SZÉCHENYI ISTVÁN EGYETEM

GÉPSZERKEZETTAN ÉS MECHANIKA TANSZÉK

9. MECHANIKA-STATIKA GYAKORLAT

Kidolgozta: Triesz Péter egy. ts.

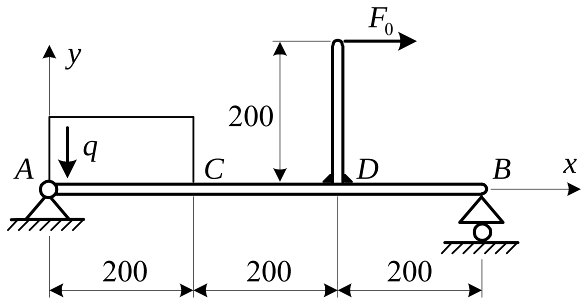
Rúdszerkezetek igénybevételei, igénybevételi ábrák

9.1. Példa

Adott:

$q = 20\,\mathrm{kN/m} \quad \rightarrow \quad F_q = 4\,\mathrm{kN}$,

$F_0 = 5\,\mathrm{kN}$.



Feladat:

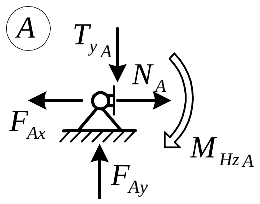
a. Határozza meg a támasztóerőket!
b. Számítsa ki az igénybevételeket az *A*, *B*, *C* és *D* keresztmetszetekben!

Megoldás:

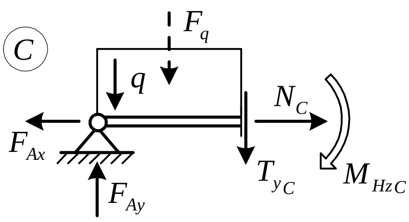
a.

$$F_x = 0 = F_{Ax} + F_0 \quad \Rightarrow \quad F_{Ax} = -F_0 = -5\,\mathrm{kN}\,(\leftarrow),$$
$$F_y = 0 = F_{Ay} - F_q + F_{By} \quad \Rightarrow \quad F_{Ay} = -1{,}6\dot{6}\,\mathrm{kN}\,(\uparrow),$$
$$M_a = 0 = -0{,}1F_q - 0{,}2F_0 + 0{,}6F_{By} \quad \Rightarrow \quad F_{By} = 2{,}3\dot{3}\,\mathrm{kNm}\,(\uparrow).$$

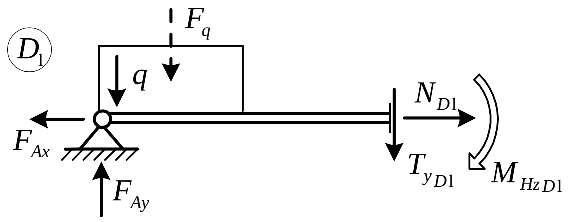
b. Az igénybevételek számításakor a rúd meghagyott részének egyensúlyát vizsgáljuk.



$$F_x = 0 = -F_{Ax} + N_A$$
$$\Rightarrow \quad N_A = F_{Ax} = 5\,\mathrm{kN} \rightarrow$$
$$F_y = 0 = F_{Ay} - T_A$$
$$\Rightarrow \quad T_A = F_{Ay} = 1{,}6\dot{6}\,\mathrm{kN} \downarrow$$
$$M_a = 0 = M_{HA}$$



$$F_x = 0 = -F_{Ax} + N_C$$
$$\Rightarrow \quad N_C = F_{Ax} = 5\,\mathrm{kN} \rightarrow$$
$$F_y = 0 = F_{Ay} - F_q - T_C$$
$$\Rightarrow \quad T_C = F_{Ay} - F_q = -2{,}3\dot{3}\,\mathrm{kN} \uparrow$$
$$M_c = 0 = -0{,}2 \cdot F_{Ay} + 0{,}1 \cdot F_q - M_{HC}$$
$$\Rightarrow \quad M_{HC} = 0{,}06\dot{6}\,\mathrm{kNm}$$



$F_x = 0 = -F_{Ax} + N_{D1}$

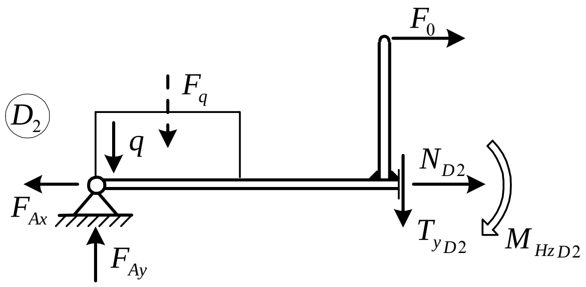
$\Rightarrow \quad N_{D1} = F_{Ax} = 5\,\text{kN} \rightarrow$

$F_y = 0 = F_{Ay} - F_q - T_{D1}$

$\Rightarrow \quad T_{D1} = F_{Ay} - F_q = -2{,}3\dot{3}\,\text{kN} \uparrow$

$M_{D1} = 0 = -0{,}4 \cdot F_{Ay} + 0{,}3 \cdot F_q - M_{HD1}$

$\Rightarrow \quad M_{HD1} = 0{,}53\dot{3}\,\text{kNm}$



$F_x = 0 = -F_{Ax} + F_0 + N_{D2}$

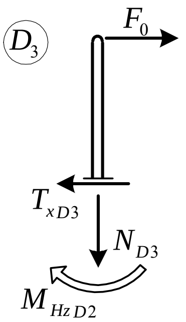
$\Rightarrow \quad N_{D2} = 0\,\text{kN}$

$F_y = 0 = F_{Ay} - F_q - T_{D2}$

$\Rightarrow \quad T_{D2} = -2{,}3\dot{3}\,\text{kN} \uparrow$

$M_{D1} = 0 = -0{,}4 \cdot F_{Ay} + 0{,}3 \cdot F_q - 0{,}2 \cdot F_0 - M_{HD2}$

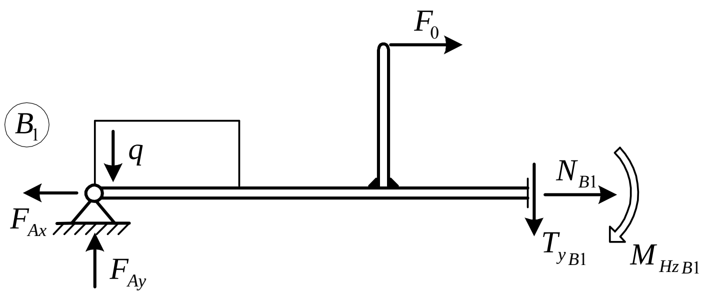
$\Rightarrow \quad M_{HD2} = -0{,}46\dot{6}\,\text{kNm}$



$N_{D3} = 0$

$T_{D3} = 5\,\text{kN} \leftarrow$

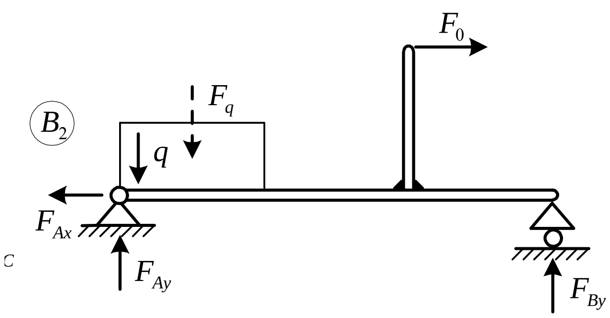
$M_{\text{HD3}} = \text{-}1\,\text{kNm}$



$N_{B1} = 0$

$T_{B1} = -2{,}3\dot{3}\,\text{kN} \uparrow$

$M_{\text{HB1}} = 0$

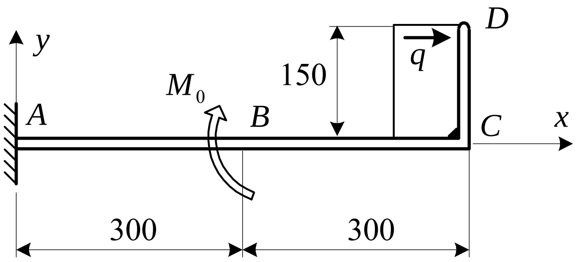


$N_{B2} = 0$

$T_{B2} = 0$

$M_{\text{HB2}} = 0$

9.2. Példa



Adott:

$q = 30\,\text{kN/m} \quad \to \quad F_q = 4{,}5\,\text{kN}$,

$M_0 = 10\,\text{kN}$.

Feladat:

a. Határozza meg a támasztóerőket!
b. Számítsa ki az igénybevételeket az A, B^-, B^+, C^- és C^+ keresztmetszetekben!

Megoldás:

a.

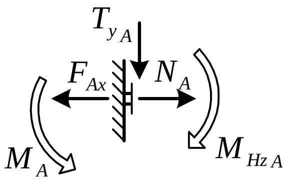
$$F_x = 0 = F_{Ax} + F_q \quad \Rightarrow \quad F_{Ax} = -F_q = -4{,}5\,\text{kN}\,(\leftarrow),$$

$$F_y = 0 = F_{Ay},$$

$$M_a = M_A - M_0 - 0{,}075 \cdot F_q \quad \Rightarrow \quad M_A = 10{,}3375\,\text{kNm}\,(\circlearrowright).$$

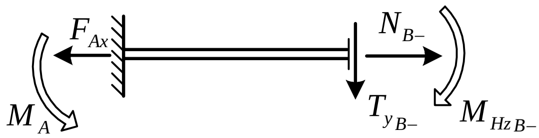
b. Az igénybevételeket ezúttal úgy határozzuk meg, hogy a rúd elhagyott részén ébredő erőrendszert redukáljuk a megmaradt rész végének súlypontjába.

A



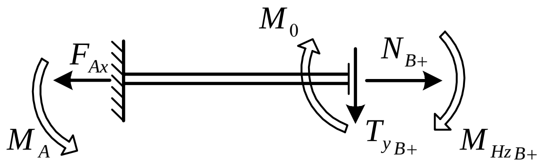
$$N_A = F_q = 4{,}5\,\text{kN} \rightarrow$$
$$T_A = 0$$
$$M_{HA} = -M_0 - 0{,}075 \cdot F_q = -10{,}3375\,\text{kNm}\circlearrowright$$

B-



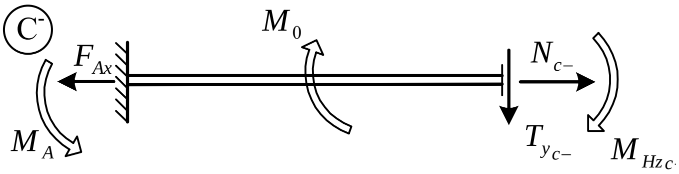
$$N_{B^-} = F_q = 4{,}5\,\text{kN} \rightarrow$$
$$T_{B^-} = 0$$
$$M_{HB^-} = -M_0 - 0{,}075 \cdot F_q = -10{,}3375\,\text{kNm}\circlearrowright$$

B+



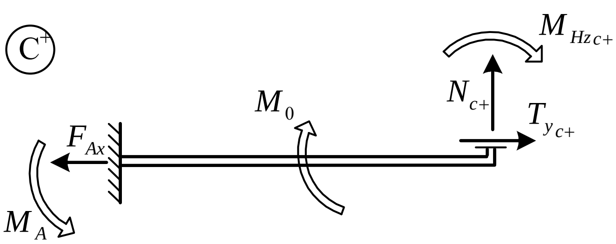
$$N_{B^+} = F_q = 4{,}5\,\text{kN} \rightarrow$$
$$T_{B^+} = 0$$
$$M_{HB^+} = -0{,}075 \cdot F_q = -0{,}3375\,\text{kNm}\circlearrowright$$

C-



$$N_{C^-} = F_q = 4{,}5\,\text{kN} \rightarrow$$
$$T_{C^-} = 0$$
$$M_{HC^-} = -0{,}075 \cdot F_q = -0{,}3375\,\text{kNm}\circlearrowright$$

C+



$$N_{C^+} = 0$$
$$0 = F_q - T_{C^+} \Rightarrow T_{C^+} = F_q = 4{,}5\,\text{kN} \leftarrow$$
$$M_{C^+} = 0 = -M_{HC^+} - 0{,}075 \cdot F_q$$
$$\Rightarrow \quad M_{HC^+} = -0{,}3375\,\text{kNm}\circlearrowright$$

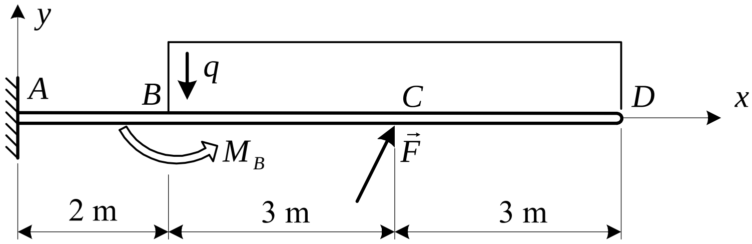
9.3. Példa

Adott:

$\vec{F}_0 = \left(5\vec{i} + 10\vec{j}\right)\text{kN}$,

$q = 2\ \text{kN/m} \quad \rightarrow \quad F_q = 12\ \text{kN}$,

$M_B = 8\ \text{kNm}$.



Feladat:

a. Határozza meg a támasztóerőket!
b. Rajzolja meg a tartó igénybevételi ábráit!

Megoldás:

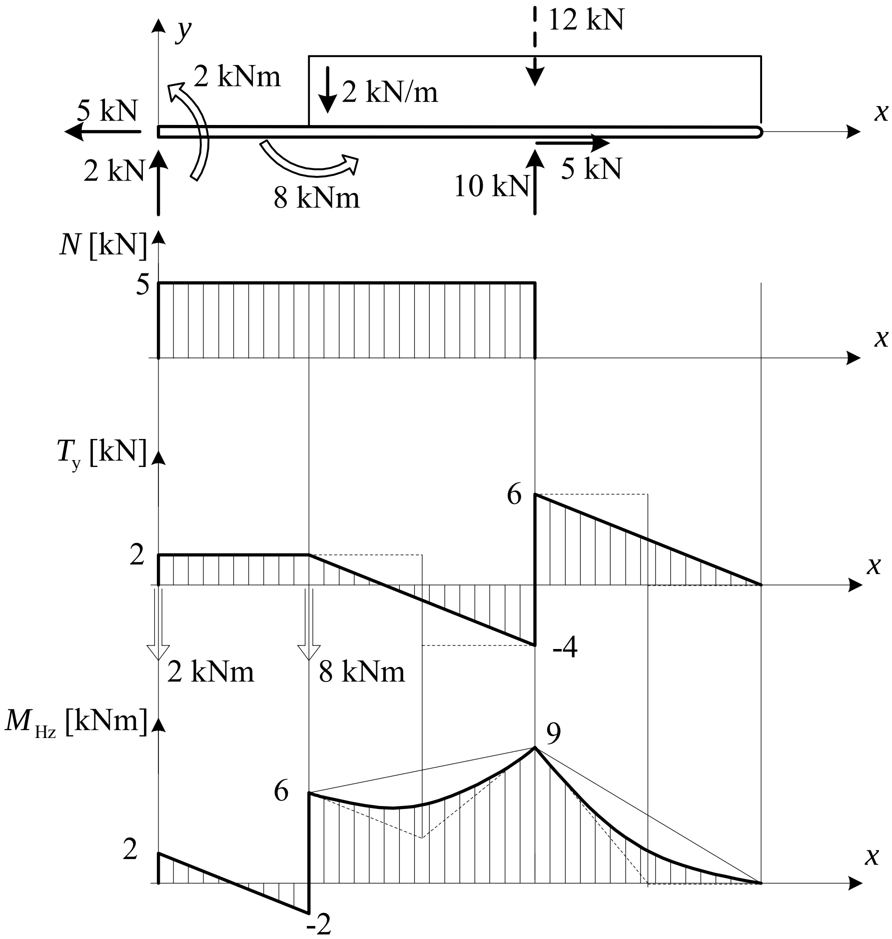
a.

$$F_x = 0 = F_{Ax} + F_{0x} \quad \Rightarrow \quad F_{Ax} = -5\ \text{kN}\ (\leftarrow)$$

$$F_y = 0 = F_{Ay} - F_q + F_{0y} \quad \Rightarrow \quad F_{Ay} = F_q - F_y = 12 - 10 = 2\ \text{kN}\ (\uparrow)$$

$$M_a = M_A + M_0 - 5 \cdot F_q + 5 \cdot F_y \quad \Rightarrow \quad M_A = 2\ \text{kNm}\ (\circlearrowright)$$

b.

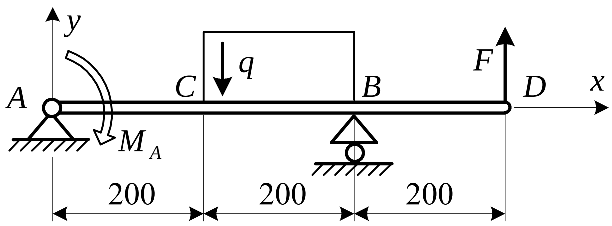


$M_{H\,\max} = 9\ \text{kNm}$.

9.4. Példa

Adott:

$q = 3\,\text{kN/m} \quad \rightarrow \quad F_q = 6\,\text{kN}$,
$F = 4\,\text{kN}$,
$M_A = 8\,\text{kNm}$.



Feladat:

a. Határozza meg a támasztóerőket!
b. Rajzolja meg a tartó igénybevételi ábráit!

Megoldás:

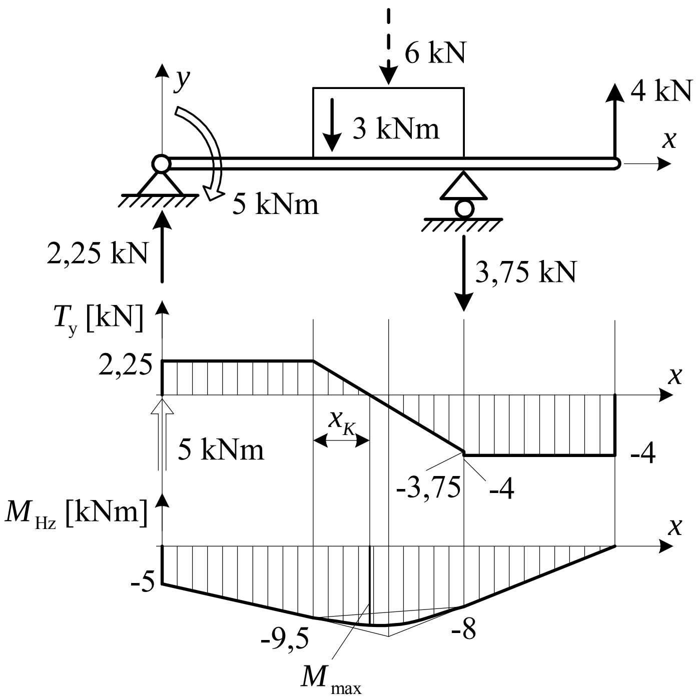
a.

$$F_x = 0 = F_{Ax}\,,$$

$$F_y = 0 = F_{Ay} - F_q + F_{By} + F_0 \Rightarrow F_{Ay} = \frac{9}{4}\,\text{kN}\,(\uparrow),$$

$$M_a = -M_0 - 3\cdot F_q + 4\cdot F_{\mathbf{B}y} + 6\cdot F_0 \Rightarrow F_{By} = \frac{M_0 + 3F_q - 6F_0}{4} = -\frac{1}{4}\,\text{kN}\,(\downarrow).$$

b.



A hajlító nyomaték szélsőérték helye meghatározható a nyíróerő ábrából, ($T_y = 0$):

$$0 = 2{,}25 - q\,x_K \quad \Rightarrow \quad x_K = \frac{2{,}25}{q} = 0{,}75\,\text{m}\,,$$

$$M_{\max} = -9{,}5 - \frac{q\,x_K^{\;2}}{2} = 10{,}34\,\text{kNm}\,.$$