# Uncertainty of indirect measurements
# Error propagation

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# Indirect measurements

- Let us suppose, the expected value and standard deviation of the random variable *X* were estimated based on measurements.
- What can we say about the parameters of a random variable *Y=f(X)*?

*Example:* We measure the diameter *d* of a bearing ball. Based on the diameter, the volume of the ball is:

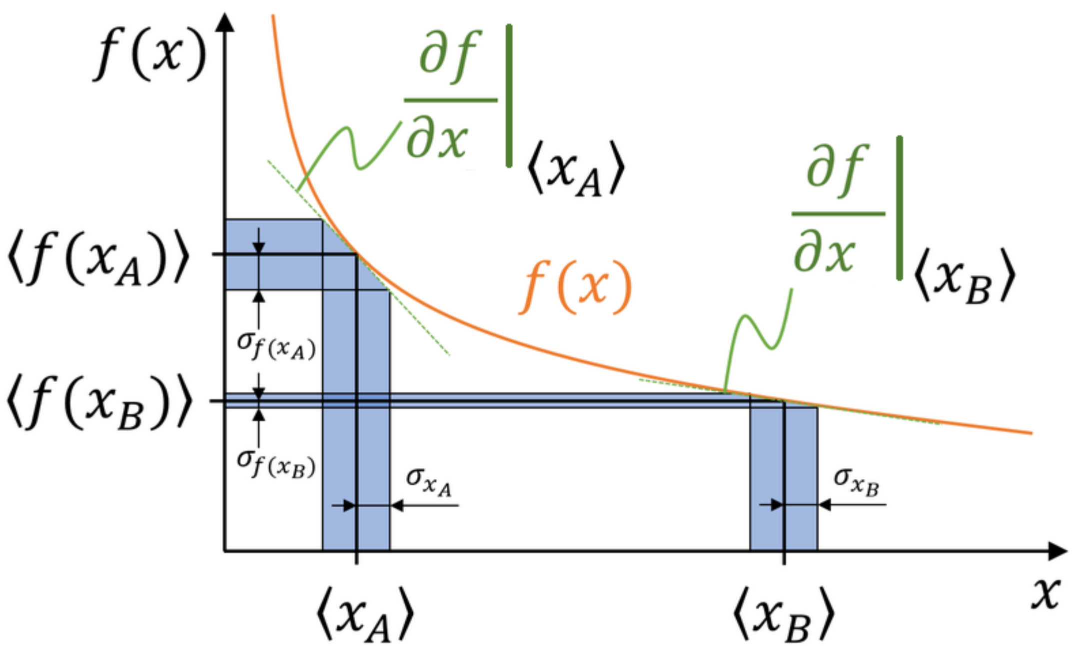
$$V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$$

If the expected value and standard deviation of the diameter are known, what can we say about the expected value and standard deviation of the volume?

If *f(x)* is a strictly monotonic function in the proximity of the directly measured value, and the standard deviation of the directly measured value is "small", then a "good" approximation of *f(x)* is its linearized Taylor series:

$$\mu(f(x)) = f(\mu(x))$$

$$\sigma(f(x)) = \left|\frac{\partial f}{\partial x}\right|_{\mu(x)} .\sigma(x)$$



# Indirect measurements

- Let's assume that the expected values and standard deviations of the independent random variables $X_1 ... X_N$ were estimated based on measurements.
- What can we say about the parameters of a random variable $Y=f(X_1 \ldots X_N)$?

*Example:* We measure the diameter *d* and mass *m* of a bearing ball. Based on the diameter and mass, the density of the ball is:

$$\rho = \frac{m}{V} = \frac{6m}{\pi d^3}$$

If the expected values and standard deviations of diameter and mass are known, what can we say about the expected value and standard deviation of density?

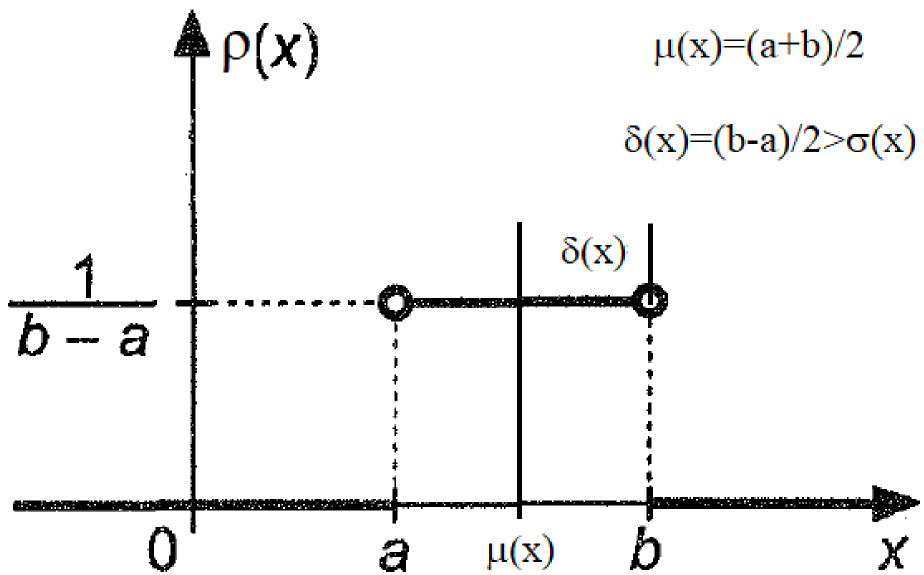
# Error propagation law by Gauss

$$\sigma(f(x_1.....x_N)) = \sqrt{\sum_{i=1}^{N} (\frac{\partial f}{\partial x_i})^2_{x_i=\mu(x_i)} . \sigma^2(x_i)}$$

*Note:* The condition for the applicability of the above error propagation law is, that the values of all partial derivatives involved in it are different from zero.

# A special case of the continuous uniform distribution - the concept of error limits

- In the case of measurements by the scale, the density function of the random error is:




δ(x) – absolute limit or absolute error

$$\delta(x) = \frac{b-a}{2} > \sigma(x) = \frac{b-a}{\sqrt{12}}$$

The absolute error is always an overestimate of the standard deviation!

# Error propagation law of measure-ments with absolute error

$$\delta(f(x_1 \ldots x_N)) = \sum_{i=1}^{N} \left| \frac{\partial f}{\partial x_i} \right|_{x_i=\mu(x_i)} .\delta(x_i)$$

*<u>Note:</u>* The condition for the applicability of the above error propagation law is, that the values of all partial derivatives included in it are different from zero.