Filters

Dr. Berta Miklós
Department of Physics and Chemistry
Széchenyi István University

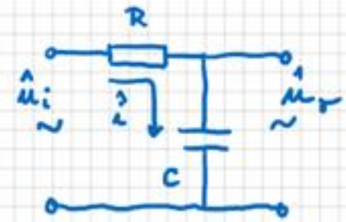
SZÉCHENYI ISTVÁN EGYETEM
SZE - GYŐR

Fizika és Kémia Tanszék

Basic idea of filtering

Filters

RC - filter



R

$X_c = \frac{1}{\omega C}$

$Z = R - jX_c$

$$\hat{H}(\omega) = \frac{\hat{u}_r(\omega)}{\hat{u}_i(\omega)}$$

$$\hat{i} = \frac{\hat{u}_i}{Z} = \frac{\hat{u}_i}{R - jX_c} \Rightarrow \hat{u}_r = \hat{i} \cdot (-jX_c) = -jX_c \cdot \frac{\hat{u}_i}{Z}$$

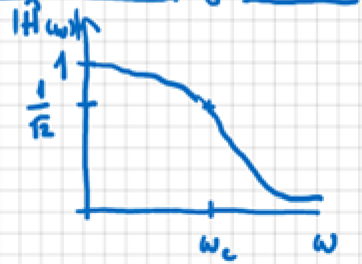
$$\hat{H}(\omega) = \frac{\hat{u}_r}{\hat{u}_i} = -j\frac{X_c}{Z} = -j\frac{X_c}{R - jX_c} \cdot \frac{R + jX_c}{R + jX_c} = \frac{-jX_cR + X_c^2}{R^2 + X_c^2} = \frac{X_c^2}{R^2 + X_c^2} - j\frac{X_cR}{R^2 + X_c^2}$$

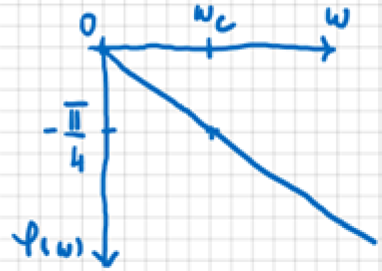
1. $$|\hat{H}(\omega)| = \sqrt{\frac{X_c^4}{(R^2 + X_c^2)^2} + \frac{X_c^2R^2}{(R^2 + X_c^2)^2}} = \sqrt{\frac{X_c^2\cancel{(R^2 + X_c^2)}}{(R^2 + X_c^2)^{\cancel{2}}}} = \sqrt{\frac{X_c^2}{R^2 + X_c^2}} = \sqrt{\frac{1}{1 + \frac{R^2}{X_c^2}}} =$$

$$= \sqrt{\frac{1}{1 + \omega^2R^2C^2}} \overset{\frac{1}{RC} = \omega_c}{=} \sqrt{\frac{1}{1 + (\frac{\omega}{\omega_c})^2}}$$

2.) $$tg\,\varphi = -\frac{\cancel{X_c}R}{X_c^{\cancel{2}}} = -\frac{R}{X_c} = -\omega RC = -\frac{\omega}{\omega_c}$$

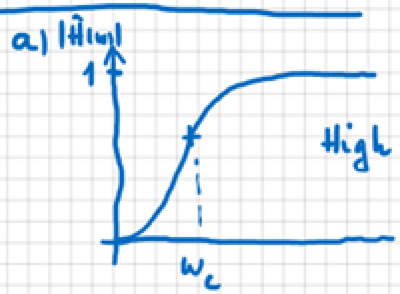
Bode - diagram



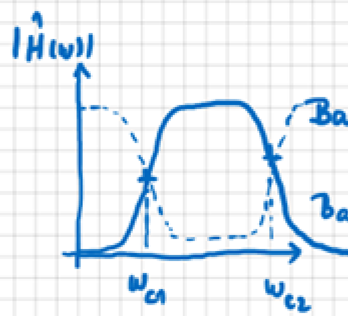


Low - pass filter

Generalization



High - pass



Band - pass —

Band - filter - - -

Every filter's transfer function can be written in the next form:

$$\hat{H}(\omega) = \frac{\hat{b}(\omega)}{\hat{a}(\omega)}$$

complex polynom

complex polynom

If	$\hat{a}(\omega) = 1$	-	then	FIR	filter	-	causal
If	$\hat{a}(\omega) \neq 1$	-	then	IIR	filter	-	non - causal

Filter’s design in MATLAB

> fdatool

Filtering and spectral analysis in MATLAB

> sptool