

FÜGGVÉNYEK

1. (a) Legyen $f(x) = 2x - 3$. Mivel egyenlő $f(4x)$? $[f(4x) = 8x - 3]$
 (b) Legyen $f(x) = 2x - 5$. Mivel egyenlő $f\left(\frac{x}{3}\right)$? $[f\left(\frac{x}{3}\right) = \frac{2}{3}x - 5]$
 (c) Legyen $f(x) = 2x + 7$. Mivel egyenlő $f\left(\frac{2}{x}\right)$? $[f\left(\frac{2}{x}\right) = \frac{4}{x} + 7]$
 (d) Legyen $f(x) = x^2 + 3x - 2$. Mivel egyenlő $f(5x)$? $[f(5x) = 25x^2 + 15x - 2]$
 (e) **B** Legyen $f(x) = x^2 + 3x - 2$. Mivel egyenlő $f\left(\frac{x}{2}\right)$? $[f\left(\frac{x}{2}\right) = \frac{1}{4}x^2 + \frac{3}{2}x - 2]$
 (f) **B** Legyen $f(x) = x^2 + 3x - 2$. Mivel egyenlő $f\left(\frac{1}{x}\right)$? $[f\left(\frac{1}{x}\right) = \frac{1}{x^2} + \frac{3}{x} - 2]$

2. Határozza meg a következő összetett függvényeket!

$$[g \circ f = g(f(x)); f \circ g = f(g(x)); f \circ f = f(f(x))]$$

- (a) **B** $f(x) = \cos x + x^2; g(x) = \sqrt{x}; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \cos(\sqrt{x}) + (\sqrt{x})^2 = \cos(\sqrt{x}) + x; g(f(x)) = \sqrt{\cos x + x^2}]$
- (b) **B** $f(x) = \sin x; g(x) = x^2; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \sin(x^2) = \sin x^2; g(f(x)) = (\sin x)^2 = \sin^2 x]$
- (c) **B** $f(x) = \sqrt{x+3}; g(x) = \sqrt{x} + 3; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \sqrt{(\sqrt{x} + 3) + 3} = \sqrt{\sqrt{x} + 6}; g(f(x)) = \sqrt{\sqrt{x+3} + 3} = \sqrt[4]{x+3} + 3]$
- (d) **B** $f(x) = \ln x + 4x^5; g(x) = e^x; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \ln(e^x) + 4(e^x)^5 = x + 4e^{5x}; g(f(x)) = e^{\ln x + 4x^5}]$
- (e) **B** $f(x) = x^2 - 3x; g(x) = \sqrt{5 - 2x}; f(g(x)) = ?; g(f(x)) = ?; f(f(x)) = ?$
 $[f(g(x)) = (\sqrt{5 - 2x})^2 - 3\sqrt{5 - 2x} = 5 - 2x - 3\sqrt{5 - 2x};$
 $g(f(x)) = \sqrt{5 - 2(x^2 - 3x)} = \sqrt{5 - 2x^2 + 6x};$
 $f(f(x)) = (x^2 - 3x)^2 - 3(x^2 - 3x) = x^4 - 6x^3 + 9x^2 - 3x^2 + 9x = x^4 - 6x^3 + 6x^2 + 9x]$
- (f) **B** $f(x) = 1 - x + x^2; g(x) = e^x; f(g(x)) = ?; g(f(x)) = ?; f(f(x)) = ?$
 $[f(g(x)) = 1 - e^x + (e^x)^2 = 1 - e^x + e^{2x}; g(f(x)) = e^{1 - x + x^2};$
 $f(f(x)) = 1 - (1 - x + x^2) + (1 - x + x^2)^2 = x^4 - 2x^3 + 2x^2 - x + 1]$
- (g) **B** $f(x) = \cos(7 - x); g(x) = x^4 - 3x + 2; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \cos(7 - (x^4 - 3x + 2)) = \cos(-x^4 + 3x + 5);$
 $g(f(x)) = (\cos(7 - x))^4 - 3\cos(7 - x) + 2 = \cos^4(7 - x) - 3\cos(7 - x) + 2]$
- (h) **B** $f(x) = \sqrt[3]{2 - 3x}; g(x) = 4x - x^3; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \sqrt[3]{2 - 3(4x - x^3)} = \sqrt[3]{2 - 12x + 3x^3};$
 $g(f(x)) = 4\sqrt[3]{2 - 3x} - (\sqrt[3]{2 - 3x})^3 = 4\sqrt[3]{2 - 3x} - 2 + 3x]$
- (i) **B** $f(x) = \sqrt[5]{4 - x}; g(x) = x^5 - 3^x; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \sqrt[5]{4 - (x^5 - 3^x)} = \sqrt[5]{4 - x^5 + 3^x};$
 $g(f(x)) = (\sqrt[5]{4 - x})^5 - 3^{\sqrt[5]{4 - x}} = 4 - x - 3^{\sqrt[5]{4 - x}}]$

(j) **B** $f(x) = (x - 2)^2; g(x) = 2 - x^2; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = [(2 - x^2) - 2]^2 = x^4;$
 $g(f(x)) = 2 - [(x - 2)^2]^2 = -x^4 + 8x^3 - 24x^2 + 32x - 14]$

3. Határozza meg a hiányzó függvényeket!

(a) $f(g(x)) = \sin(x + 4); f(x) = \sin x; g(x) = ?$ $[g(x) = x + 4]$

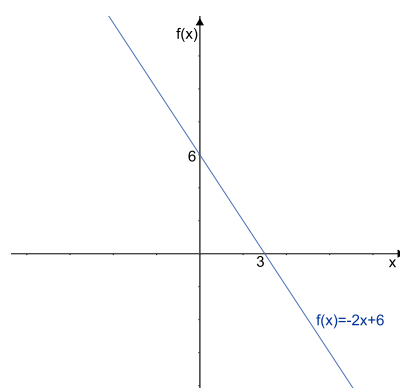
(b) **B** $f(g(x)) = \cos^4 x + 3 \cos x; g(x) = \cos x; f(x) = ?$ $[f(x) = x^4 + 3x]$

(c) **B** $g(f(x)) = x - e^{\sqrt{x}}; g(x) = x^2 - e^x; f(x) = ?$ $[f(x) = \sqrt{x}]$

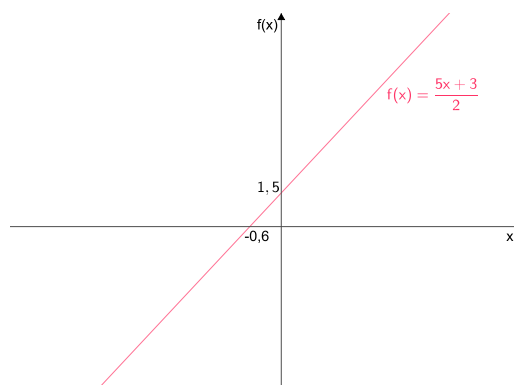
(d) **B** $g(f(x)) = \frac{x}{1 + x^4}; f(x) = x^2; g(x) = ?$ $[g(x) = \frac{\sqrt{x}}{1+x^2}]$

4. Ábrázolja az alábbi $f : \mathbb{R} \rightarrow \mathbb{R}$ függvényeket, majd az ábra alapján határozza meg a függvények értelmezési tartományát és értékkészletét!

(a) $f(x) = -2x + 6$ $[D_f = \mathbb{R}; R_f = \mathbb{R}]$

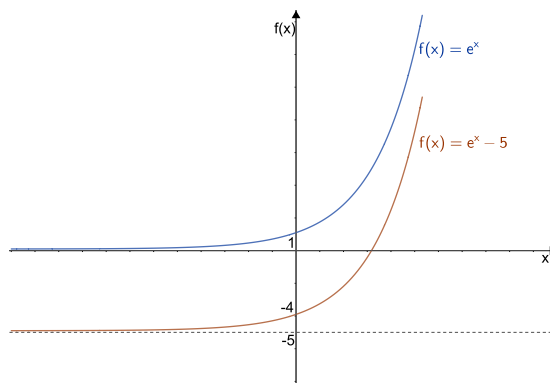


(b) $f(x) = \frac{5x + 3}{2}$ $[f(x) = \frac{5x+3}{2} = \frac{5}{2}x + \frac{3}{2}, D_f = \mathbb{R}; R_f = \mathbb{R}]$



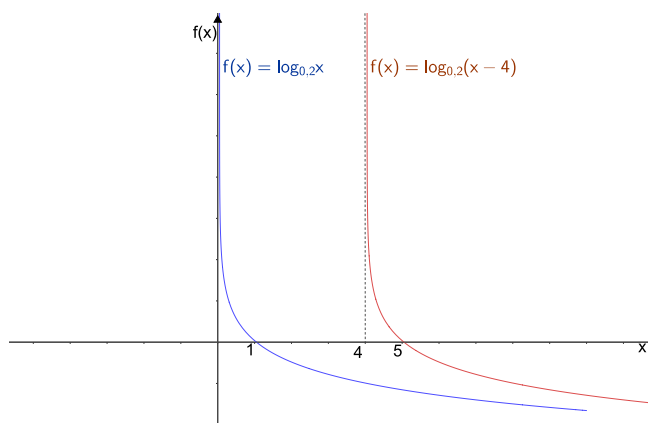
(c) $f(x) = e^x - 5$

$[D_f = \mathbb{R}; R_f =] - 5; \infty[]$



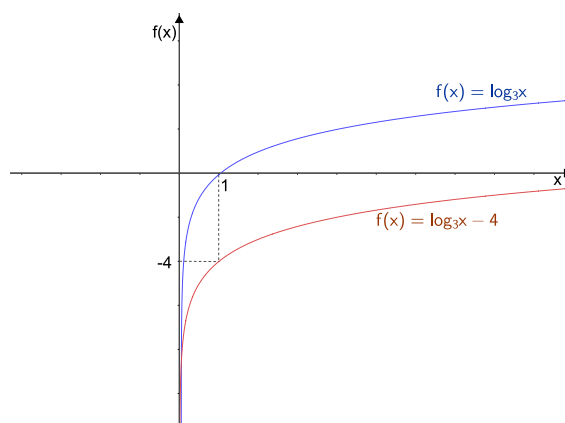
(d) $f(x) = \log_{0,2}(x - 4)$

$[D_f =]4; \infty[; R_f = \mathbb{R}]$



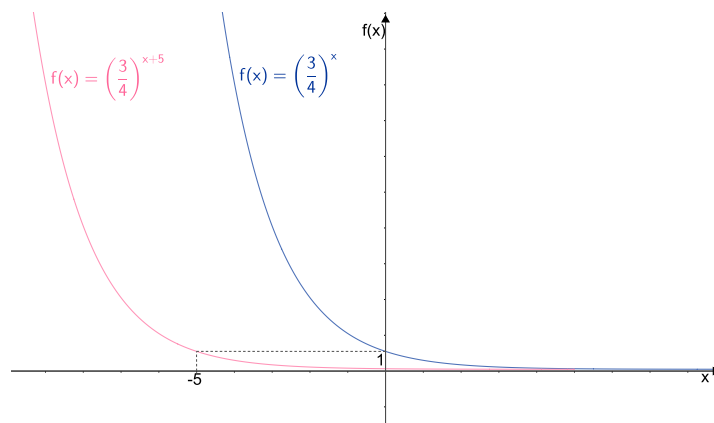
(e) $f(x) = \log_3 x - 4$

$[D_f =]0; \infty[; R_f = \mathbb{R}]$



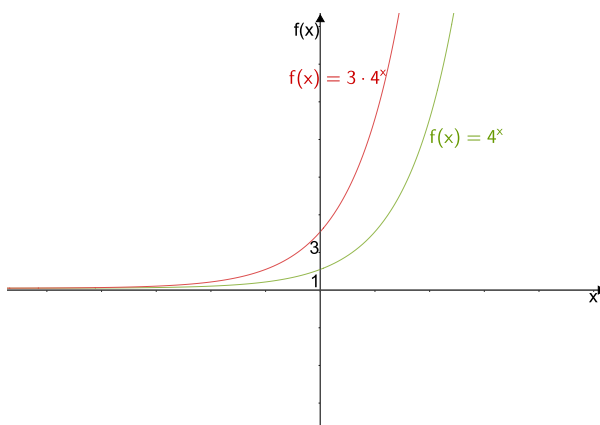
(f) $f(x) = \left(\frac{3}{4}\right)^{x+5}$

$[D_f = \mathbb{R}; R_f =]0; \infty[]$



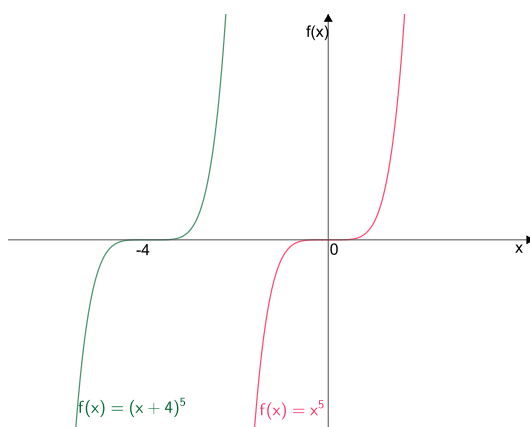
(g) $f(x) = 3 \cdot 4^x$

$[D_f = \mathbb{R}; R_f =]0; \infty[]$



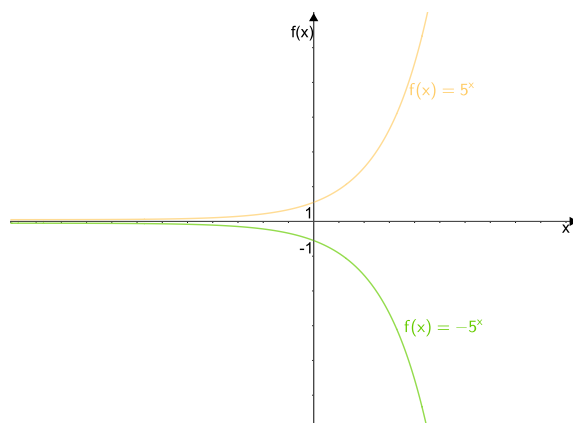
(h) $f(x) = (x+4)^5$

$[D_f = \mathbb{R}; R_f = \mathbb{R}]$



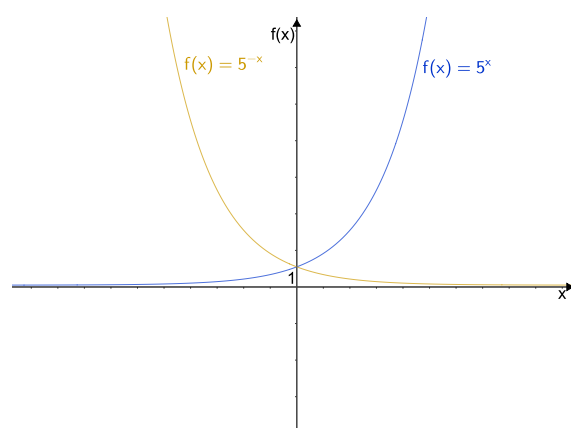
(i) $f(x) = -5^x$

$[D_f = \mathbb{R}; R_f =] - \infty; 0[]$



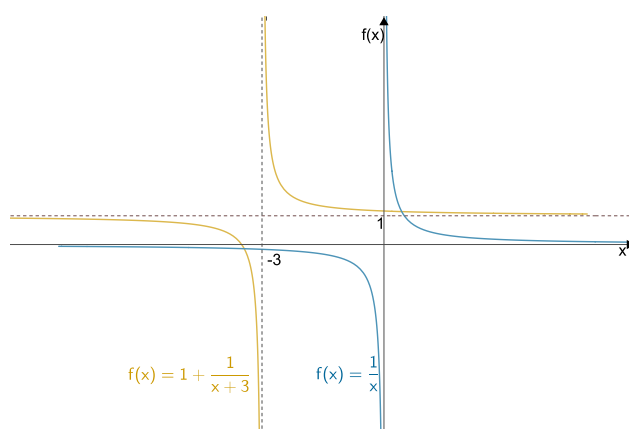
(j) $f(x) = 5^{-x}$

$[D_f = \mathbb{R}; R_f =]0; \infty[]$



(k) $f(x) = 1 + \frac{1}{x+3}$

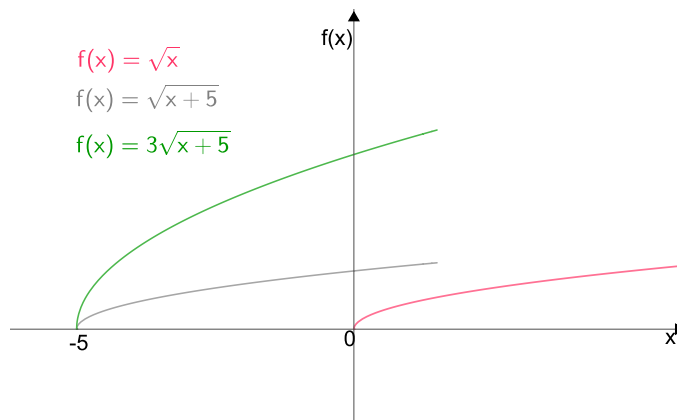
$[D_f = \mathbb{R} \setminus \{-3\}; R_f = \mathbb{R} \setminus \{1\}]$



5. Ábrázolja az alábbi $f : \mathbb{R} \rightarrow \mathbb{R}$ függvényeket, majd az ábra alapján határozza meg a függvények értelmezési tartományát és értékkészletét!

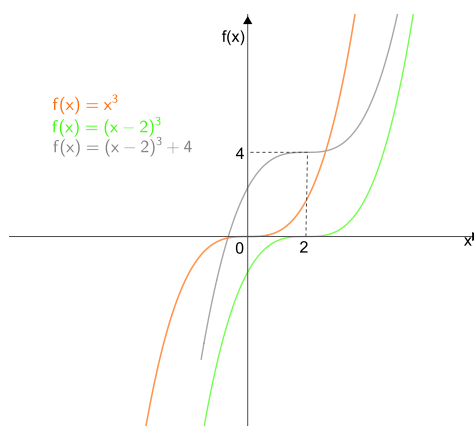
(a) **B** $f(x) = 3\sqrt{x+5}$

$[D_f = [-5, \infty[; R_f = [0; \infty[]$



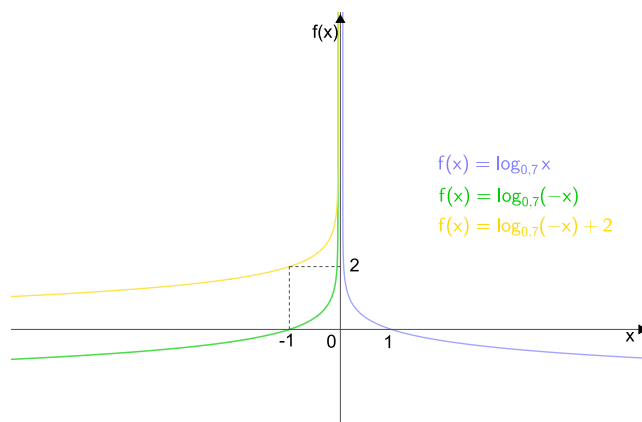
(b) **B** $f(x) = (x-2)^3 + 4$

$[D_f = \mathbb{R}; R_f = \mathbb{R}]$



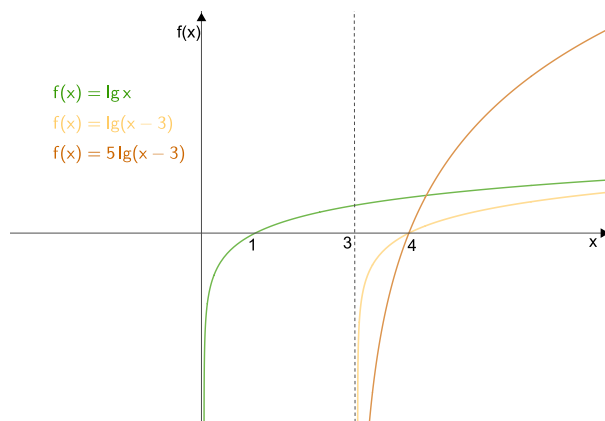
(c) **B** $f(x) = \log_{0,7}(-x) + 2$

$[D_f =]-\infty, 0[; R_f = \mathbb{R}]$



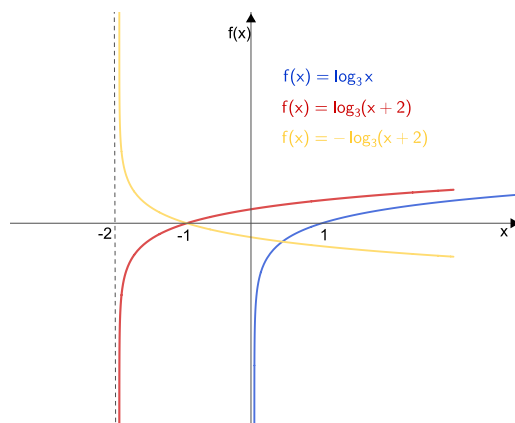
(d) **B** $f(x) = 5 \lg(x - 3)$

$[D_f =]3, \infty[; R_f = R]$



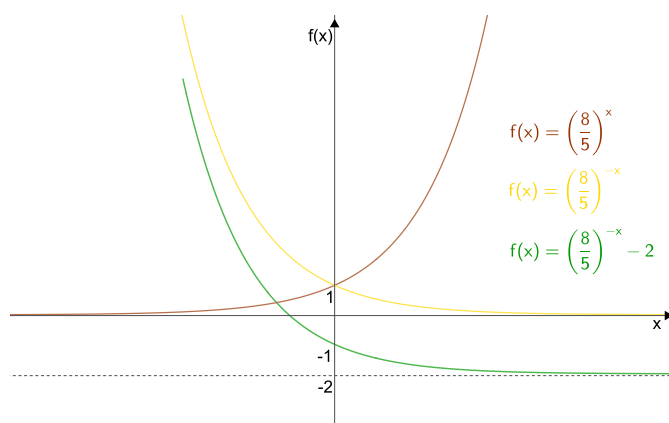
(e) **B** $f(x) = -\log_3(x + 2)$

$[D_f =]-2, \infty[; R_f = R]$



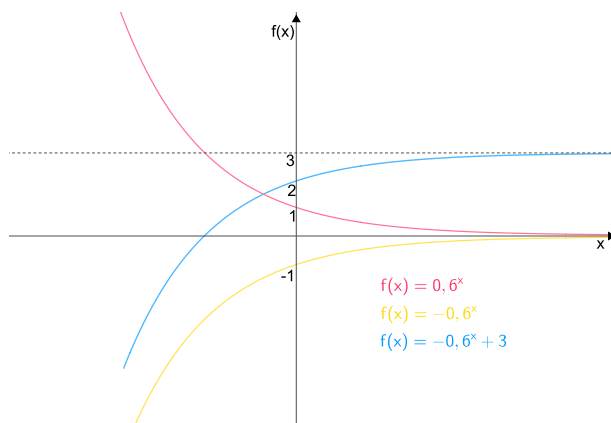
(f) **B** $f(x) = \left(\frac{8}{5}\right)^{-x} - 2$

$[D_f = R; R_f =]-2, \infty[]$



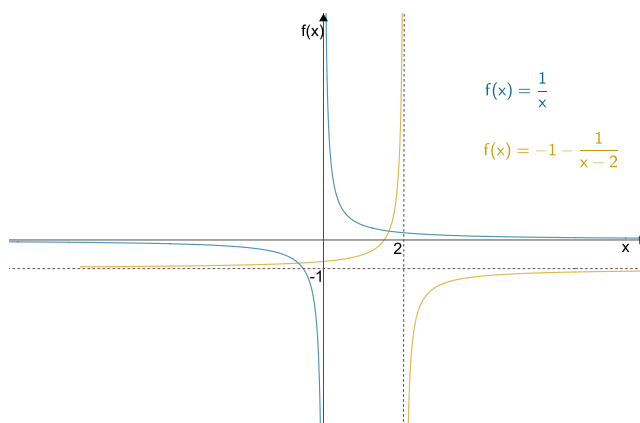
(g) **B** $f(x) = -0,6^x + 3$

$[D_f = \mathbb{R}; R_f =] - \infty, 3[]$



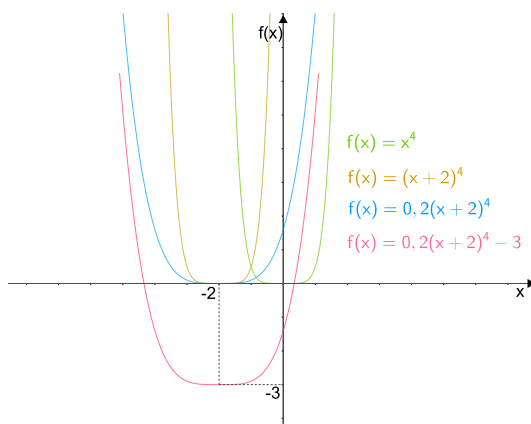
(h) **B** $f(x) = -1 - \frac{1}{x-2}$

$[D_f = \mathbb{R} \setminus \{2\}; R_f = \mathbb{R} \setminus \{-1\}]$

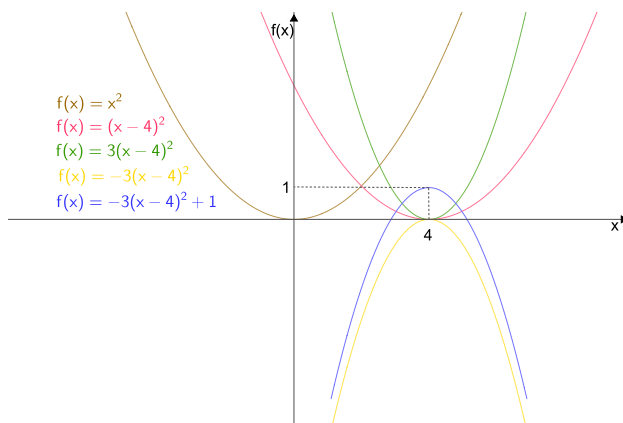


(i) **V** $f(x) = 0,2(x+2)^4 - 3$

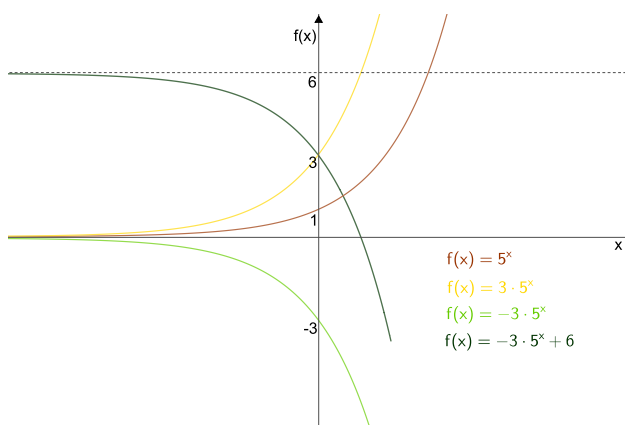
$[D_f = \mathbb{R}; R_f = [-3; \infty[]$



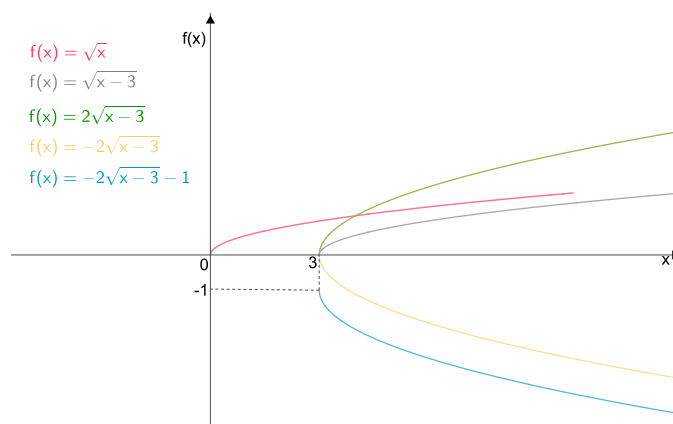
(j) $\nabla f(x) = -3(x-4)^2 + 1$ $[D_f = \mathbb{R}; R_f =]-\infty; 1]$



(k) $\nabla f(x) = -3 \cdot 5^x + 6$ $[D_f = \mathbb{R}; R_f =]-\infty; 6[$

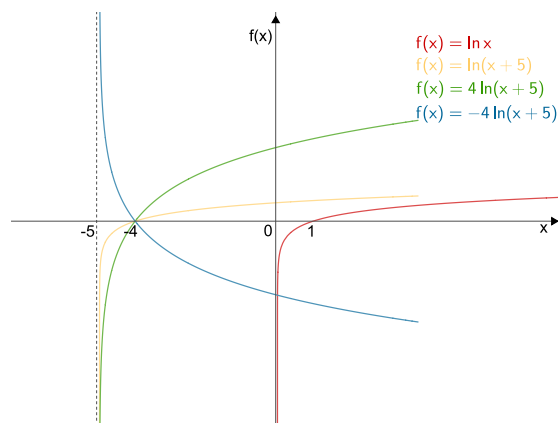


(l) $\nabla f(x) = -2\sqrt{x-3} - 1$ $[D_f = [3, \infty[; R_f =]-\infty; -1]$



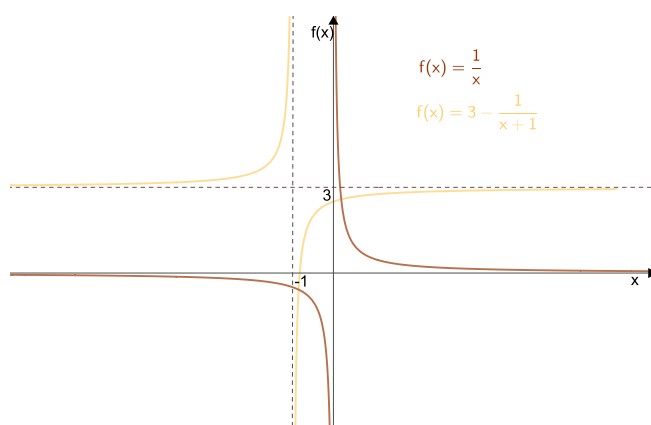
(m) $\nabla f(x) = -4 \ln(x + 5)$

$[D_f =] - 5, \infty[; R_f = R]$



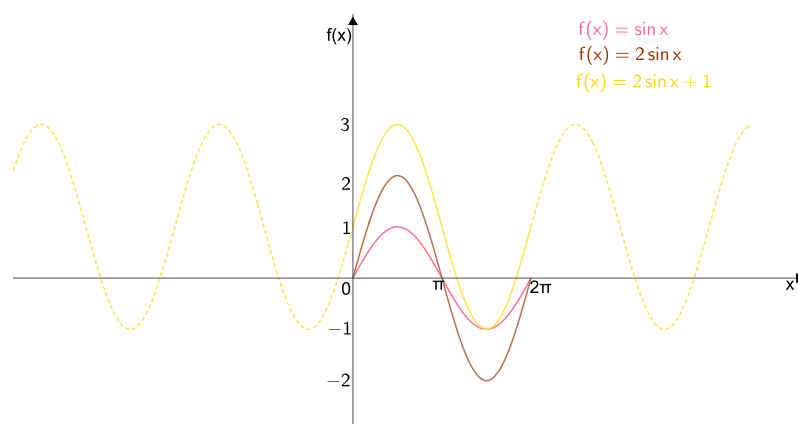
(n) $\nabla f(x) = \frac{3x+2}{x+1}$

$\left[f(x) = \frac{3x+2}{x+1} = 3 - \frac{1}{x+1}; D_f = R \setminus \{-1\}; R_f = R \setminus \{3\} \right]$



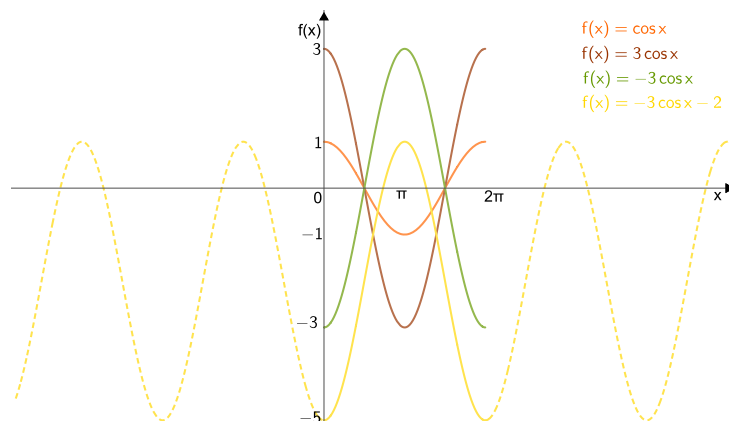
(o) $\nabla f(x) = 2 \sin x + 1$

$[D_f = R; R_f = [-1; 3]]$

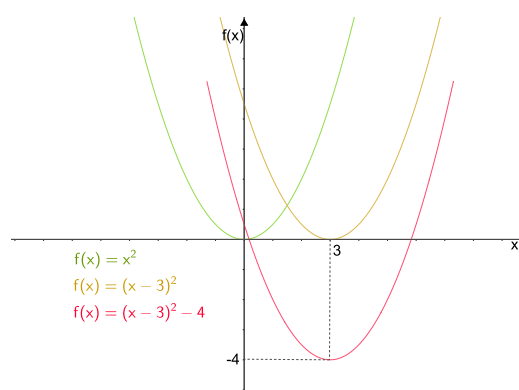


(p) **V** $f(x) = -3 \cos x - 2$

$[D_f = \mathbb{R}; R_f = [-5; 1]]$

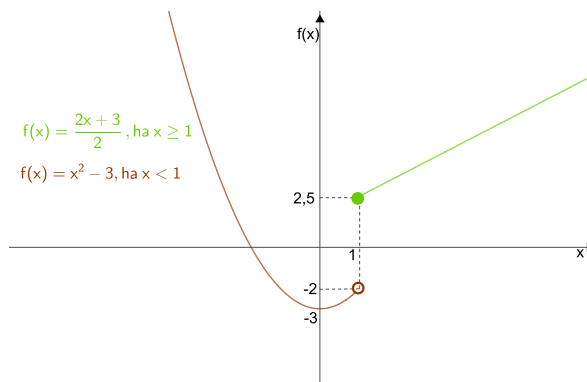


(r) **V** $f(x) = x^2 - 6x + 5$ $[f(x) = x^2 - 6x + 5 = (x - 3)^2 - 4; D_f = \mathbb{R}; R_f = [-4, \infty[]$

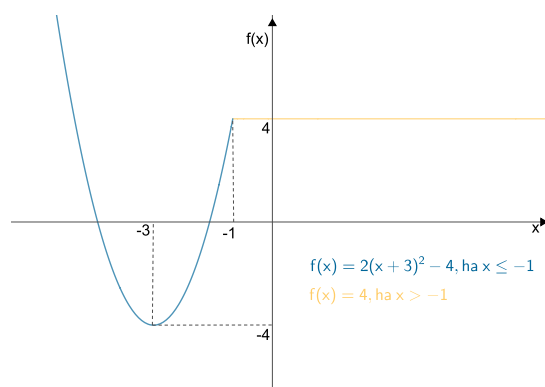


6. Ábrázolja az alábbi $f : \mathbb{R} \rightarrow \mathbb{R}$ függvényeket, majd az ábra alapján határozza meg a függvények értékkészletét!

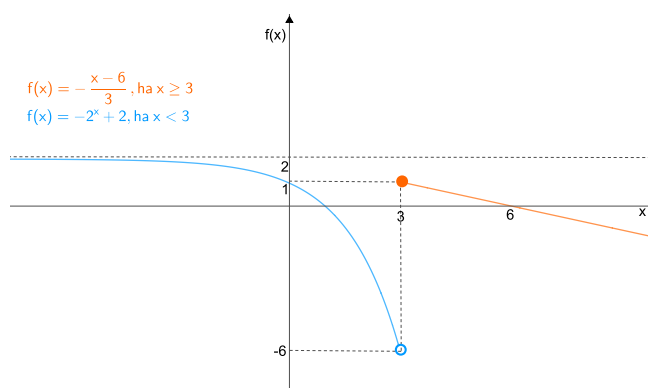
(a) **B, V** $f(x) = \begin{cases} \frac{2x+3}{2}, & \text{ha } x \geq 1 \\ x^2 - 3, & \text{ha } x < 1 \end{cases}$ $[R(f) = [-3; \infty[]$



(b) **B, V** $f(x) = \begin{cases} 2(x+3)^2 - 4, & \text{ha } x \leq -1 \\ 4, & \text{ha } x > -1 \end{cases} \quad [R(f) = [-4, \infty[]$



(c) **V** $f(x) = \begin{cases} -2^x + 2, & \text{ha } x < 3 \\ -\frac{x-6}{3}, & \text{ha } x \geq 3 \end{cases} \quad [R(f) =] -\infty, 2[]$



7. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

- (a) $f(x) = \sqrt{4x-8}$ $[4x-8 \geq 0; D_f = [2; \infty[]$
- (b) $g(x) = \log_8(9-5x)$ $[9-5x > 0; D_g =] -\infty; \frac{9}{5}[]$
- (c) $h(x) = \sqrt[7]{4x+7}$ $[D_h = \mathbb{R}]$
- (d) $f(x) = \frac{3x+7}{8x+9}$ $[8x+9 \neq 0; D_f = \mathbb{R} \setminus \{-\frac{9}{8}\}]$
- (e) $g(x) = e^{6x-7}$ $[D_g = \mathbb{R}]$
- (f) $f(x) = 3^{\frac{4x-3}{-x+7}}$ $[-x+7 \neq 0; D_f = \mathbb{R} \setminus \{7\}]$
- (g) $h(x) = \frac{8x}{\sqrt[5]{4x-3}}$ $[\sqrt[5]{4x-3} \neq 0; D_h = \mathbb{R} \setminus \{\frac{3}{4}\}]$
- (h) $f(x) = \frac{4x-5}{\sqrt{3x+6}}$ $[\sqrt{3x+6} \neq 0; 3x+6 \geq 0; D_f =] -2; \infty[]$
- (i) $h(x) = \sqrt[4]{x^2+3x-10}$ $[x^2+3x-10 \geq 0; D_h =] -\infty; -5] \cup [2; \infty[]$

$$(j) f(x) = \frac{3x^5 - 2}{\sqrt[3]{x^2 - 2x - 3}} \quad \left[\sqrt[3]{x^2 - 2x - 3} \neq 0; D_f = R \setminus \{-1; 3\} \right]$$

$$(k) f(x) = \frac{\sqrt{x}}{x^2 - 5x + 6} \quad [x \geq 0; x^2 - 5x + 6 \neq 0; D_f = [0; \infty[\setminus \{2; 3\}]]$$

8. Határozza meg a következő $f : R \rightarrow R$ függvények legbővebb értelmezési tartományát!

$$(a) \text{ B } f(x) = \sqrt{-x^2 + 6x - 8} \quad [-x^2 + 6x - 8 \geq 0; D_f = [2; 4]]$$

$$(b) \text{ B } f(x) = \log_7(-x^2 + 4x + 12) \quad [-x^2 + 4x + 12 > 0; D_f =] - 2; 6[]$$

$$(c) \text{ B } f(x) = \frac{3 - 2x}{\sqrt{x^2 - x - 20}} \quad \left[\sqrt{x^2 - x - 20} \neq 0; x^2 - x - 20 \geq 0; D_f =] - \infty; -4[\cup] 5; \infty[\right]$$

$$(d) \text{ B } f(x) = \frac{6^{3x}}{\log_3(x + 5)} \quad [x + 5 > 0; \log_3(x + 5) \neq 0; D_f =] - 5; \infty[\setminus \{-4\}]$$

$$(e) \text{ B } f(x) = -\frac{\lg(11 - 4x)}{8x - 3} \quad \left[11 - 4x > 0; 8x - 3 \neq 0; D_f =] - \infty; \frac{11}{4}[\setminus \left\{ \frac{3}{8} \right\} \right]$$

$$(f) \text{ B } f(x) = \frac{\sqrt[4]{7 + 4x}}{3x - 12} \quad \left[7 + 4x \geq 0; 3x - 12 \neq 0; D_f = \left[-\frac{7}{4}; \infty[\setminus \{4\} \right] \right]$$

$$(g) \text{ B } g(x) = \frac{5}{4 - \sqrt{5x + 3}} \quad \left[5x + 3 \geq 0; 4 - \sqrt{5x + 3} \neq 0; D_g = \left[-\frac{3}{5}; \infty[\setminus \left\{ \frac{13}{5} \right\} \right] \right]$$

$$(h) \text{ B } f(x) = \log_7(-5x + 7) + \sqrt{3x} \quad [-5x + 7 > 0; 3x \geq 0; D_f = [0; \frac{7}{5}[]]$$

$$(i) \text{ B } f(x) = \sqrt{\frac{8 - 3x}{5}} \quad \left[\frac{8 - 3x}{5} \geq 0 \Leftrightarrow 8 - 3x \geq 0, D_f =] - \infty, \frac{8}{3}[\right]$$

$$(j) \text{ B } f(x) = \log_3\left(\frac{-7}{4x + 6}\right) \quad \left[\frac{-7}{4x + 6} > 0 \Leftrightarrow 4x - 6 < 0, D_f =] - \infty, -\frac{3}{2}[\right]$$

$$(k) \text{ B } h(x) = \frac{1}{\sqrt{5 - x}} + \lg(x + 1) \quad [5 - x \geq 0; 5 - x \neq 0; x + 1 > 0; D_h =] - 1, 5[]$$

$$(l) \text{ B } f(x) = \frac{\ln(3 - 2x)}{\sqrt{4x - x^2}} \quad [4x - x^2 \geq 0; 4x - x^2 \neq 0; 3 - 2x > 0; D_f =] 0, \frac{3}{2}[]$$

$$(m) \text{ B } f(x) = \lg(15 - 3x) + \sqrt{x^2 + 8x - 9} \quad [15 - 3x > 0; x^2 + 8x - 9 \geq 0; D_f =] - \infty, -9[\cup [1, 5[]]$$

9. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

$$(a) \text{ V } f(x) = \sqrt{\log_2 x + 5} \quad [\log_2 x + 5 \geq 0 (\log_2 x - \text{szigorúan monoton növekvő}); x > 0; D(f) = [2^{-5}; \infty] = \left[\frac{1}{32}; \infty \right]]$$

$$(b) \text{ V } f(x) = \sqrt[6]{\log_{0,4} x + 2} \quad [\log_{0,4} x + 2 \geq 0 (\log_{0,4} x - \text{szigorúan monoton csökkenő}); x > 0; D(f) =] 0; 0, 4^{-2}[=] 0; 6, 25[]]$$

$$(c) \text{ V } f(x) = \sqrt{3^x - 81} \quad [3^x - 81 \geq 0 (3^x - \text{szigorúan monoton növekvő}); D(f) = [4; \infty[]]$$

- (d) $\mathbf{V} \quad f(x) = \frac{-7}{5 - \sqrt{x^2 - 9}}$
 $[x^2 - 9 \geq 0; 5 - \sqrt{x^2 - 9} \neq 0; D(f) =] - \infty; -3[\cup] 3; \infty[\setminus \{-\sqrt{34}; \sqrt{34}\}]$
- (e) $\mathbf{V} \quad f(x) = \ln(x^2 - x) + \sqrt{16 - 4x^2}$
 $[x^2 - x > 0; 16 - 4x^2 \geq 0; D(f) = [-2; 0[\cup] 1; 2]$
- (f) $\mathbf{V} \quad f(x) = x + \sqrt{5 - x - \frac{6}{x}}$
 $[5 - x - \frac{6}{x} \geq 0; D_f =] - \infty, 0[\cup] 2, 3]$
- (g) $\mathbf{V} \quad f(x) = \frac{x + 1}{\lg(1 - 2x)} + \sqrt{1 - x^2}$
 $[\lg(1 - 2x) \neq 0; 1 - 2x > 0; 1 - x^2 \geq 0; D_f = [-1; \frac{1}{2}[\setminus \{0\}]$
- (h) $\mathbf{V} \quad f(x) = \frac{x}{5} \cdot \ln(16 - x^2) - \sqrt{\frac{x - 2}{x + 3}}$
 $[16 - x^2 > 0; \frac{x - 2}{x + 3} \geq 0; D_f =] - 4, -3[\cup] 2, 4[]$
- (i) $\mathbf{V} \quad f(x) = \frac{\lg x - 8}{\sqrt{35 + 2x - x^2}} + \frac{5}{x - 4}$
 $[x > 0; \sqrt{35 + 2x - x^2} \neq 0; 35 + 2x - x^2 \geq 0, x - 4 \neq 0; D_f =] 0, 4[\cup] 4, 7[]$
- (j) $\mathbf{V} \quad f(x) = \frac{\sqrt{x^2 - x - 2}}{\ln x} - \frac{3 + 6x}{8}$
 $[x^2 - x - 2 \geq 0; \ln x \neq 0; x > 0; D_f = [2, \infty[]$
- (k) $\mathbf{V} \quad f(x) = \frac{2x}{5} + \frac{\sqrt{16 - x^2}}{\lg(8 - 2x)}$
 $[16 - x^2 \geq 0; \lg(8 - 2x) \neq 0; 8 - 2x > 0; D_f = [-4, 4[\setminus \{\frac{7}{2}\} = [-4, \frac{7}{2}[\cup] \frac{7}{2}, 4[]$
- (l) $\mathbf{V} \quad f(x) = \frac{\sqrt{x^2 - 1}}{x^2 - 1} + \frac{2}{\sqrt{1 - \lg(2x)}}$
 $[x^2 - 1 \geq 0; x^2 - 1 \neq 0; 1 - \lg(2x) \geq 0; \sqrt{1 - \lg(2x)} \neq 0; 2x > 0; D_f =] 1, 5[]$
- (m) $\mathbf{V} \quad f(x) = \sqrt{\frac{3}{2x - 5}} + \ln(6 + 11x - 2x^2)$
 $[\frac{3}{2x - 5} \geq 0; 6 + 11x - 2x^2 > 0; D_f =] \frac{5}{2}, 6[]$
- (n) $\mathbf{V} \quad f(x) = 3 \cdot \lg\left(\frac{x + 1}{x - 5}\right) - \frac{\sqrt{5x - 10}}{x^2 - 36}$
 $[\frac{x + 1}{x - 5} > 0; 5x - 10 \geq 0; x^2 - 36 \neq 0; D_f =] 5, \infty[\setminus] 6, 6[\cup] 6, \infty[]$
- (o) $\mathbf{V} \quad f(x) = \ln\left(\frac{x^2 - x - 2}{x^2 + x - 2}\right)$
 $[\frac{x^2 - x - 2}{x^2 + x - 2} > 0; D_f =] - \infty, -2[\cup] - 1, 1[\cup] 2, \infty[]$

10. Az alábbi $f : R \rightarrow R$ függvényeknek létezik inverze. Határozza meg az inverz függvény hozzárendelési utasítását!

(a) **B** $f(x) = 4x - 7$ $[f^{-1}(x) = \frac{x+7}{4} = \frac{1}{4}x + \frac{7}{4}]$

(b) **B** $f(x) = \frac{3x-4}{x+2}$ $[f^{-1}(x) = \frac{2x+4}{3-x}]$

(c) **B** $f(x) = 6 - 5(x-1)^5$ $[f^{-1}(x) = \sqrt[5]{\frac{x-6}{-5}} + 1]$

(d) **B** $f(x) = 3(4+6x)^7 - 2$ $[f^{-1}(x) = \frac{\sqrt[7]{\frac{x+2}{3}} - 4}{6}]$

(e) **B** $f(x) = 5\sqrt[3]{x-8} + 9$ $[f^{-1}(x) = \left(\frac{x-9}{5}\right)^3 + 8]$

(f) **B** $f(x) = -2\sqrt[4]{5x+3} - 7$ $[f^{-1}(x) = \frac{\left(\frac{x+7}{-2}\right)^4 - 3}{5}]$

(g) **B** $f(x) = 3e^{2x-7} + 5$ $[f^{-1}(x) = \frac{\ln\left(\frac{x-5}{3}\right)+7}{2} = \frac{1}{2}\ln\left(\frac{x-5}{3}\right) + \frac{7}{2}]$

(h) **B** $f(x) = 5^{2-\frac{x}{3}} - 1$ $[f^{-1}(x) = -3(\log_5(x+1) - 2)]$

(i) **B** $f(x) = 5\log_2(6-4x) + 2$ $[f^{-1}(x) = \frac{2^{\frac{x-2}{5}} - 6}{-4} = -\frac{1}{4} \cdot 2^{\frac{x-2}{5}} + \frac{3}{2}]$

(l) **B** $f(x) = 4\ln(3+2x) - 7$ $[f^{-1}(x) = \frac{e^{\frac{x+7}{4}} - 3}{2} = \frac{1}{2}e^{\frac{x+7}{4}} - \frac{3}{2}]$

11. Határozza meg a következő $f : R \rightarrow R$ függvények legbővebb értelmezési tartományát és értékkészletét! Határozza meg az inverz függvényt és annak legbővebb értelmezési tartományát és értékkészletét !

(a) **V** $f(x) = 4(3x+7)^5 + 6$
 $[D_f = R; R_f = R; f^{-1}(x) = \frac{\sqrt[5]{\frac{x-6}{4}} - 7}{3} = \frac{1}{3}\sqrt[5]{\frac{x-6}{4}} - \frac{7}{3};$
 $D_{f^{-1}} = R; R_{f^{-1}} = R]$

(b) **V** $f(x) = 4\sqrt{6-2x} + 12$
 $[D_f =] - \infty; 3]; R_f = [12; \infty[; f^{-1}(x) = \frac{\left(\frac{x-12}{4}\right)^2 - 6}{-2} = -\frac{1}{2}\left(\frac{x-12}{4}\right)^2 + 3;$
 $D_{f^{-1}} = [12; \infty[; R_{f^{-1}} =] - \infty; 3]]$

(c) **V** $f(x) = 2e^{5x+3} - 4$
 $[D_f = R; R_f =] - 4; \infty[; f^{-1}(x) = \frac{\ln\left(\frac{x+4}{2}\right) - 3}{5} = \frac{1}{5}\ln\left(\frac{x+4}{2}\right) - \frac{3}{5};$
 $D_{f^{-1}} =] - 4; \infty[; R_{f^{-1}} = R]$

(d) **V** $f(x) = 3 - 2^{3x-5}$
 $[D_f = R; R_f =] - \infty; 3[; f^{-1}(x) = \frac{\log_2\left(\frac{x-3}{-1}\right) + 5}{3} = \frac{\log_2(3-x) + 5}{3} = \frac{1}{3}\log_2(3-x) + \frac{5}{3};$
 $D_{f^{-1}} =] - \infty; 3[; R_{f^{-1}} = R]$

(e) **V** $f(x) = 4\ln(2x+5) - 8$
 $[D_f =] - 2, 5; \infty[; R_f = R; f^{-1}(x) = \frac{e^{\frac{x+8}{4}} - 5}{2} = \frac{1}{2}e^{\frac{x+8}{4}} - \frac{5}{2};$
 $D_{f^{-1}} = R; R_{f^{-1}} =] - 2, 5; \infty[]$

(f) **V** $f(x) = 4 \log_3(7x - 14) + 11$

$$[D_f =]2; \infty[; R_f = R; f^{-1}(x) = \frac{3^{\frac{x-11}{4}} + 14}{7} = \frac{1}{7} \cdot 3^{\frac{x-11}{4}} + 2;$$

$$D_{f^{-1}} = R; R_{f^{-1}} =]2; \infty[]$$

(g) **V** $f(x) = \frac{3x - 15}{x + 4}$

$$[f(x) = \frac{3x-15}{x+4} = 3 - \frac{27}{x+4}; D_f = R \setminus \{-4\}; R_f = R \setminus \{3\}; f^{-1}(x) = \frac{-15-4x}{x-3};$$

$$D_{f^{-1}} = R \setminus \{3\}; R_{f^{-1}} = R \setminus \{-4\}]$$