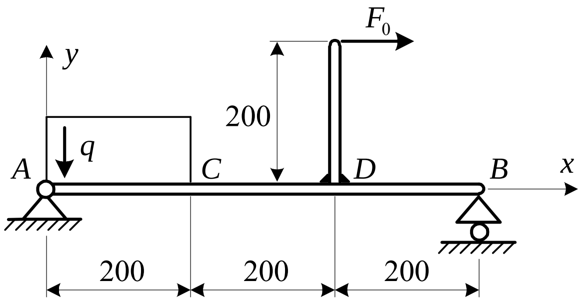
SZÉCHENYI ISTVÁN EGYETEM | **ALKALMAZOTT MECHANIKA TANSZÉK**

9. MECHANIKA-STATIKA GYAKORLAT
(kidolgozta: Triesz Péter, egy. ts.; Tarnai Gábor, mérnöktanár)

Rúdszerkezetek igénybevételei

9.1. Példa

Adott:



$q = 20\,\text{kN/m} \quad \rightarrow \quad F_q = 4\,\text{kN}$,
$F_0 = 5\,\text{kN}$.

Feladat:

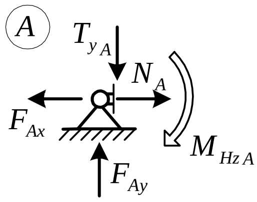
a. Határozza meg a támasztóerőket!
b. Számítsa ki az igénybevételeket az *A*, *B*, *C* és *D* keresztmetszetekben!

Megoldás:

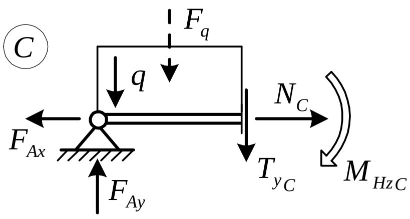
a.

$$F_x = 0 = F_{Ax} + F_0 \quad \Rightarrow \quad F_{Ax} = -F_0 = -5\,\text{kN}\,(\leftarrow),$$
$$F_y = 0 = F_{Ay} - F_q + F_{By} \quad \Rightarrow \quad F_{Ay} = 1{,}6\dot{6}\,\text{kN}\,(\uparrow),$$
$$M_a = 0 = -0{,}1F_q - 0{,}2F_0 + 0{,}6F_{By} \quad \Rightarrow \quad F_{By} = 2{,}3\dot{3}\,\text{kN}\,(\uparrow).$$

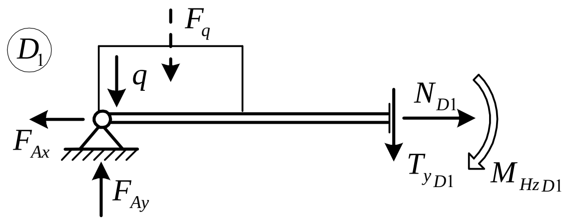
b. Az igénybevételek számításakor <u>a rúd meghagyott részének</u> egyensúlyát vizsgáljuk.



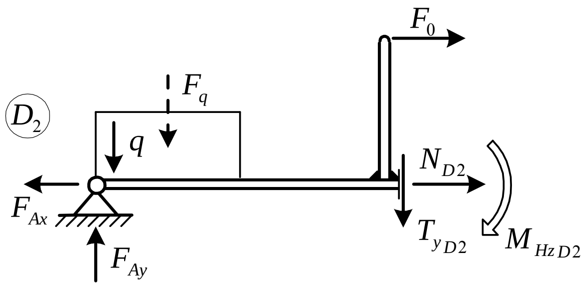
$$F_x = 0 = -F_{Ax} + N_A$$
$$\Rightarrow \quad N_A = F_{Ax} = 5\,\text{kN} \rightarrow$$
$$F_y = 0 = F_{Ay} - T_A$$
$$\Rightarrow \quad T_A = F_{Ay} = 1{,}6\dot{6}\,\text{kN} \downarrow$$
$$M_a = 0 = M_{HA}$$



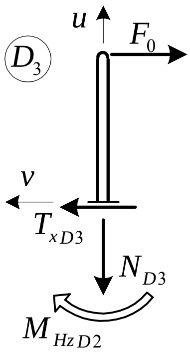
$$F_x = 0 = -F_{Ax} + N_C$$
$$\Rightarrow \quad N_C = F_{Ax} = 5\,\text{kN} \rightarrow$$
$$F_y = 0 = F_{Ay} - F_q - T_C$$
$$\Rightarrow \quad T_C = F_{Ay} - F_q = -2{,}3\dot{3}\,\text{kN} \uparrow$$
$$M_c = 0 = -0{,}2 \cdot F_{Ay} + 0{,}1 \cdot F_q - M_{HC}$$
$$\Rightarrow \quad M_{HC} = 0{,}06\dot{6}\,\text{kNm}$$



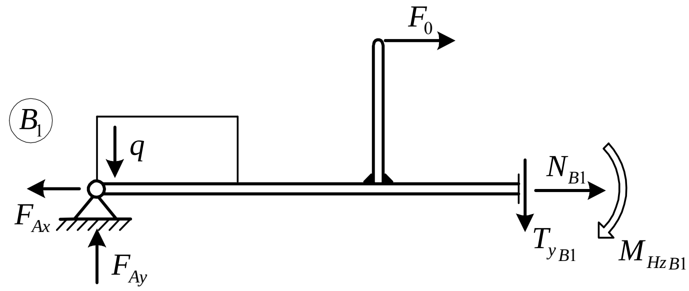
$$F_x = 0 = -F_{Ax} + N_{D1}$$
$$\Rightarrow \quad N_{D1} = F_{Ax} = 5\,\mathrm{kN} \rightarrow$$
$$F_y = 0 = F_{Ay} - F_q - T_{D1}$$
$$\Rightarrow \quad T_{D1} = F_{Ay} - F_q = -2{,}3\dot{3}\,\mathrm{kN} \uparrow$$
$$M_{D1} = 0 = -0{,}4 \cdot F_{Ay} + 0{,}3 \cdot F_q - M_{HD1}$$
$$\Rightarrow \quad M_{HD1} = 0{,}53\dot{3}\,\mathrm{kNm}$$



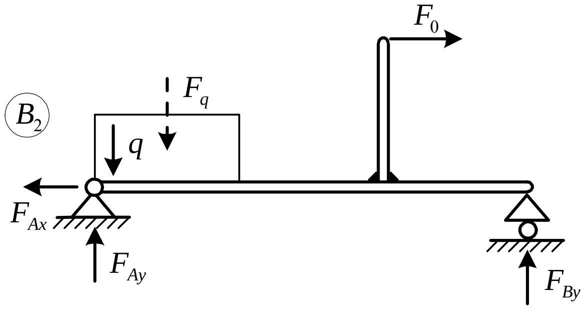
$$F_x = 0 = -F_{Ax} + F_0 + N_{D2}$$
$$\Rightarrow \quad N_{D2} = 0\,\mathrm{kN}$$
$$F_y = 0 = F_{Ay} - F_q - T_{D2}$$
$$\Rightarrow \quad T_{D2} = -2{,}3\dot{3}\,\mathrm{kN} \uparrow$$
$$M_{D2} = 0 = -0{,}4 \cdot F_{Ay} + 0{,}3 \cdot F_q - 0{,}2 \cdot F_0 - M_{HD2}$$
$$\Rightarrow \quad M_{HD2} = -0{,}46\dot{6}\,\mathrm{kNm}$$



$$F_u = 0 = -N_{D3} \quad \Rightarrow \quad N_{D3} = 0$$
$$F_v = 0 = T_{D3} - F_0 \quad \Rightarrow T_{D3} = 5\,\mathrm{kN} \leftarrow$$
$$M_{D3} = 0 = -M_{HzD3} - 0{,}2F_0$$
$$\Rightarrow M_{\mathrm{HzD3}} = -1\,\mathrm{kNm}$$



$$F_x = 0 = -F_{Ax} + F_0 + N_{B1}$$
$$\Rightarrow N_{B1} = 0$$
$$Fy = 0 = F_{Ay} - F_q - T_{yB1}$$
$$\Rightarrow T_{B1} = -2{,}3\dot{3}\,\mathrm{kN} \uparrow$$
$$M_{HzB1} = 0 = -0{,}6F_{Ay} + 0{,}5F_q -$$
$$-0{,}2F_0 - M_{HzB1}$$
$$\Rightarrow M_{HzB1} = 0$$



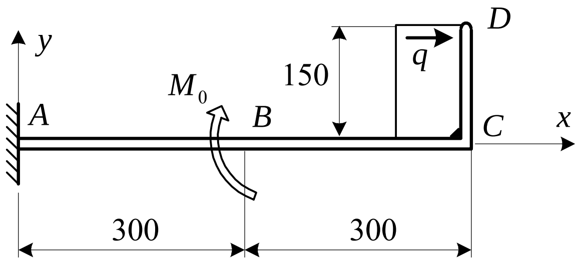
$$N_{B2} = 0$$
$$T_{B2} = 0$$
$$M_{\mathrm{HB2}} = 0$$

9.2. Példa

Adott:

$q = 30\,\text{kN/m} \quad \rightarrow \quad F_q = 4{,}5\,\text{kN}$,

$M_0 = 10\ \text{kNm}$.



Feladat:

a. Határozza meg a támasztóerőket!
b. Számítsa ki az igénybevételeket az A, B^-, B^+, C^- és C^+ keresztmetszetekben!

Megoldás:

a.

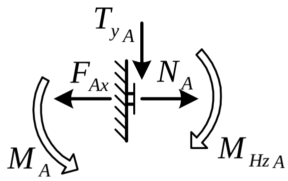
$$F_x = 0 = F_{Ax} + F_q \quad \Rightarrow \quad F_{Ax} = -F_q = -4{,}5\,\text{kN}\,(\leftarrow),$$

$$F_y = 0 = F_{Ay},$$

$$M_a = 0 = M_A - M_0 - 0{,}075 \cdot F_q \quad \Rightarrow \quad M_A = 10{,}3375\ \text{kNm}\,(\ \ )$$

b. Az igénybevételeket ezúttal úgy határozzuk meg, hogy a rúd elhagyott részén ébredő erőrendszert redukáljuk a megmaradt rész végének súlypontjába.

A

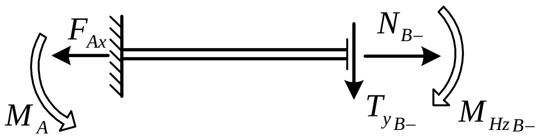


$$N_A = F_q = 4{,}5\,\text{kN} \rightarrow$$

$$T_A = 0$$

$$M_{HzA} = -M_0 - 0{,}075 \cdot F_q = -10{,}3375\ \text{kNm}$$

B^-

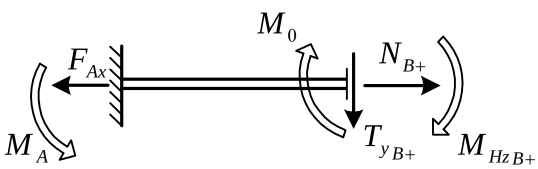


$$N_{B^-} = F_q = 4{,}5\,\text{kN} \rightarrow$$

$$T_{B^-} = 0$$

$$M_{HzB^-} = -M_0 - 0{,}075 \cdot F_q = -10{,}3375\ \text{kNm}$$

B^+

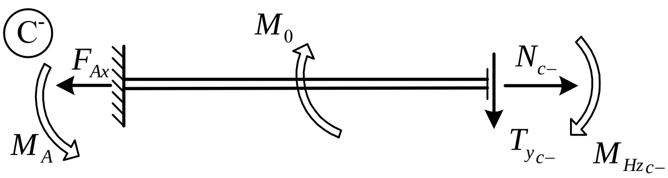


$$N_{B^+} = F_q = 4{,}5\,\text{kN} \rightarrow$$

$$T_{B^+} = 0$$

$$M_{HzB^+} = -0{,}075 \cdot F_q = -0{,}3375\ \text{kNm}$$

C^-

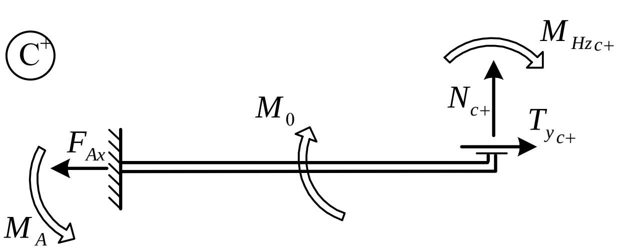


$$N_{C^-} = F_q = 4{,}5\,\text{kN} \rightarrow$$

$$T_{C^-} = 0$$

$$M_{HzC^-} = -0{,}075 \cdot F_q = -0{,}3375\ \text{kNm}$$

C^+



$$N_{C^+} = 0$$

$$T_{C^+} = F_q = 4{,}5\,\text{kN} \rightarrow$$

$$M_{HzC^+} = -0{,}075 \cdot F_q = -0{,}3375\ \text{kNm}$$

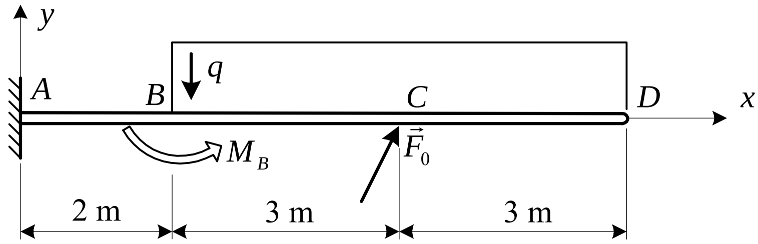
9.3. Példa

Adott:

$\vec{F}_0 = \left(5\vec{i} + 10\vec{j}\right)\text{kN}$,

$q = 2\,\text{kN/m} \quad \rightarrow \quad F_q = 12\,\text{kN}$,

$M_B = 8\,\text{kNm}$.



Feladat:

a. Határozza meg a támasztóerőket!
b. Számítsa ki az igénybevételeket az A, B^-, B^+, C^-, C^+ és D keresztmetszetekben!

Megoldás:

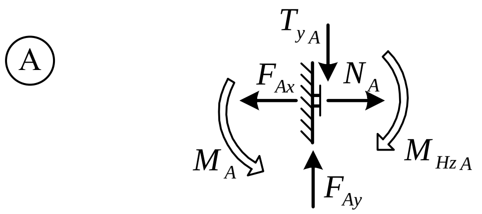
a.

$$F_x = 0 = F_{Ax} + F_{0x} \quad \Rightarrow \quad F_{Ax} = -5\,\text{kN}\,(\leftarrow)$$

$$F_y = 0 = F_{Ay} - F_q + F_{0y} \quad \Rightarrow \quad F_{Ay} = F_q - F_{0y} = 12 - 10 = 2\,\text{kN}\,(\uparrow)$$

$$M_a = 0 = M_A + M_0 - 5 \cdot F_q + 5 \cdot F_{0y} \quad \Rightarrow \quad M_A = 2\,\text{kNm}\,(\circlearrowright)$$

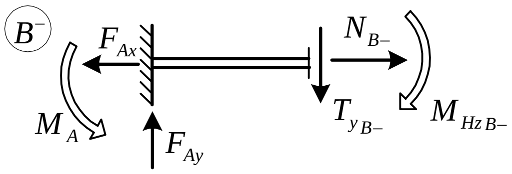
b. Az igénybevételeket határozzuk meg úgy, hogy a rúd elhagyott részén ébredő erőrendszert redukáljuk a megmaradt rész végének súlypontjába



$$N_A = F_{0x} = 5\,\text{kN} \rightarrow$$

$$T_{yA} = -F_q + F_{0y} = -2\,\text{kN} \downarrow$$

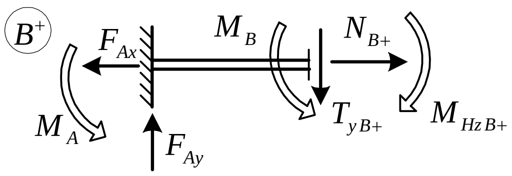
$$M_{HzA} = M_B - 5 \cdot F_q + 5 \cdot F_{0y} = -2\,\text{kNm}$$



$$N_{B^-} = F_{0x} = 5\,\text{kN} \rightarrow$$

$$T_{yB^-} = -F_q + F_{0y} = -2\,\text{kN} \downarrow$$

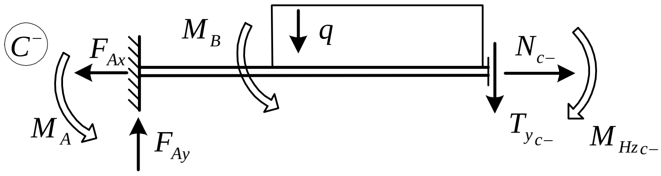
$$M_{HzB^-} = M_B - 3 \cdot F_q + 3 \cdot F_{0y} = 2\,\text{kNm}$$



$$N_{B^+} = F_{0x} = 5\,\text{kN} \rightarrow$$

$$T_{yB^+} = -F_q + F_{0y} = -2\,\text{kN} \downarrow$$

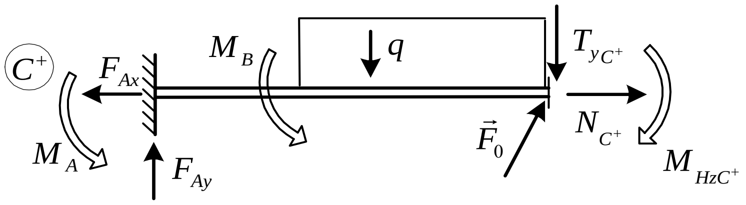
$$M_{HzB^+} = -3 \cdot F_q + 3 \cdot F_{0y} = -6\,\text{kNm}$$



$$N_{C^-} = F_{0x} = 5\,\text{kN} \rightarrow$$

$$T_{yC^-} = -\frac{F_q}{2} + F_{0y} = 4\,\text{kN} \uparrow$$

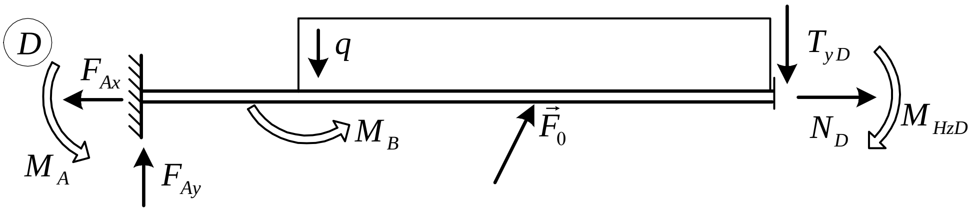
$$M_{HzC^-} = -1{,}5 \cdot \frac{F_q}{2} = -9\,\text{kNm}$$



$$N_{C^+} = 0 \text{ kN}$$

$$T_{yC^-} = -\frac{F_q}{2} = -6 \text{ kN} \downarrow$$

$$M_{HzC^-} = -1,5 \cdot \frac{F_q}{2} = -9 \text{ kNm}$$



$$N_D = 0 \text{ kN}$$

$$T_D = 0 \text{ kN}$$

$$M_{HzD} = 0 \text{ kNm}$$

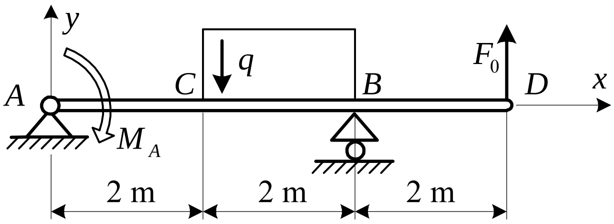
9.4. Példa

Adott:

$q = 3\,\text{kN/m} \quad \rightarrow \quad F_q = 6\,\text{kN}$,

$F_0 = 4\,\text{kN}$,

$M_A = 5\,\text{kNm}$.



Feladat:

a. Határozza meg a támasztóerőket!
b. feladat rész kiírása inkább: Számítsa ki az igénybevételeket az A, B^-, B^+, C^-, és D^-, D^+ keresztmetszetekben!

Megoldás:

a.

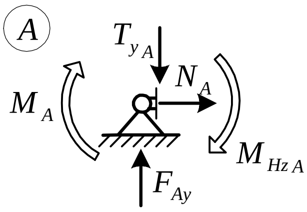
$$F_x = 0 = F_{Ax},$$

$$F_y = 0 = F_{Ay} - F_q + F_{By} + F_0 \Rightarrow F_{Ay} = \frac{9}{4}\,\text{kN}\,(\uparrow),$$

$$M_a = 0 = -M_A - 3\cdot F_q + 4\cdot F_{\mathbf{B}y} + 6\cdot F_0 \Rightarrow F_{By} = \frac{M_A + 3F_q - 6F_0}{4} = -\frac{1}{4}\,\text{kN}\,(\downarrow).$$

b. Most az igénybevételek számításakor <u>a rúd meghagyott részének</u> egyensúlyát vizsgáljuk.

A



$$F_x = 0 = N_A \quad \Rightarrow \quad N_A = 0$$

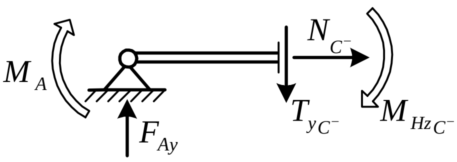
$$F_y = 0 = F_{Ay} - T_A$$

$$\Rightarrow \quad T_A = F_{Ay} = 2{,}25\,\text{kN}\downarrow$$

$$M_a = 0 = -M_A - M_{HzA}$$

$$\Rightarrow M_{HzA} = -M_A = -5\,\text{kN}$$

C^-



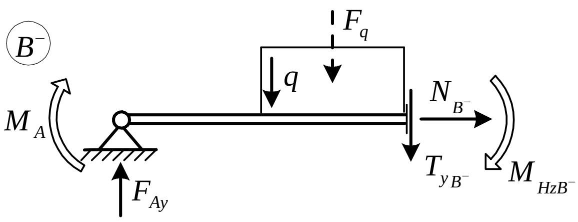
$$F_x = 0 = N_{C^-} \quad \Rightarrow \quad N_{C^-} = 0$$

$$F_y = 0 = F_{Ay} - T_{yC^-} \Rightarrow \quad T_{yC^-} = F_{Ayq} = 2{,}25\,\text{kN}\downarrow$$

$$M_{c^-} = 0 = -M_A - 2F_{Ay} - M_{HzC^-}$$

$$\Rightarrow \quad M_{HzC^-} = -M_A - 2F_{Ay} = -9{,}5\,\text{kN}$$

B^-

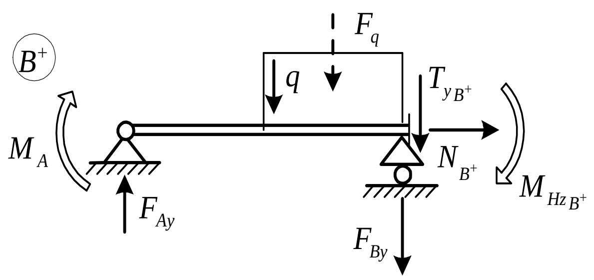


$$N_{B^-} = 0; \qquad F_y = 0 = F_{Ay} - F_q - T_{B^-}$$

$$\Rightarrow \quad T_{B^-} = F_{Ay} - F_q = -3{,}75\,\text{kN}\downarrow$$

$$M_{B^-} = 0 = -4\cdot F_{Ay} + 1\cdot F_q - M_{HzB^-}$$

$$\Rightarrow \quad M_{HzB^-} = -M_A - 4F_{Ay} + F_q = -8\,\text{kN}$$



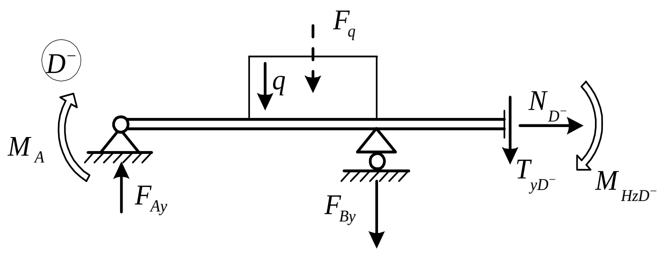
$N_{B^+} = 0;$

$F_y = 0 = F_{Ay} - F_q - F_{By} - T_{yB^+}$

$\Rightarrow \quad T_{yB^+} = -4 \text{ kN}$

$M_{B^+} = 0 = -M_A - 4 \cdot F_{Ay} + 1 \cdot F_q - M_{HzB^+}$

$\Rightarrow \quad M_{HzB^+} = -8 \text{ kNm}$



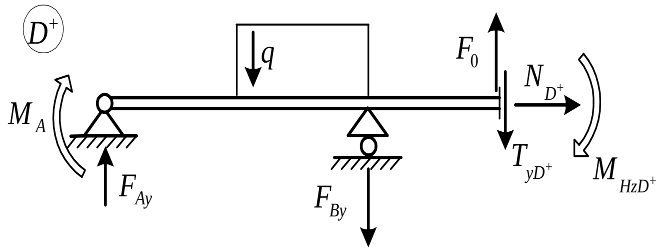
$N_{D^-} = 0$

$F_y = 0 = F_{Ay} - F_q - F_{By} - T_{yD^-}$

$\Rightarrow \quad T_{yD^-} = -4 \text{ kN}$

$M_{D^-} = 0 = -M_A - 6F_{Ay} + 3F_q + 2F_{By} - M_{HzD^-}$

$\Rightarrow \quad M_{HzD^-} = 0 \text{ kNm}$



$N_{D^-} = 0$

$F_y = 0 = F_{Ay} - F_q - F_{By} + F_0 - T_{yD^+}$

$\Rightarrow \quad T_{yD^+} = 0 \text{ kN}$

$M_{D^+} = 0 = -M_A - 6F_{Ay} + 3F_q + 2F_{By} - M_{HzD^+}$

$\Rightarrow \quad M_{HzD^+} = 0 \text{ kNm}$