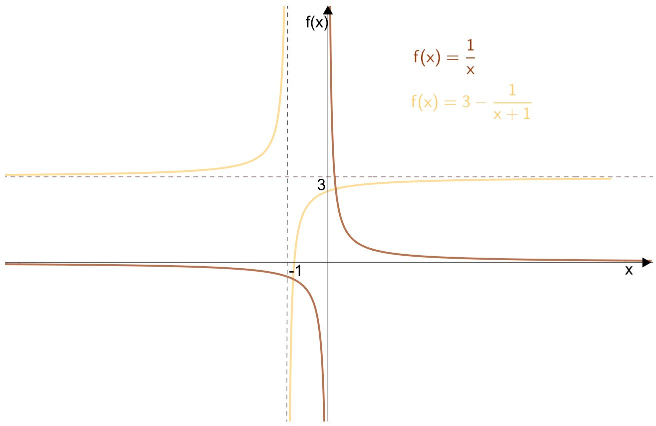
(l) V $f(x) = -2\sqrt{x-3} - 1$ $[D_f = [3, \infty[; R_f =]-\infty; -1]\ ]$



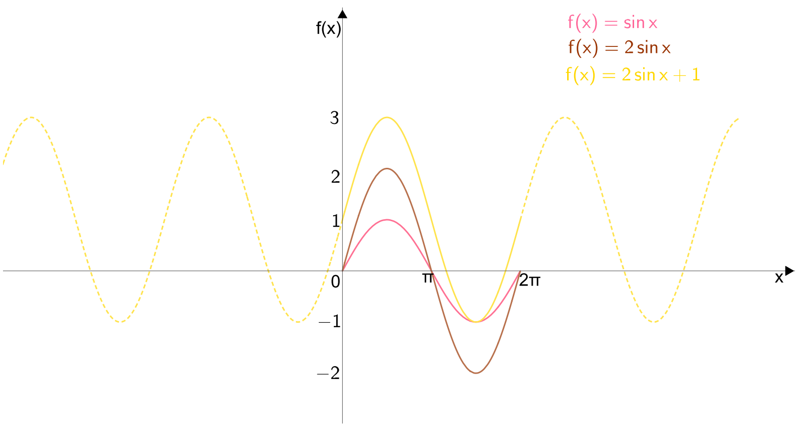
(m) V $f(x) = -4\ln(x+5)$ $[D_f =]-5, \infty[; R_f = R\ ]$



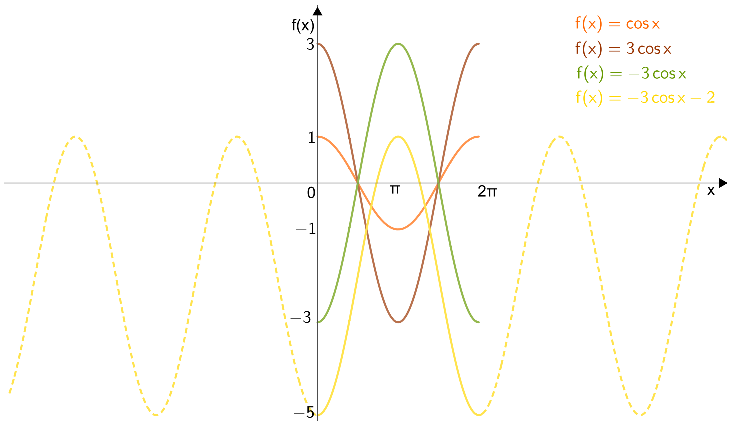
(n) V $f(x) = \dfrac{3x+2}{x+1}$ $\left[f(x) = \dfrac{3x+2}{x+1} = 3 - \dfrac{1}{x+1}; D_f = R \setminus \{-1\}; R_f = R\setminus\{3\}\ \right]$



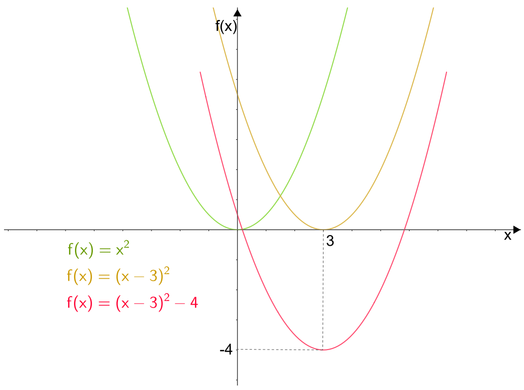
(o) V $f(x) = 2\sin x + 1$ $[D_f = R; R_f = [-1;3]\ ]$



(p) V $f(x) = -3\cos x - 2$ $[D_f = R; R_f = [-5; 1]\ ]$



(r) V $f(x) = x^2 - 6x + 5$ $\left[f(x) = x^2 - 6x + 5 = (x-3)^2 - 4; D_f = R; R_f = [-4, \infty[\ \right]$



6. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

(a) $f(x) = \sqrt{4x-8}$ $[4x - 8 \geq 0; D_f = [2; \infty[\ ]$

(b) $g(x) = \log_8(9-5x)$ $\left[9 - 5x > 0; D_g =]-\infty; \frac{9}{5}[\ \right]$

(c) $h(x) = \sqrt[7]{4x+7}$ $[D_h = R\ ]$

(d) $f(x) = \dfrac{3x+7}{8x+9}$ $\left[8x + 9 \neq 0; D_f = R \setminus \{-\frac{9}{8}\}\ \right]$

(e) $g(x) = \mathrm{e}^{6x-7}$ $[D_g = R\ ]$

(f) $f(x) = 3^{\frac{4x-3}{-x+7}}$ $[-x + 7 \neq 0; D_f = R \setminus \{7\}\ ]$

(g) $h(x) = \dfrac{8x}{\sqrt[5]{4x-3}}$ $\left[\sqrt[5]{4x-3} \neq 0; D_h = R \setminus \{\frac{3}{4}\}\ \right]$

(h) $f(x) = \dfrac{4x-5}{\sqrt{3x+6}}$ $\left[\sqrt{3x+6} \neq 0; 3x + 6 \geq 0; D_f =]-2; \infty[\ \right]$

(i) $h(x) = \sqrt[4]{x^2+3x-10}$ $[x^2 + 3x - 10 \geq 0; D_h =]-\infty; -5] \cup [2; \infty[\ ]$

(j) $f(x) = \dfrac{3x^5 - 2}{\sqrt[3]{x^2 - 2x - 3}}$ $\left[\sqrt[3]{x^2 - 2x - 3} \neq 0; D_f = R \setminus \{-1; 3\}\ \right]$

(k) $f(x) = \dfrac{\sqrt{x}}{x^2 - 5x + 6}$ $[x \geq 0; x^2 - 5x + 6 \neq 0; D_f = [0; \infty[\setminus\{2; 3\}\ ]$

7. Határozza meg a következő $f : R \to R$ függvények legbővebb értelmezési tartományát!

(a) B $f(x) = \sqrt{-x^2 + 6x - 8}$ $[-x^2 + 6x - 8 \geq 0; D_f = [2; 4]\ ]$

(b) B $f(x) = \log_7(-x^2 + 4x + 12)$ $[-x^2 + 4x + 12 > 0; D_f =]-2; 6[\ ]$

(c) B $f(x) = \dfrac{3 - 2x}{\sqrt{x^2 - x - 20}}$
$\left[\sqrt{x^2 - x - 20} \neq 0; x^2 - x - 20 \geq 0; D_f =]-\infty; -4[\cup]5; \infty[\ \right]$

(d) B $f(x) = \dfrac{6^{3x}}{\log_3(x + 5)}$ $[x + 5 > 0; \log_3(x + 5) \neq 0; D_f =]-5; \infty[\setminus\{-4\}\ ]$

(e) B $f(x) = -\dfrac{\lg(11 - 4x)}{8x - 3}$ $\left[11 - 4x > 0; 8x - 3 \neq 0; D_f =]-\infty; \frac{11}{4}[\setminus\{\frac{3}{8}\}\ \right]$

(f) B $f(x) = \dfrac{\sqrt[4]{7 + 4x}}{3x - 12}$ $\left[7 + 4x \geq 0; 3x - 12 \neq 0; D_f = [-\frac{7}{4}; \infty[\setminus\{4\}\ \right]$

(g) B $g(x) = \dfrac{5}{4 - \sqrt{5x + 3}}$ $\left[5x + 3 \geq 0; 4 - \sqrt{5x + 3} \neq 0; D_g = [-\frac{3}{5}; \infty[\setminus\{\frac{13}{5}\}\ \right]$

(h) B $f(x) = \log_7(-5x + 7) + \sqrt{3x}$ $\left[-5x + 7 > 0; 3x \geq 0; D_f = [0; \frac{7}{5}[\ \right]$

(i) B $f(x) = \sqrt{\dfrac{8 - 3x}{5}}$ $\left[\frac{8-3x}{5} \geq 0 \Leftrightarrow 8 - 3x \geq 0, D_f =]-\infty, \frac{8}{3}]\ \right]$

(j) B $f(x) = \log_3\left(\dfrac{-7}{4x + 6}\right)$ $\left[\frac{-7}{4x+6} > 0 \Leftrightarrow 4x - 6 < 0, D_f =]-\infty, -\frac{3}{2}[\ \right]$

(k) B $h(x) = \dfrac{1}{\sqrt{5 - x}} + \lg(x + 1)$ $[5 - x \geq 0; 5 - x \neq 0; x + 1 > 0; D_h =]-1, 5[\ ]$

(l) B $f(x) = \dfrac{\ln(3 - 2x)}{\sqrt{4x - x^2}}$ $\left[4x - x^2 \geq 0; 4x - x^2 \neq 0; 3 - 2x > 0; D_f =]0, \frac{3}{2}[\ \right]$

(m) B $f(x) = \lg(15 - 3x) + \sqrt{x^2 + 8x - 9}$
$[15 - 3x > 0; x^2 + 8x - 9 \geq 0; D_f =]-\infty, -9] \cup [1, 5[\ ]$

8. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

(a) V $f(x) = \sqrt{\log_2 x + 5}$
$[\log_2 x + 5 \geq 0(\log_2 x-$szigorúan monoton növekvő$); x > 0; D(f) = [2^{-5}; \infty] = [\frac{1}{32}; \infty]\ ]$

(b) V $f(x) = \sqrt[6]{\log_{0,4} x + 2}$
$[\log_{0,4} x + 2 \geq 0(\log_{0,4} x-$szigorúan monoton csökkenő$); x > 0;$
$D(f) =]0; 0,4^{-2}] =]0; 6,25]]$

(c) V $f(x) = \sqrt{3^x - 81}$
$[3^x - 81 \geq 0(3^x-$szigorúan monoton növekvő$); D(f) = [4; \infty[\ ]$

(d) V $f(x) = \dfrac{-7}{5-\sqrt{x^2-9}}$

$\left[x^2-9 \geq 0; 5-\sqrt{x^2-9} \neq 0; D(f) =]-\infty; -3] \cup [3; \infty[\setminus \{-\sqrt{34}; \sqrt{34}\}\ \right]$

(e) V $f(x) = \ln(x^2-x) + \sqrt{16-4x^2}$

$[x^2-x > 0; 16-4x^2 \geq 0; D(f) = [-2; 0[\cup]1; 2]\ ]$

(f) V $f(x) = x + \sqrt{5-x-\dfrac{6}{x}}$

$\left[5-x-\dfrac{6}{x} \geq 0; D_f =]-\infty, 0[\cup[2, 3]\ \right]$

(g) V $f(x) = \dfrac{x+1}{\lg(1-2x)} + \sqrt{1-x^2}$

$\left[\lg(1-2x) \neq 0; 1-2x > 0; 1-x^2 \geq 0; D_f = [-1; \frac{1}{2}[\setminus\{0\}\ \right]$

(h) V $f(x) = \dfrac{x}{5} \cdot \ln(16-x^2) - \sqrt{\dfrac{x-2}{x+3}}$

$\left[16-x^2 > 0; \frac{x-2}{x+3} \geq 0; D_f =]-4, -3[\cup[2, 4[\ \right]$

(i) V $f(x) = \dfrac{\lg x - 8}{\sqrt{35+2x-x^2}} + \dfrac{5}{x-4}$

$\left[x > 0; \sqrt{35+2x-x^2} \neq 0; 35+2x-x^2 \geq 0, x-4 \neq 0; D_f =]0, 4[\cup]4, 7[\ \right]$

(j) V $f(x) = \dfrac{\sqrt{x^2-x-2}}{\ln x} - \dfrac{3+6x}{8}$

$[x^2-x-2 \geq 0; \ln x \neq 0; x > 0; D_f = [2, \infty[\ ]$

(k) V $f(x) = \dfrac{2x}{5} + \dfrac{\sqrt{16-x^2}}{\lg(8-2x)}$

$\left[16-x^2 \geq 0; \lg(8-2x) \neq 0; 8-2x > 0; D_f = [-4, 4[\setminus \left\{\frac{7}{2}\right\} = \left[-4, \frac{7}{2}\right[\cup \left]\frac{7}{2}, 4\right[\ \right]$

(l) V $f(x) = \dfrac{\sqrt{x^2-1}}{x^2-1} + \dfrac{2}{\sqrt{1-\lg(2x)}}$

$\left[x^2-1 \geq 0; x^2-1 \neq 0; 1-\lg(2x) \geq 0; \sqrt{1-\lg(2x)} \neq 0; 2x > 0; D_f =]1, 5[\ \right]$

(m) V $f(x) = \sqrt{\dfrac{3}{2x-5}} + \ln(6+11x-2x^2)$

$\left[\frac{3}{2x-5} \geq 0; 6+11x-2x^2 > 0; D_f = \left]\frac{5}{2}, 6\right[\ \right]$

(n) V $f(x) = 3 \cdot \lg\left(\dfrac{x+1}{x-5}\right) - \dfrac{\sqrt{5x-10}}{x^2-36}$

$\left[\frac{x+1}{x-5} > 0; 5x-10 \geq 0; x^2-36 \neq 0; D_f =]5, \infty[\setminus 6 =]5, 6[\cup]6, \infty[\ \right]$

(o) V $f(x) = \ln\left(\dfrac{x^2-x-2}{x^2+x-2}\right)$

$\left[\frac{x^2-x-2}{x^2+x+2} > 0; D_f =]-\infty, -2[\cup]-1, 1[\cup]2, \infty[\ \right]$

9. Az alábbi $f : R \to R$ függvényeknek létezik inverze. Határozza meg az inverz függvény hozzárendelési utasítását!

   (a) B $f(x) = 4x - 7$ $\left[f^{-1}(x) = \frac{x+7}{4} = \frac{1}{4}x + \frac{7}{4}\right]$

   (b) B $f(x) = \dfrac{3x-4}{x+2}$ $\left[f^{-1}(x) = \frac{2x+4}{3-x}\right]$

   (c) B $f(x) = 6 - 5(x-1)^5$ $\left[f^{-1}(x) = \sqrt[5]{\frac{x-6}{-5}} + 1\right]$

   (d) B $f(x) = 3(4+6x)^7 - 2$ $\left[f^{-1}(x) = \frac{\sqrt[7]{\frac{x+2}{3}}-4}{6}\right]$

   (e) B $f(x) = 5\sqrt[3]{x-8} + 9$ $\left[f^{-1}(x) = \left(\frac{x-9}{5}\right)^3 + 8\right]$

   (f) B $f(x) = -2\sqrt[4]{5x+3} - 7$ $\left[f^{-1}(x) = \frac{\left(\frac{x+7}{-2}\right)^4-3}{5}\right]$

   (g) B $f(x) = 3e^{2x-7} + 5$ $\left[f^{-1}(x) = \frac{\ln\left(\frac{x-5}{3}\right)+7}{2} = \frac{1}{2}\ln\left(\frac{x-5}{3}\right) + \frac{7}{2}\right]$

   (h) B $f(x) = 5^{2-\frac{x}{3}} - 1$ $\left[f^{-1}(x) = -3(\log_5(x+1) - 2)\right]$

   (i) B $f(x) = 5\log_2(6-4x) + 2$ $\left[f^{-1}(x) = \frac{2^{\frac{x-2}{5}}-6}{-4} = -\frac{1}{4}\cdot 2^{\frac{x-2}{5}} + \frac{3}{2}\right]$

   (l) B $f(x) = 4\ln(3+2x) - 7$ $\left[f^{-1}(x) = \frac{e^{\frac{x+7}{4}}-3}{2} = \frac{1}{2}e^{\frac{x+7}{4}} - \frac{3}{2}\right]$

   (k) B,V $f(x) = \dfrac{\arcsin(9x-4)}{3}$ $\left[f^{-1}(x) = \frac{\sin(3x)+4}{9}\right]$

   (l) B,V $f(x) = \dfrac{2}{3}\mathrm{arctg}(2x) + 4$ $\left[f^{-1}(x) = \frac{\mathrm{tg}\left(\frac{3}{2}(x-4)\right)}{2}\right]$

   (m) B,V $f(x) = \cos(1-3x) + 2$
   [értelmezési tartomány leszűkítése: $\frac{1-\pi}{3} \le x \le \frac{1}{3}$, $f^{-1}(x) = \frac{\arccos(x-2)-1}{-3}$]

   (n) B,V $f(x) = 7 - 2\sin\left(\dfrac{x}{2}\right)$
   [értelmezési tartomány leszűkítése: $-\frac{5\pi}{2} \le x \le \frac{5\pi}{2}$, $f^{-1}(x) = 5\arcsin\left(\frac{7-x}{2}\right)$]

10. Határozza meg a következő $f : R \to R$ függvények legbővebb értelmezési tartományát és értékkészletét! Határozza meg az inverz függvényt és annak legbővebb értelmezési tartományát és értékkészletét !

   (a) V $f(x) = 4(3x+7)^5 + 6$
   [$D_f = R$; $R_f = R$; $f^{-1}(x) = \frac{\sqrt[5]{\frac{x-6}{4}}-7}{3} = \frac{1}{3}\sqrt[5]{\frac{x-6}{4}} - \frac{7}{3}$;
   $D_{f^{-1}} = R$; $R_{f^{-1}} = R$]

   (b) V $f(x) = 4\sqrt{6-2x} + 12$
   [$D_f =]-\infty; 3]$; $R_f = [12; \infty[$; $f^{-1}(x) = \frac{\left(\frac{x-12}{4}\right)^2-6}{-2} = -\frac{1}{2}\left(\frac{x-12}{4}\right)^2 + 3$;
   $D_{f^{-1}} = [12; \infty[$; $R_{f^{-1}} =]-\infty; 3]$]

(c) V $f(x) = 2e^{5x+3} - 4$
$[D_f = R; R_f =]-4;\infty[; f^{-1}(x) = \frac{\ln\left(\frac{x+4}{2}\right)-3}{5} = \frac{1}{5}\ln\left(\frac{x+4}{2}\right) - \frac{3}{5};$
$D_{f^{-1}} =]-4;\infty[; R_{f^{-1}} = R]$

(d) V $f(x) = 3 - 2^{3x-5}$
$[D_f = R; R_f =]-\infty;3[; f^{-1}(x) = \frac{\log_2\left(\frac{x-3}{-1}\right)+5}{3} = \frac{\log_2(3-x)+5}{3} = \frac{1}{3}\log_2(3-x) + \frac{5}{3};$
$D_{f^{-1}} =]-\infty;3[; R_{f^{-1}} = R]$

(e) V $f(x) = 4\ln(2x+5) - 8$
$[D_f =]-2,5;\infty[; R_f = R; f^{-1}(x) = \frac{e^{\frac{x+8}{4}}-5}{2} = \frac{1}{2}e^{\frac{1}{4}x+2} - \frac{5}{2};$
$D_{f^{-1}} = R; R_{f^{-1}} =]-2,5;\infty[\ ]$

(f) V $f(x) = 4\log_3(7x-14) + 11$
$[D_f =]2;\infty[; R_f = R; f^{-1}(x) = \frac{3^{\frac{x-11}{4}}+14}{7} = \frac{1}{7}\cdot 3^{\frac{x-11}{4}} + 2;$
$D_{f^{-1}} = R; R_{f^{-1}} =]2;\infty[\ ]$

(g) V $f(x) = \frac{3x-15}{x+4}$
$[f(x) = \frac{3x-15}{x+4} = 3 - \frac{27}{x+4}; D_f = R\setminus\{-4\}; R_f = R\setminus\{3\}; f^{-1}(x) = \frac{-15-4x}{x-3};$
$D_{f^{-1}} = R\setminus\{3\}; R_{f^{-1}} = R\setminus\{-4\}]$

(h) V $f(x) = 3 + \arccos\left(1 - \frac{3x}{4}\right)$
$[D_f = \left[0;\frac{8}{3}\right]; R_f = [3;3+\pi]; f^{-1}(x) = \frac{4-4\cos(x-3)}{3};$
$D_{f^{-1}} = [3;3+\pi]; R_{f^{-1}} = \left[0;\frac{8}{3}\right]]$

(i) V $f(x) = 5\arcsin\left(\frac{2+x}{4}\right) - 1$
$[D_f = [-6;2]; R_f = \left[-1-\frac{5\pi}{2}; -1+\frac{5\pi}{2}\right]; f^{-1}(x) = 4\sin\left(\frac{x+1}{5}\right) - 2;$
$D_{f^{-1}} = \left[-1-\frac{5\pi}{2}; -1+\frac{5\pi}{2}\right]; R_{f^{-1}} = [-6;2]]$

(j) V $f(x) = \frac{3\text{arctg}(2x+1)}{4}$
$[D_f = R; R_f = \left[-\frac{3\pi}{8}; \frac{3\pi}{8}\right]; f^{-1}(x) = \frac{\text{tg}\left(\frac{4}{3}\right)-1}{2}$
$D_{f^{-1}} = \left[-\frac{3\pi}{8}; \frac{3\pi}{8}\right]; R_{f^{-1}} = R]$

(k) V $f(x) = -6\sin\left(\frac{x}{5} + 1\right)$
$[D_f = R;$leszűkítése:$D_f = \left[-\frac{5\pi}{2} - 5; \frac{5\pi}{2} - 5\right]; R_f = [-6;6];$
$f^{-1}(x) = 5\arcsin\left(-\frac{x}{6}\right) - 5; D_{f^{-1}} = [-6;6]; R_{f^{-1}} = \left[-\frac{5\pi}{2} - 5; \frac{5\pi}{2} - 5\right]]$