Evaluation of time series in the frequency domain

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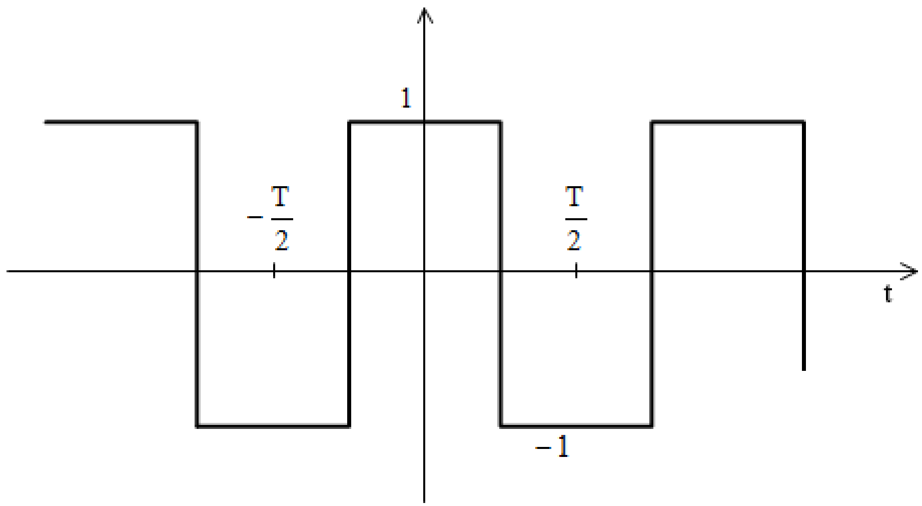
Fizika és Kémia Tanszék

Fourier - series

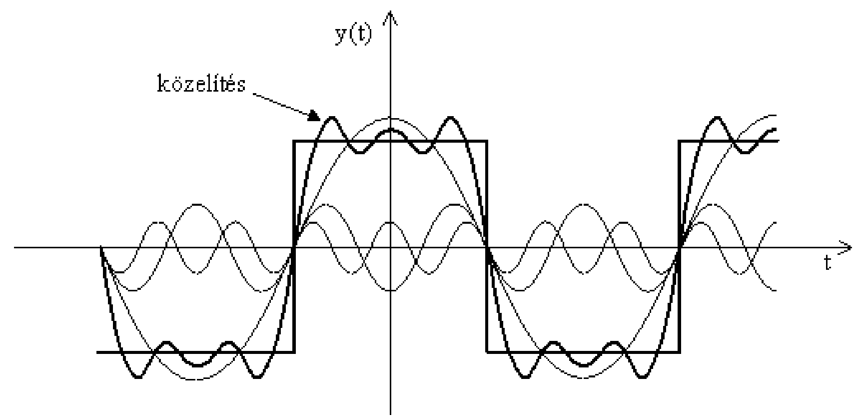
- Every T periodic, quadratically integrable time signal can be written as an infinite series of harmonic functions.

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{ik\omega_k t}$$

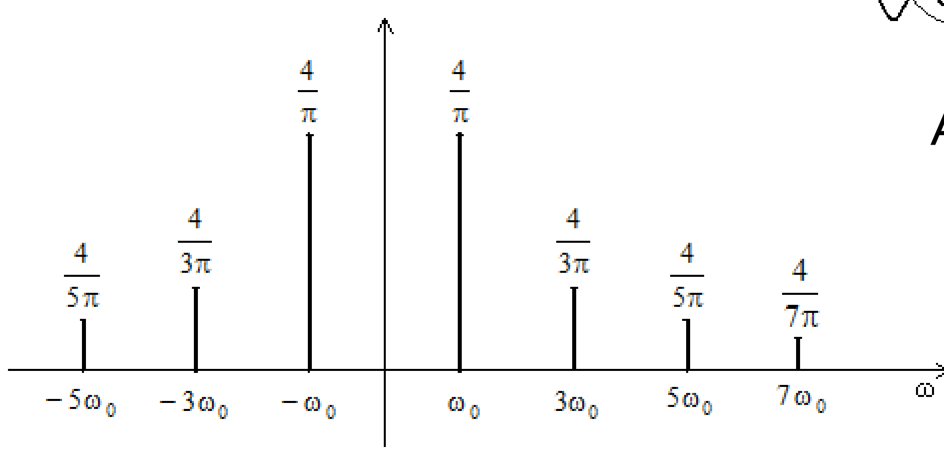
$$c_k(\omega_k) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-ik\omega_k t}$$



Boxcar signal



Approximation of boxcar signal with finite sum of harmonic functions



Spectra of boxcar signal

Fourier series of sampled signals (DFT)

- If our time signal is sampled with frequency F_s, i.e. at intervals $\Delta t = 1/F_s$, and contains N samples, then for the coefficients of the Fourier series holds:

$$c_k = \sum_{l=0}^{N-1} f(l.\Delta t) e^{-i\frac{2\pi kl}{N}}, k = -\frac{N}{2}, \ldots\ldots \frac{N}{2}.$$

Sampling theorem (Shannon)

- If a time signal is sampled with a frequency of F_s, i.e. at intervals of $\Delta t = 1/F_s$, then a component with a maximum frequency of $F_N = F_s/2$ (*Nyquist – frequency*) can be identified in the sampled time series!
- All components with frequency $f > F_N$ appear as *alias* below the Nyquist frequency!
- An *analog antialiasing filter* is required even before sampling! (In ADCs already included.)

Fast Fourier transform (FFT)

- In 1965, *Cooley and Tukey* found an algorithm for calculating the coefficients of the Fourier components that is *much faster than the definition. (FFT)*

Auto and cross spectra (APSD, CPSD)

- Let $F(\omega)$ is the Fourier transform of $f(t)$.
- The $APSD(\omega) = |F(\omega)|^2$ *frequency function is called* *autospectra* of time serie.
- *Cross – spectra* of two time series:

$$CPSD(\omega) = F_1^*(\omega)\, F_2(\omega).$$

- Dimension of spectra:

$$[\text{spectra}]=[\text{amplitude}]^2 / [\text{frequency}]$$