FÜGGVÉNYEK

1. Határozza meg a következő összetett függvényeket!
   $[g \circ f = g(f(x)); f \circ g = f(g(x))]$

   (a) B $f(x) = \cos x + x^2; g(x) = \sqrt{x}; f(g(x)) =?; g(f(x)) =?$
   $\left[f(g(x)) = \cos(\sqrt{x}) + (\sqrt{x})^2 = \cos(\sqrt{x}) + x; g(f(x)) = \sqrt{\cos x + x^2}\right]$

   (b) B $f(x) = \sin x; g(x) = x^2; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \sin(x^2) = \sin x^2; g(f(x)) = (\sin x)^2 = \sin^2 x]$

   (c) B $f(x) = \dfrac{x}{1+x^2}; g(x) = \mathrm{tg}x; f(g(x)) =?; g(f(x)) =?$
   $\left[f(g(x)) = \frac{\mathrm{tg}x}{1+(\mathrm{tg}x)^2} = \frac{\mathrm{tg}x}{1+\mathrm{tg}^2x}; g(f(x)) = \mathrm{tg}\left(\frac{x}{1+x^2}\right)\right]$

   (d) B $f(x) = \ln x + 4x^5; g(x) = \mathrm{e}^x; f(g(x)) =?; g(f(x)) =?$
   $\left[f(g(x)) = \ln(e^x) + 4(\mathrm{e}^x)^5 = x + 4\mathrm{e}^{5x}; g(f(x)) = \mathrm{e}^{\ln x + 4x^5}\right]$

   (e) B $f(x) = x^2 - 3x; g(x) = \sqrt{5-2x}; f(g(x)) =?; g(f(x)) =?$
   $\Big[f(g(x)) = \left(\sqrt{5-2x}\right)^2 - 3\sqrt{5-2x} = 5 - 2x - 3\sqrt{5-2x};$
   $g(f(x)) = \sqrt{5 - 2(x^2-3x)} = \sqrt{5 - 2x^2 + 6x}]$

   (f) B $f(x) = 1 - x + x^2; g(x) = \mathrm{e}^x; f(g(x)) =?; g(f(x)) =?$
   $\left[f(g(x)) = 1 - \mathrm{e}^x + (\mathrm{e}^x)^2 = 1 - \mathrm{e}^x + \mathrm{e}^{2x}; g(f(x)) = \mathrm{e}^{1-x+x^2}\right]$

   (g) B $f(x) = \cos(7-x); g(x) = x^4 - 3x + 2; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \cos(7 - (x^4 - 3x + 2)) = \cos(-x^4 + 3x + 5);$
   $g(f(x)) = (\cos(7-x))^4 - 3\cos(7-x) + 2 = \cos^4(7-x) - 3\cos(7-x) + 2]$

   (i) B $f(x) = \sqrt[3]{2-3x}; g(x) = 4x - x^3; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \sqrt[3]{2 - 3(4x - x^3)} = \sqrt[3]{2 - 12x + 3x^3};$
   $g(f(x)) = 4\sqrt[3]{2-3x} - (\sqrt[3]{2-3x})^3 = 4\sqrt[3]{2-3x} - 2 + 3x]$

   (h) B $f(x) = \sqrt[5]{4-x}; g(x) = x^5 - 3^x; f(g(x)) =?; g(f(x)) =?$
   $[f(g(x)) = \sqrt[5]{4 - (x^5 - 3^x)} = \sqrt[5]{4 - x^5 + 3^x};$
   $g(f(x)) = (\sqrt[5]{4-x})^5 - 3^{\sqrt[5]{4-x}} = 4 - x - 3^{\sqrt[5]{4-x}}]$

2. Határozza meg a hiányzó függvényeket!

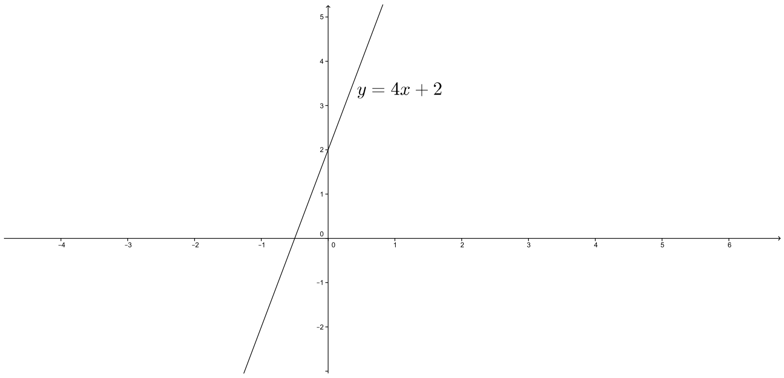
   (a) $f(x) = \sin x; g(x) =?; f(g(x)) = \sin(x+4)$ $[g(x) = x + 4]$

   (b) B $f(x) =?; g(x) = \cos x; f(g(x)) = \cos^4 x + 3\cos x$ $[f(x) = x^4 + 3x]$

   (c) B $f(x) =?; g(x) = x^2 - \mathrm{e}^x; g(f(x)) = x - \mathrm{e}^{\sqrt{x}}$ $[f(x) = \sqrt{x}]$

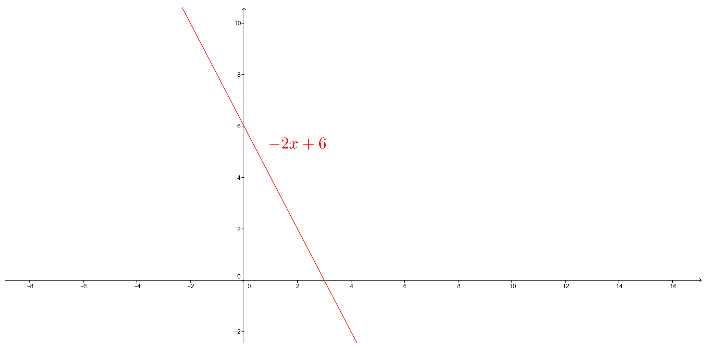
   (d) B $f(x) = x^2; g(x) =?; g(f(x)) = \dfrac{x}{1+x^4}$ $\left[g(x) = \frac{\sqrt{x}}{1+x^2}\right]$

3. Ábrázolja az alábbi $f : R \to R$ függvényeket, majd olvassa le a függvények értelmezési tartományát és értékkészletét!

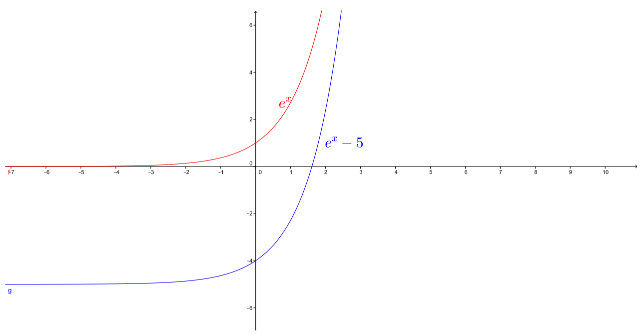


(a) $f(x) = 4x + 2$
$[D(f) = R; R(f) = R\ ]$

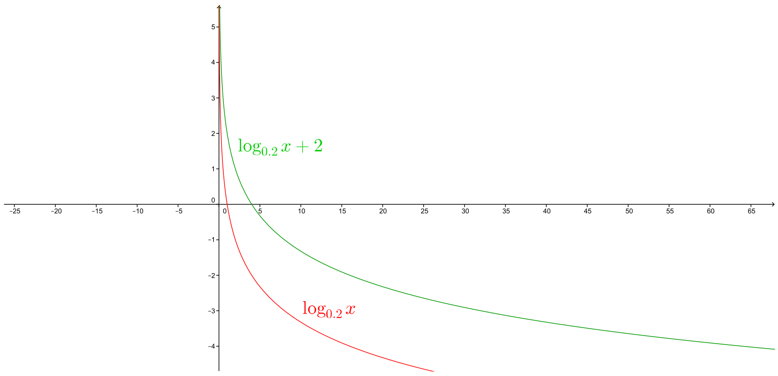
(b) $f(x) = -2x + 6$
$[D(f) = R; R(f) = R\ ]$



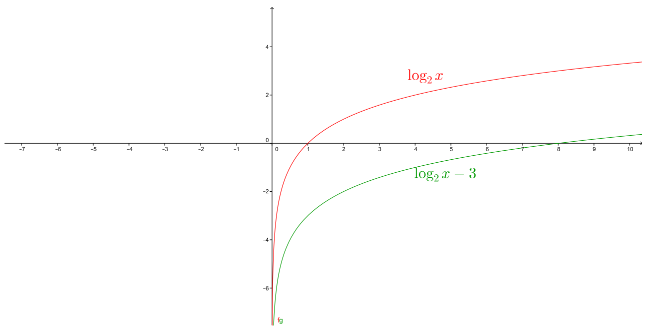
(c) $f(x) = \mathrm{e}^x - 5$
$[D(f) = R; R(f) =]-5; \infty[\ ]$



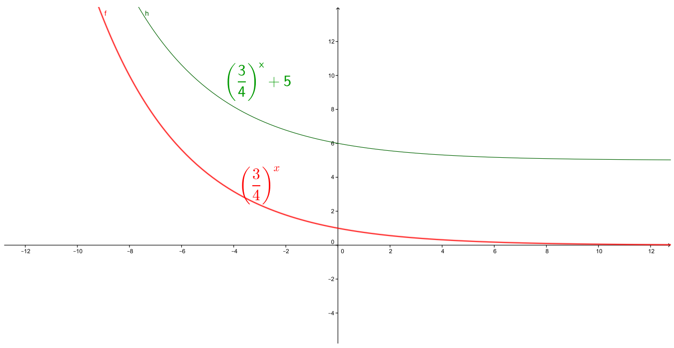
(d) $f(x) = \log_{0,2} x + 2$
$[D(f) =]0; \infty[; R(f) = R\ ]$



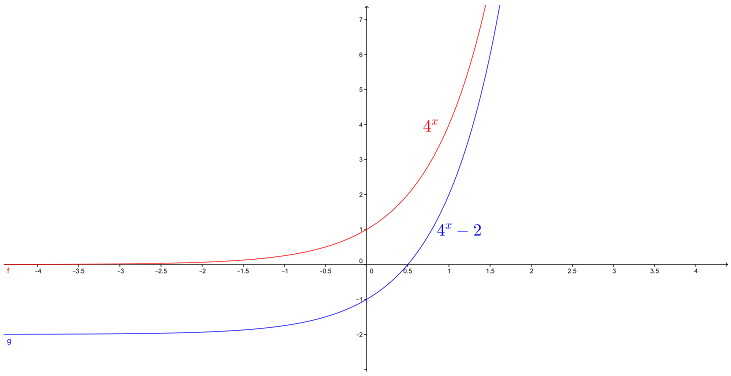
(e) $f(x) = \log_2 x - 3$
$[D(f) =]0; \infty[; R(f) = R\ ]$



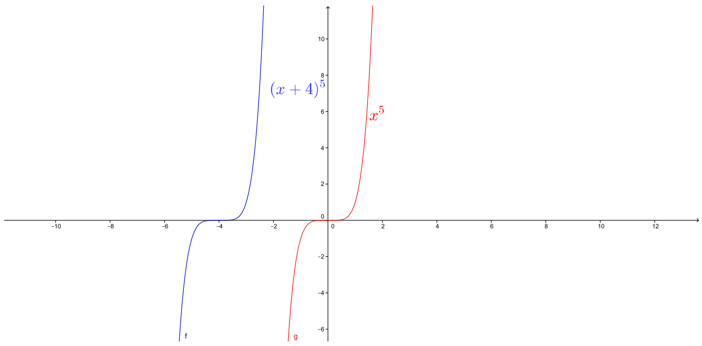
(f) $f(x) = \left(\frac{3}{4}\right)^x + 5$
$[D(f) = R; R(f) =]5; \infty[\ ]$



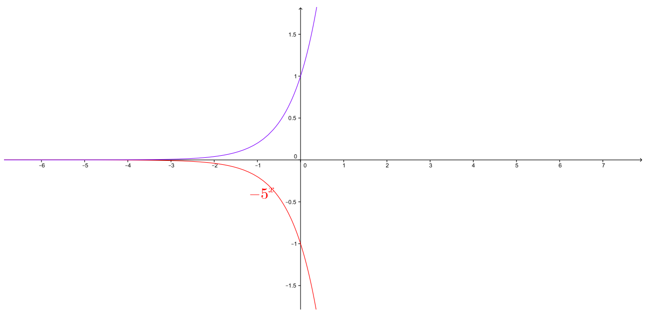
(g) $f(x) = 4^x - 2$
$[D(f) = R; R(f) =] - 2; \infty[\ ]$



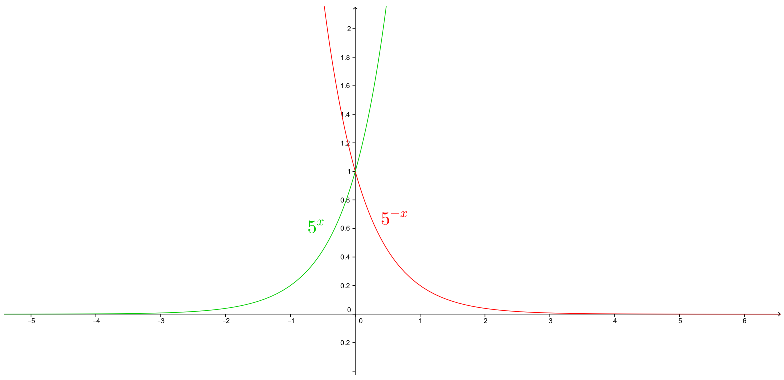
(h) $f(x) = (x+4)^5$
$[D(f) = R; R(f) = R\ ]$



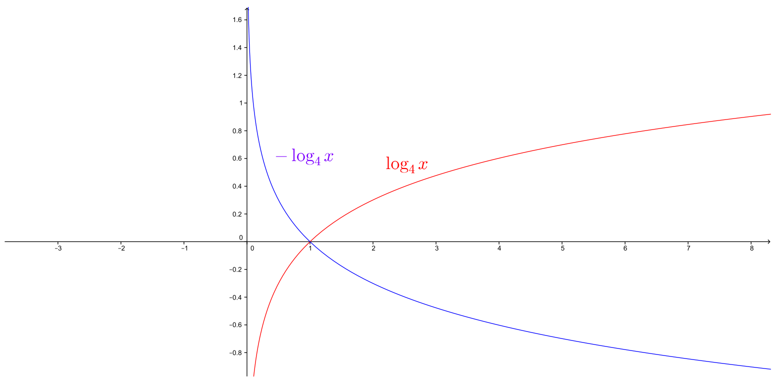
(i) $f(x) = -5^x$
$[D(f) = R; R(f) =]-\infty; 0[\ ]$



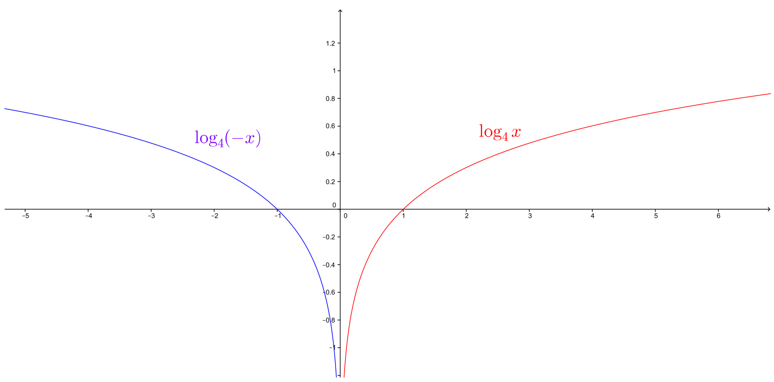
(j) $f(x) = 5^{-x}$
$[D(f) = R; R(f) =]0; \infty[\ ]$



(k) $f(x) = -\log_4 x$
$[D(f) =]0; \infty[; R(f) = R\ ]$

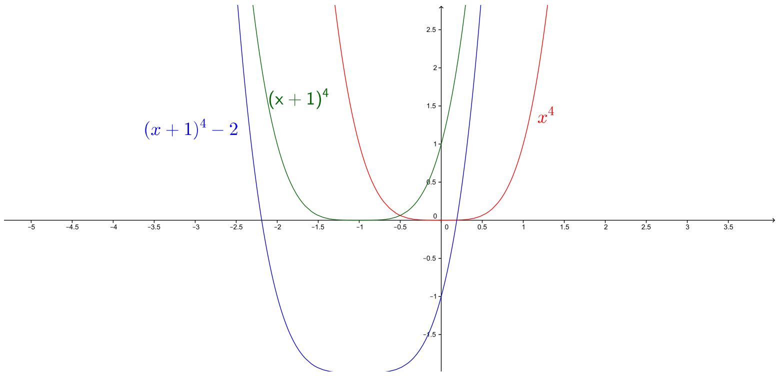


(l) $f(x) = \log_4(-x)$
$[D(f) =] -\infty; 0[; R(f) = R\ ]$

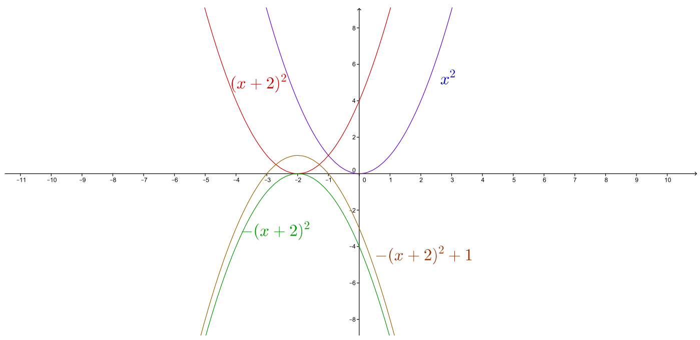


4. Ábrázolja az alábbi $f : R \to R$ függvényeket, majd olvassa le a függvények értelmezési tartományát és értékkészletét!

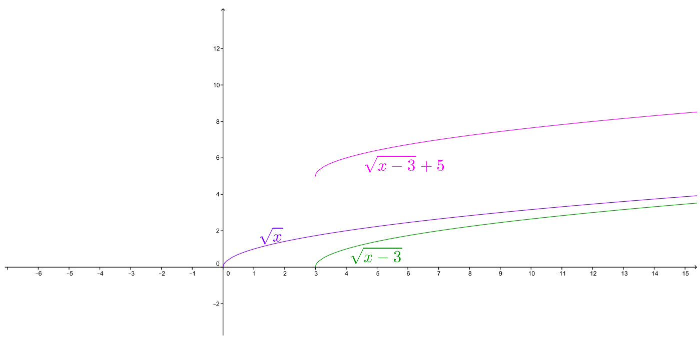
   (a) B $f(x) = (x+1)^4 - 2$
   $[D(f) = R; R(f) = [-2; \infty[\ ]$



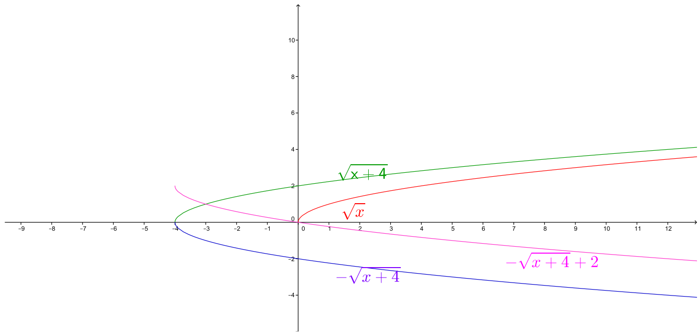
(b) V $f(x) = -(x+2)^2 + 1$
$[D(f) = R; R(f) =]-\infty; 1]$]



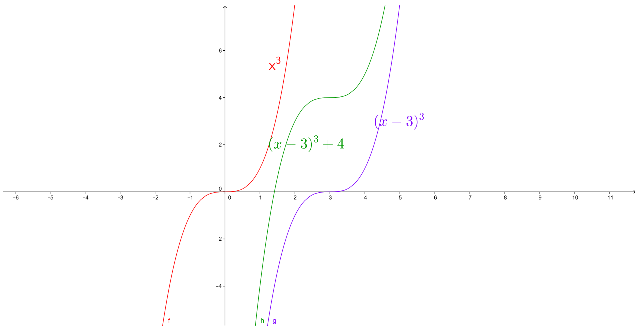
(c) B $f(x) = \sqrt{x-3} + 5$
$[D(f) = [3; \infty[; R(f) = [5; \infty[$]



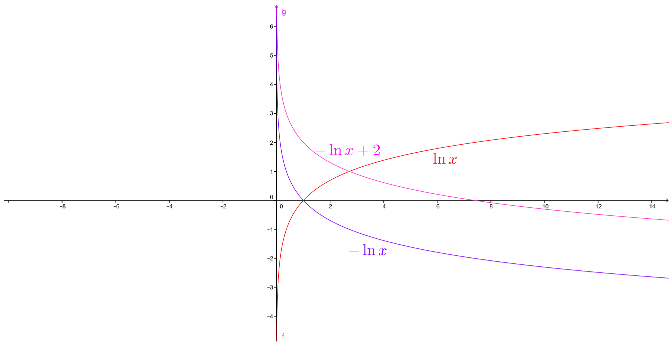
(d) V $f(x) = -\sqrt{x+4} + 2$
$[D(f) = [-4; \infty[; R(f) =]-\infty; 2]$]



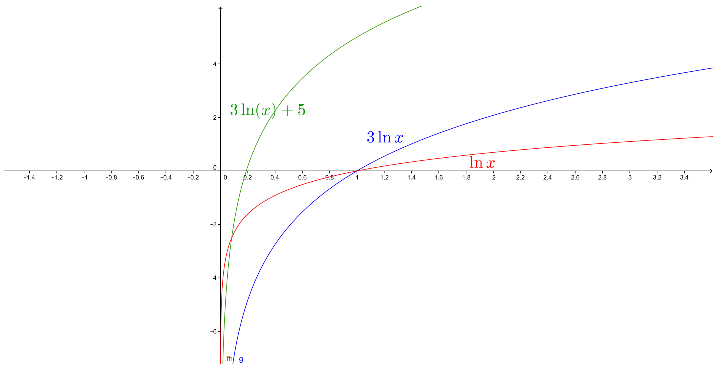
(e) B $f(x) = (x-3)^3 + 4$
$[D(f) = R; R(f) = R\ ]$



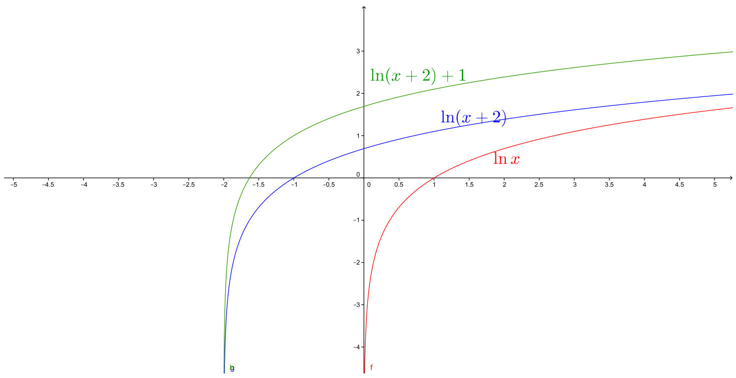
(f) B $f(x) = -\ln x + 2$
$[D(f) =]0; \infty[; R(f) = R\ ]$



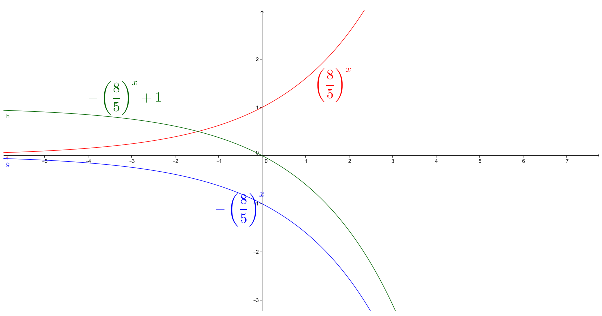
(g) B $f(x) = 3\ln x + 5$
$[D(f) =]0; \infty[; R(f) = R\ ]$



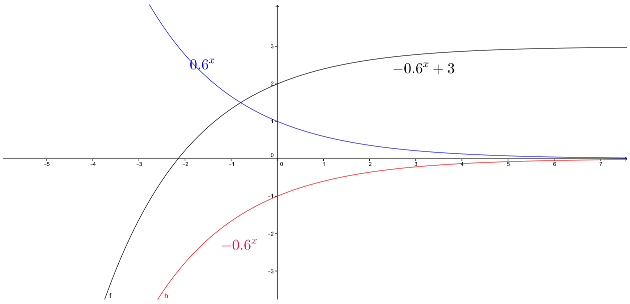
(h) B $f(x) = \ln(x+2) + 1$
$[D(f) =]-2; \infty[; R(f) = R\ \ ]$



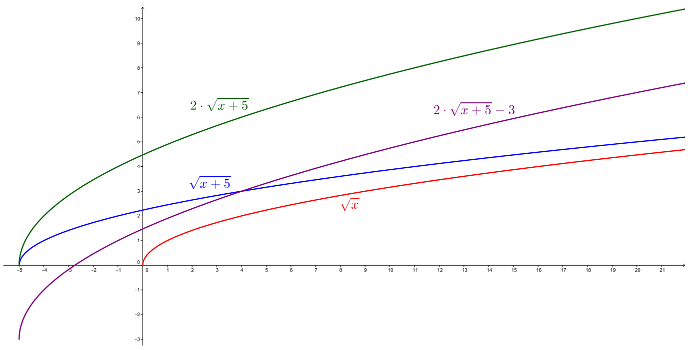
(i) B $f(x) = -\left(\frac{8}{5}\right)^x + 1$
$[D(f) = R; R(f) =]-\infty; 1[\ \ ]$



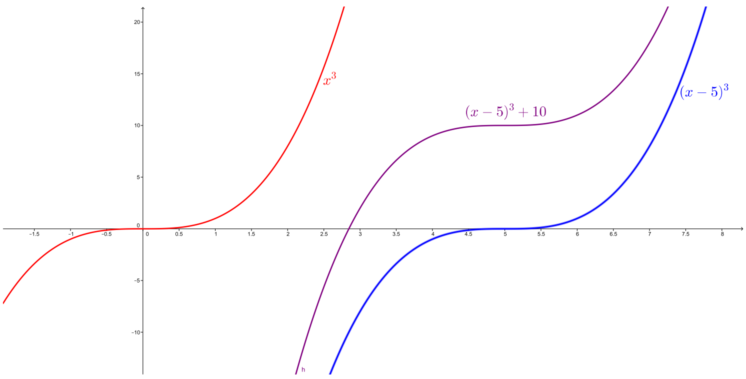
(j) B $f(x) = -0,6^x + 3$
$[D(f) = R; R(f) =]-\infty; 3[\ \ ]$



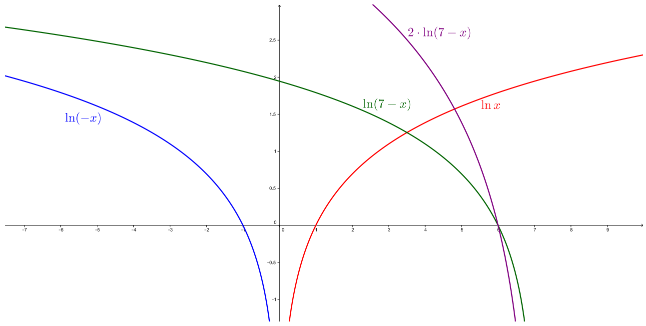
(k) V $f(x) = 2\sqrt{x+5} - 3$
$[D(f) = [-5; \infty[; R(f) = [-3; \infty[\ ]$



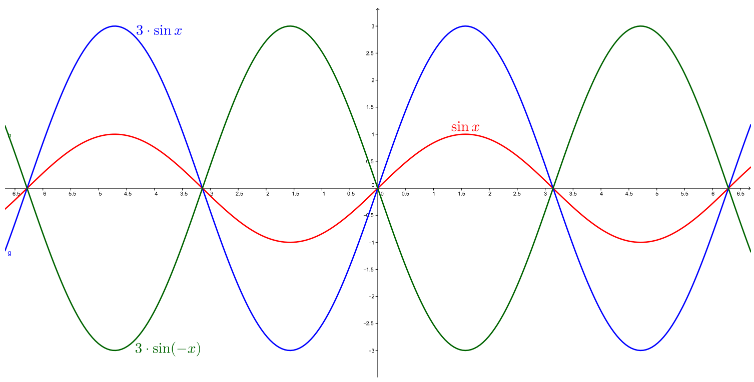
(l) B $f(x) = (x-5)^3 + 10$
$[D(f) = R; R(f) = R\ ]$



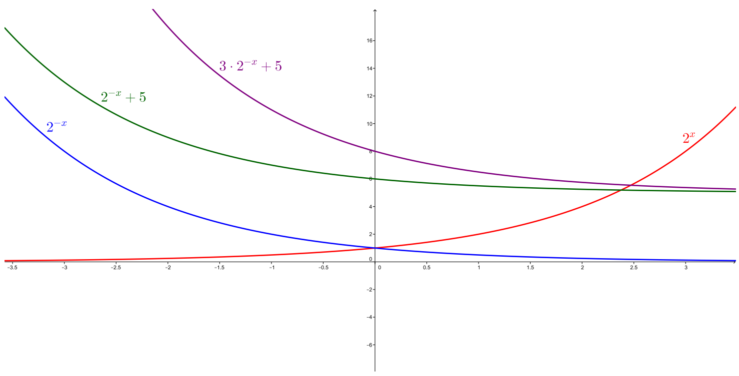
(m) V $f(x) = 2\ln(7-x)$
$[D(f) =]-\infty; 7[; R(f) = R\ ]$



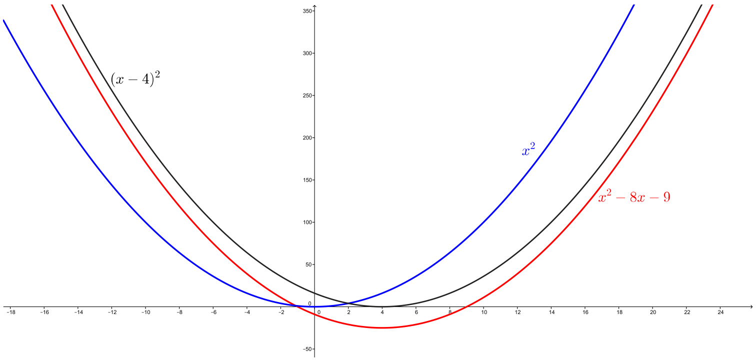
(n) B $f(x) = 3\sin(-x)$
$[D(f) = R; R(f) = [-3; 3]\ \ ]$



(o) V $f(x) = 3\cdot 2^{-x} + 5$
$[D(f) = R; R(f) =]5; \infty[\ \ ]$

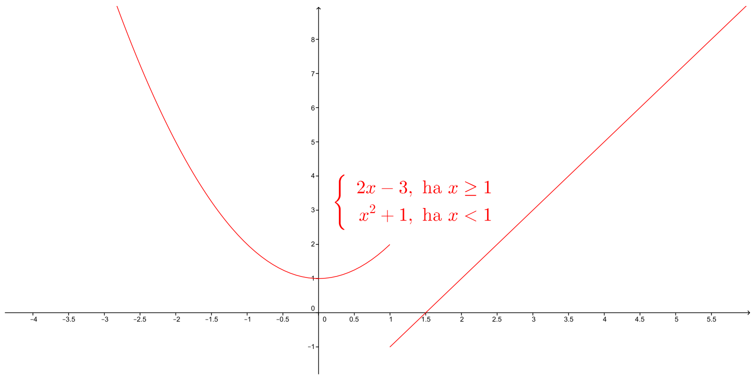


(p) V $f(x) = x^2 - 8x - 9$
$[x^2 - 8x - 9 = (x-4)^2 - 25; D(f) = R; R(f) = [-25; \infty[\ \ ]$

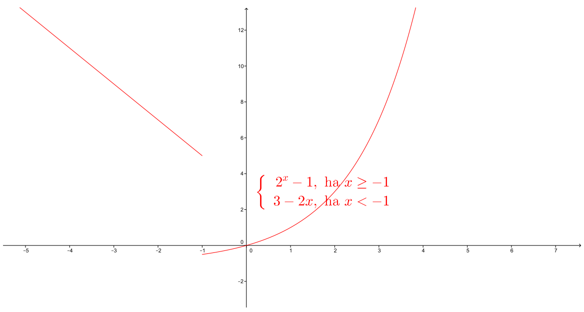


5. Ábrázolja az alábbi $f : R \to R$ függvényeket, majd olvassa le a függvények értékkészletét!

   (a) B $f(x) = \begin{cases} 2x-3 & \text{ha} x \geq 1 \\ x^2+1 & \text{ha } x < 1 \end{cases}$
   $[R(f) = [-1; \infty[\ ]$

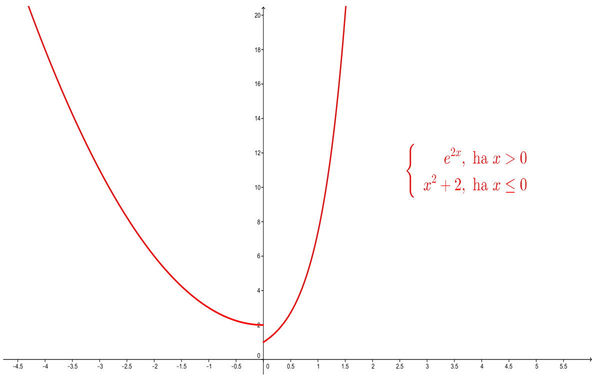


   (b) V $f(x) = \begin{cases} 2^x-1 & \text{ha} x \geq -1 \\ 3-2x & \text{ha } x < -1 \end{cases}$
   $[R(f) = [-0,5; \infty[\ ]$



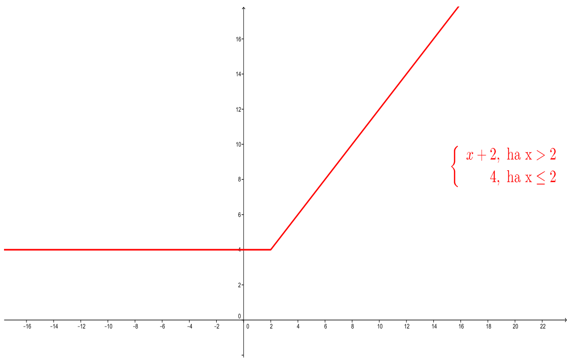
(c) B $f(x) = \begin{cases} e^{2x} & \text{ha} x > 0 \\ x^2 + 2 & \text{ha } x \leq 0 \end{cases}$

$[R(f) =]1; \infty[\ ]$



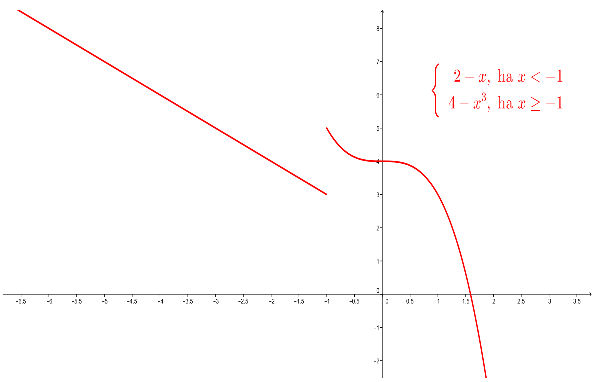
(d) B $f(x) = \begin{cases} x + 2 & \text{ha} x > 2 \\ 4 & \text{ha } x \leq 2 \end{cases}$

$[R(f) = [4; \infty[\ ]$



(e) V $f(x) = \begin{cases} 2 - x & \text{ha} x < -1 \\ 4 - x^3 & \text{ha } x \geq -1 \end{cases}$

$[R(f) = R\ ]$



6. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

   (a) $f(x) = \sqrt{4x-8}$ $\qquad [4x-8 \geq 0; D(f) = [2; \infty[\ ]$

   (b) $g(x) = \log_8(9-5x)$ $\qquad \left[9-5x > 0; D(g) =]-\infty; \frac{9}{5}[\ \right]$

   (c) $h(x) = \sqrt[7]{4x+7}$ $\qquad [D(h) = R\ ]$

   (d) $f(x) = \dfrac{3x+7}{8x+9}$ $\qquad \left[8x+9 \neq 0; D(f) = R \setminus \{-\frac{9}{8}\}\ \right]$

   (e) $g(x) = \mathrm{e}^{6x-7}$ $\qquad [D(g) = R\ ]$

   (f) $h(x) = \dfrac{8x}{\sqrt[5]{4x-3}}$ $\qquad \left[\sqrt[5]{4x-3} \neq 0; D(h) = R \setminus \{\frac{3}{4}\}\ \right]$

   (g) $f(x) = \dfrac{4x-5}{\sqrt{3x+6}}$ $\qquad \left[\sqrt{3x+6} \neq 0; 3x+6 \geq 0; D(f) =]-2; \infty[\ \right]$

   (h) $g(x) = \dfrac{5^{-x+7}}{8-5x}$ $\qquad \left[8-5x \neq 0; D(g) = R \setminus \{\frac{8}{5}\}\ \right]$

   (i) B $\ h(x) = \sqrt[4]{x^2+3x-10}$ $\qquad [x^2+3x-10 \geq 0; D(h) =]-\infty; -5] \cup [2; \infty[\ ]$

   (j) B $\ f(x) = \dfrac{3x^5-2}{\sqrt[3]{x^2-2x-3}}$ $\qquad \left[\sqrt[3]{x^2-2x-3} \neq 0; D(f) = R \setminus \{-1; 3\}\ \right]$

   (k) B $\ f(x) = \dfrac{2}{\sqrt{x^2-x-20}}$
   $\left[\sqrt{x^2-x-20} \neq 0; x^2-x-20 \geq 0; D(f) =]-\infty; -4[\cup]5; \infty[\ \right]$

   (l) B $\ f(x) = \sqrt{-x^2+6x-8}$ $\qquad [-x^2+6x-8 \geq 0; D(f) = [2; 4]\ ]$

7. Határozza meg a következő $f : R \to R$ függvények legbővebb értelmezési tartományát!

   (a) B $\ f(x) = \log_7(-x^2+4x+12)$ $\qquad [-x^2+4x+12 > 0; D(f) =]-2; 6[\ ]$

   (b) B $\ f(x) = 3^{\frac{4x-3}{-x+7}}$ $\qquad [-x+7 \neq 0; D(f) = R \setminus \{7\}\ ]$

   (c) B $\ f(x) = \dfrac{6}{\log_3(x+5)}$ $\qquad [x+5 > 0; \log_3(x+5) \neq 0; D(f) =]-5; \infty[\setminus\{-4\}\ ]$

   (d) B $\ f(x) = \dfrac{\sqrt{x}}{x^2-5x+6}$ $\qquad [x \geq 0; x^2-5x+6 \neq 0; D(f) = [0; \infty[\setminus\{2; 3\}\ ]$

   (e) B $\ f(x) = -\dfrac{\ln(3x-2)}{4x+7}$ $\qquad \left[3x-2 > 0; 4x+7 \neq 0; D(f) =]\frac{2}{3}; \infty[\ \right]$

   (f) B $\ f(x) = \ln(3-2x-x^2)$ $\qquad [3-2x-x^2 > 0; D(f) =]-3; 1[\ ]$

   (g) B $\ f(x) = \dfrac{\sqrt{7+4x}}{3x-12}$ $\qquad \left[7+4x \geq 0; 3x-12 \neq 0; D(f) = [-\frac{7}{4}; \infty[\setminus\{4\}\ \right]$

   (h) B $\ f(x) = \dfrac{\lg(11-4x)}{8x-3}$ $\qquad \left[11-4x > 0; 8x-3 \neq 0; D(f) =]-\infty; \frac{11}{4}[\setminus\{\frac{3}{8}\}\ \right]$

   (i) B $\ f(x) = \dfrac{5}{4-\sqrt{5x+3}}$ $\qquad \left[5x+3 \geq 0; 4-\sqrt{5x+3} \neq 0; D(f) = [-\frac{3}{5}; \infty[\setminus\{\frac{13}{5}\}\ \right]$

(j) B $f(x)=\ln(10-2x)\sqrt{4x-5}$ $\left[10-2x>0; 4x-5\geq 0; D(f)=[\frac{5}{4};5[\ \right]$

(k) B $f(x)=\log_7(-5x+7)+\sqrt{3x}$ $\left[-5x+7>0; 3x\geq 0; D(f)=[0;\frac{7}{5}[\ \right]$

(l) B $f(x)=\dfrac{\log_6(5x+8)}{\sqrt[5]{3x-5}}$ $\left[5x+8>0; \sqrt[5]{3x-5}\neq 0; D(f)=]-\frac{8}{5};\infty[\setminus\{\frac{5}{3}\}\ \right]$

(m) B $f(x)=\dfrac{-3x+8}{\sqrt[6]{x^2-6x-7}}$
$\left[\sqrt[6]{x^2-6x-7}\neq 0; x^2-6x-7\geq 0; D(f)=]-\infty;-1[\cup]7;\infty[\ \right]$

(n) B $f(x)=\dfrac{4x-7}{\ln(3x-6)}$ $\left[3x-6>0; \ln(3x-6)\neq 0; D(f)=]2;\infty[\setminus\{\frac{7}{3}\}\ \right]$

8. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

(a) V $f(x)=\dfrac{x}{\sqrt[4]{x^2-9}-2}$
$\left[x^2-9\geq 0; \sqrt[4]{x^2-9}-2\neq 0; D(f)=]-\infty;-3]\cup[3;\infty[\setminus\{-5;5\}\ \right]$

(b) V $f(x)=\sqrt{\dfrac{4x-5}{1-3x}}$ $\left[\frac{4x-5}{1-3x}\geq 0; D(f)=]\frac{1}{3};\frac{5}{4}]\ \right]$

(c) V $f(x)=\sqrt{\log_2 x+5}$
$[\log_2 x+5\geq 0(\log_2 x-$szigorúan monoton növekvő$); x>0; D(f)=[2^{-5};\infty]=[\frac{1}{32};\infty]\ ]$

(d) V $f(x)=\sqrt[6]{\log_{0,4} x+2}$
$[\log_{0,4} x+2\geq 0(\log_{0,4} x-$szigorúan monoton csökkenő$); x>0;$
$D(f)=]0;0,4^{-2}]=]0;6,25]\ ]$

(e) V $f(x)=\dfrac{2}{4-\sqrt{x^2+7}}$ $\left[x^2+7\geq 0; 4-\sqrt{x^2+7}\neq 0; D(f)=R\setminus\{-3;+3\}\ \right]$

(f) V $f(x)=\dfrac{-7}{5-\sqrt{x^2-9}}$
$\left[x^2-9\geq 0; 5-\sqrt{x^2-9}\neq 0; D(f)=]-\infty;-3]\cup[3;\infty[\setminus\{-\sqrt{34};\sqrt{34}\}\ \right]$

(g) V $f(x)=\dfrac{4}{\ln(5x)+3}$
$\left[5x>0; \ln(5x)+3\neq 0; D(f)=]0;\infty[\setminus\{\frac{e^{-3}}{5}\}=]0;\infty[\setminus\{\frac{1}{5e^3}\}\ \right]$

(h) V $f(x)=\sqrt{3^x-81}$
$[3^x-81\geq 0(3^x-$szigorúan monoton növekvő$); D(f)=[4;\infty[\ ]$

(i) V $f(x)=\ln(x^2-x)\sqrt{16-4x^2}$
$[x^2-x>0; 16-4x^2\geq 0; D(f)=[-2;0[\cup]1;2]\ ]$

(j) V $f(x)=\dfrac{x}{\ln(x^2+x-12)}$
$[x^2+x-12>0; \ln(x^2+x-12)\neq 0; D(f)=]-\infty;-4[\cup]3;\infty[\setminus\{-4,14;3,14\}\ ]$

(k) V $f(x)=\sqrt{\dfrac{2x-4}{7+x}}$ $\left[\frac{2x-4}{7+x}\geq 0; D(f)=]-\infty;-7[\cup[2;\infty[\ \right]$

(k) V $f(x) = \dfrac{\ln(x^2-20)}{x-6}$
$\left[x^2-20>0; x-6\neq 0; D(f) =]-\infty; -\sqrt{20}[\cup]\sqrt{20}; \infty[\setminus\{6\}\ \right]$

9. Az alábbi $f : R \to R$ függvényeknek létezik inverze. Határozza meg az inverz függvény hozzárendelési utasítását!

(a) B $f(x) = 4x-7$ $\left[f^{-1}(x) = \frac{x+7}{4}\right]$

(b) B $f(x) = \dfrac{x-5}{4-2x}$ $\left[f^{-1}(x) = \frac{4x+5}{1+2x}\right]$

(c) B $f(x) = \dfrac{3x-4}{x+2}$ $\left[f^{-1}(x) = \frac{2x+4}{3-x}\right]$

(d) B $f(x) = 3(x+4)^7-2$ $\left[f^{-1}(x) = \sqrt[7]{\frac{x+2}{3}}-4\right]$

(e) B $f(x) = 6-5(x-1)^5$ $\left[f^{-1}(x) = \sqrt[5]{\frac{x-6}{-5}}+1\right]$

(f) B $f(x) = 5\sqrt[3]{x-8}+9$ $\left[f^{-1}(x) = \left(\frac{x-9}{5}\right)^3+8\right]$

(g) B $f(x) = -2\sqrt[4]{x+3}-7$ $\left[f^{-1}(x) = \left(\frac{x+7}{-2}\right)^4-3\right]$

(h) B $f(x) = 3\mathrm{e}^{2x-7}+5$ $\left[f^{-1}(x) = \frac{\ln\left(\frac{x-5}{3}\right)+7}{2} = \frac{1}{2}\ln\left(\frac{x-5}{3}\right)+\frac{7}{2}\right]$

(i) B $f(x) = 8^{x+2}-4$ $[f^{-1}(x) = \log_8(x+4)-2]$

(j) B $f(x) = 5^{2-\frac{x}{3}}-1$ $[f^{-1}(x) = -3(\log_5(x+1)-2)]$

(k) B $f(x) = \log_6(4x-8)+3$ $\left[f^{-1}(x) = \frac{6^{x-3}+8}{4}\right]$

(i) B $f(x) = 5\log_2(6-4x)+2$ $\left[f^{-1}(x) = \frac{2^{\frac{x-2}{5}}-6}{-4} = -\frac{1}{4}2^{\frac{x-2}{5}}+\frac{3}{2}\right]$

(l) B $f(x) = 4\ln(3+2x)-7$ $\left[f^{-1}(x) = \frac{\mathrm{e}^{\frac{x+7}{4}}-3}{2} = \frac{1}{2}\mathrm{e}^{\frac{x+7}{4}}-\frac{3}{2}\right]$

10. Határozza meg a következő $f : R \to R$ függvények legbővebb értelmezési tartományát és értékkészletét! Határozza meg az inverz függvényt és annak legbővebb értelmezési tartományát és értékkészletét !

(a) V $f(x) = \ln(2x+5)-8$
$[D(f) =]-2,5; \infty[; R(f) = R; f^{-1}(x) = \frac{\mathrm{e}^{x+8}-5}{2};$
$D(f^{-1}) = R; R(f^{-1}) =]-2,5; \infty[\ ]$

(b) V $f(x) = 4\sqrt{6-2x}+12$
$[D(f) =]-\infty; 3]; R(f) = [12; \infty[; f^{-1}(x) = \frac{\left(\frac{x-12}{4}\right)^2-6}{-2} = -\frac{1}{2}\left(\frac{x-12}{4}\right)^2+3;$
$D(f^{-1}) = [12; \infty[; R(f^{-1}) =]-\infty; 3]\ ]$

(c) V $f(x) = \mathrm{e}^{5x+3}-4$
$[D(f) = R; R(f) =]-4; \infty[; f^{-1}(x) = \frac{\ln(x+4)-3}{5};$
$D(f^{-1}) =]-4; \infty[; R(f^{-1}) = R]$

(d) V $f(x) = \dfrac{3x-15}{4+x}$
$[D(f) = R \setminus \{-4\}; R(f) = R \setminus \{3\}; f^{-1}(x) = \frac{-15-4x}{x-3};$
$D(f^{-1}) = R \setminus \{3\}; R(f^{-1}) = R \setminus \{-4\}]$

(e) V $f(x) = 3 - 2^{3x-5}$
$[D(f) = R; R(f) =]-\infty; 3[; f^{-1}(x) = \frac{\log_2(3-x)+5}{3};$
$D(f^{-1}) =]-\infty; 3[; R(f^{-1}) = R]$

(f) V $f(x) = 4(3x+7)^5 + 6$
$[D(f) = R; R(f) = R; f^{-1}(x) = \frac{\sqrt[5]{\frac{x-6}{4}}-7}{3};$
$D(f^{-1}) = R; R(f^{-1}) = R]$

(g) V $f(x) = 4\log_3(7x-14) + 11$
$[D(f) =]2; \infty[; R(f) = R; f^{-1}(x) = \frac{3^{\frac{x-11}{4}}+14}{7};$
$D(f^{-1}) = R; R(f^{-1}) =]2; \infty[\ ]$