Movement of electrons in a crystal lattice
Drude - model of electron gas

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SZÉCHENYI ISTVÁN EGYETEM
SZE - GYŐR

Fizika és Kémia Tanszék

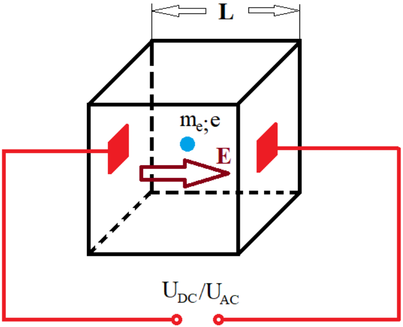
Drude's electron gas

Based on experience, metals have free charge carriers. We also know from experience that these are electrons.

$$m_e = 9 \cdot 10^{-31}\,\text{kg}, \;\; q_e = -e = -1,6 \cdot 10^{-19}\,\text{C}$$

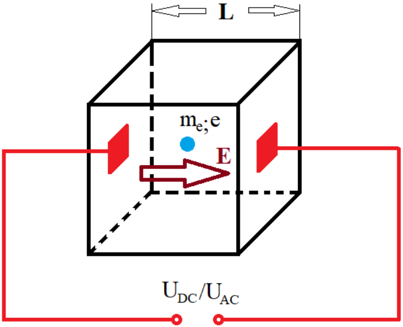
The Drude's electron gas in a crystal lattice is characterized by the following two parameters:

- n_e – electron concentration - specifies how many electrons are present in a unit volume
- τ – average collision time - average time between two consecutive collisions of electrons with the centers of the crystal lattice.



Let's examine the direct current case first! A constant accelerating force acts on the electron between two collisions, the magnitude of which is:

$$F = m_e a = eE = e\frac{U}{L} \rightarrow a = \frac{eU}{m_e L}$$



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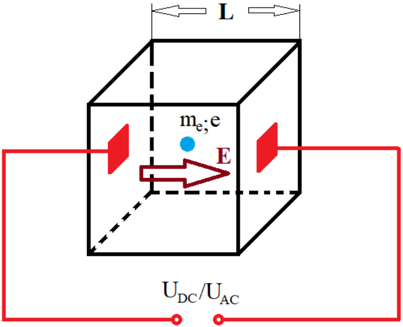

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So the resistance is:

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In the system, a constant accelerating force eE acts on the electron, while in average every τ time, the electron loses in a collision its momentum acquired during the acceleration. So the collision can be understood as an average braking force equal to $\frac{p}{\tau}$. Let's write the dynamic equation of this process:

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From here:

$$\boldsymbol{\hat{v}(\omega) = \frac{e\tau}{m_e(1 + i\omega\tau)}\hat{E}(\omega)}.$$

Let us consider, that:

$$\boldsymbol{J = \frac{en_e v}{2}!}$$

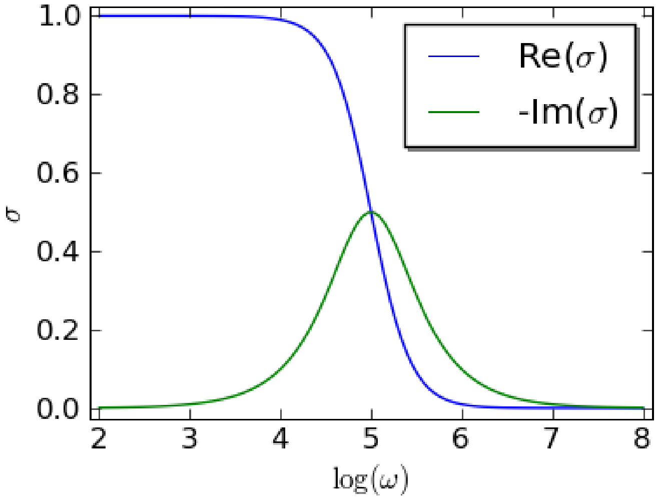
So:

$$\hat{J}(\omega) = \frac{\sigma_0}{1 + i\omega\tau}\hat{E}(\omega) = \hat{\sigma}(\omega)\hat{E}(\omega)$$

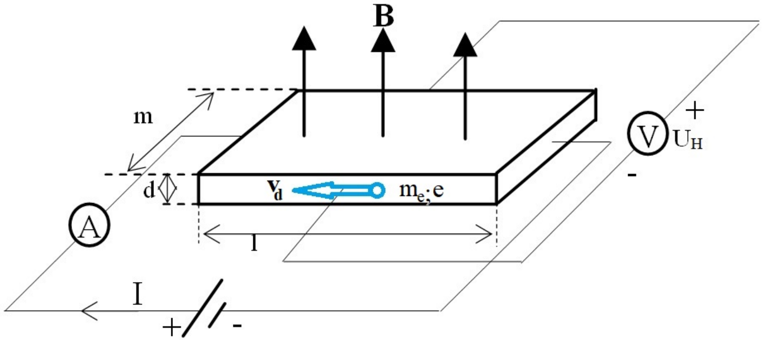
It can be seen that the differential Ohm's law holds also in this case with the complex specific conductivity:

$$\hat{\sigma} = \frac{\sigma_0}{1 + i\omega\tau} = \frac{\sigma_0}{1 + i\frac{\omega}{\omega_{co}}}$$

where $\omega_{co} = \frac{1}{\tau}$ is a cut–off frequency.


1.0
0.8
0.6
0.4
0.2
0.0
σ
2
3
4
5
6
7
8
$\log(\omega)$
Re(σ)
-Im(σ)

The Hall - effect



As a result of the applied electric field, the electrons in the wafer drift at a speed of $\boldsymbol{v_d}$. Their speed is perpendicular to the homogeneous magnetic field $\boldsymbol{B}$, so they are subject to a Lorentz force perpendicular to both the electric field and the magnetic field. The Lorentz force pushes the charges in the direction of the sides perpendicular to the direction of the current, so that these sides are charged to $\boldsymbol{U_H}$ Hall voltage. The process continues until the effect of the Hall voltage compensates the effect of the Lorentz force:

$$\boldsymbol{F_{Lorentz} = F_{Hall}}.$$

$$\boldsymbol{ev_d B = e\frac{U_H}{m}}.$$

Let us consider, that:

$$\boldsymbol{v_d = \frac{I}{en_e md}},$$

thus

$$\boldsymbol{U_H = \frac{1}{en_e}\frac{IB}{d} = R_H\frac{IB}{d}}.$$

The concentration of charge carriers in the wafer can be determined from the Hall constant $\boldsymbol{R_H}$, while the sign of the charge can be determined from the polarity of $\boldsymbol{U_H}$!

Thank You!