IMPROPRIUS INTEGRÁL

1. $\displaystyle\int_2^\infty \frac{4}{x^3}\,dx$

$$\left[\lim_{b\to\infty}\int_2^b 4x^{-3}\,dx = \lim_{b\to\infty}\left[-2\cdot\frac{1}{x^2}\right]_2^b = 0,5\right]$$

2. $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x}}\,dx$

$$\left[\lim_{b\to\infty}\int_1^b x^{-\frac{3}{2}}\,dx = \lim_{b\to\infty}\left[-2\cdot\frac{1}{\sqrt{x}}\right]_1^b = 2\right]$$

3. $\displaystyle\int_8^\infty \frac{1}{\sqrt[3]{x+4}}\,dx$

$$\left[\lim_{b\to\infty}\int_8^b (x+4)^{-\frac{1}{3}}\,dx = \lim_{b\to\infty}\left[\frac{3}{2}\cdot(x+4)^{\frac{2}{3}}\right]_8^b = \infty(\text{az improprius integrál nem létezik, divergens})\right]$$

4. $\displaystyle\int_0^\infty \mathrm{e}^{-3x}\,dx$

$$\left[\lim_{b\to\infty}\int_0^b \mathrm{e}^{-3x}\,dx = \lim_{b\to\infty}\left[-\frac{1}{3}\cdot\frac{1}{\mathrm{e}^{3x}}\right]_0^b = \frac{1}{3}\right]$$

5. $\displaystyle\int_{-\infty}^1 \frac{1}{\sqrt{2-x}}\,dx$

$$\left[\lim_{a\to-\infty}\int_a^1 (2-x)^{-\frac{1}{2}}\,dx = \lim_{a\to-\infty}\left[-2\sqrt{2-x}\right]_a^1 = \infty(\text{az improprius integrál nem létezik, divergens})\right]$$

6. $\displaystyle\int_{-\infty}^{-1} \frac{3x}{(1+x^2)^4}\,dx$

$$\left[\lim_{a\to-\infty}\int_a^{-1} 3x\cdot(1+x^2)^{-4}\,dx = \lim_{a\to-\infty}\left[3\cdot\frac{1}{2}\cdot\frac{(1+x^2)^{-3}}{-3}\right]_a^{-1} = \lim_{a\to-\infty}\left[-\frac{1}{2}\cdot\frac{1}{(1+x^2)^3}\right]_a^{-1} = -\frac{1}{16}\right]$$

7. $\displaystyle\int_1^\infty \frac{6}{\sqrt[7]{x^{10}}}\,dx$

$$\left[\lim_{b\to\infty}\int_1^b 6x^{-\frac{10}{7}}\,dx = \lim_{b\to\infty}\left[-14\cdot\frac{1}{\sqrt[7]{x^3}}\right]_1^b = 14\right]$$

8. $\displaystyle\int_{-\infty}^6 \frac{6}{7x+11}\,dx$

$$\left[\lim_{a\to-\infty}\int_a^6 \frac{6}{7x+11}\,dx = \lim_{a\to-\infty}\left[\frac{6}{7}\cdot\ln|7x+11|\right]_a^6 = \infty(\text{az improprius integrál nem létezik, divergens})\right]$$

9. $\displaystyle\int_0^\infty \frac{9}{(x+4)^2}\,dx$

$$\left[\lim_{b\to\infty}\int_0^b 9(x+4)^{-2}\,dx = \lim_{b\to\infty}\left[-9\cdot\frac{1}{x+4}\right]_0^b = \frac{9}{4}\right]$$

10. $\displaystyle\int_{-\infty}^{-4} \frac{8}{(3x+9)^5}\,dx$

$$\left[\lim_{a\to-\infty}\int_a^{-4} 8(3x+9)^{-5}\,dx = \lim_{a\to-\infty}\left[8\cdot\frac{1}{3}\cdot\frac{(3x+9)^{-4}}{-4}\right]_a^{-4} = \lim_{a\to-\infty}\left[-\frac{2}{3}\cdot\frac{1}{(3x+9)^4}\right]_a^{-4} = -\frac{2}{243}\right]$$

11. $\displaystyle\int_1^\infty \mathrm{e}^{1-5x}\,dx$

$$\left[\lim_{b\to\infty}\int_1^b \mathrm{e}^{1-5x}\,dx = \lim_{b\to\infty}\left[-\frac{1}{5}\cdot\mathrm{e}^{1-5x}\right]_1^b = -\frac{1}{5}\cdot\frac{1}{\mathrm{e}^4} = 0,003663\right]$$

12. $\displaystyle\int_{-\infty}^0 \mathrm{e}^{4x-3}\,dx$

$$\left[\lim_{a\to-\infty}\int_a^0 \mathrm{e}^{4x-3}\,dx = \lim_{a\to-\infty}\left[\frac{1}{4}\cdot\mathrm{e}^{4x-3}\right]_a^0 = \frac{1}{4}\cdot\frac{1}{\mathrm{e}^3} = 0,012447\right]$$

13. $\displaystyle\int_1^\infty \frac{6}{\sqrt[5]{(x+2)^3}}\,dx$

$$\left[\lim_{b\to\infty}\int_1^b 6(x+2)^{-\frac{3}{5}}\,dx = \lim_{b\to\infty}\left[15(x+2)^{\frac{2}{5}}\right]_1^b = \infty\text{(az improprius integrál nem létezik, divergens)}\right]$$

14. $\displaystyle\int_{-\infty}^\infty \mathrm{e}^{\frac{x}{3}}\,dx$

$$\left[\int \mathrm{e}^{\frac{1}{3}x}\,dx = 3\mathrm{e}^{\frac{1}{3}x}+c\right]$$

$$\left[\lim_{b\to\infty}3\mathrm{e}^{\frac{1}{3}b} - \lim_{a\to-\infty}3\mathrm{e}^{\frac{1}{3}a} = \infty\text{(az improprius integrál nem létezik, divergens)}\right]$$

15. Határozza meg a $\displaystyle\int_{-\infty}^\infty f(x)\,dx$ improprius integrált, ha

$$f(x) = \begin{cases} 0 & \text{ha } x \le 0 \\ \frac{1}{(x+1)^2} & \text{ha } x > 0 \end{cases}$$

$$\left[\int_{-\infty}^0 0\,dx + \int_0^\infty \frac{1}{(x+1)^2}\,dx = \lim_{b\to\infty}\int_0^b (x+1)^{-2}\,dx = \lim_{b\to\infty}\left[-\frac{1}{x+1}\right]_0^b = 1\right]$$

16. Határozza meg a $\displaystyle\int_{-\infty}^\infty f(x)\,dx$ improprius integrált, ha

$$f(x) = \begin{cases} \frac{x+5}{4} & \text{ha } 2 < x < 10 \\ 0 & \text{különben} \end{cases}$$

$$\left[\int_{-\infty}^{2} 0\,dx + \int_{2}^{10} \left(\frac{1}{4}x + \frac{5}{4}\right)\,dx + \int_{10}^{\infty} 0\,dx = \left[\frac{1}{8}x^2 + \frac{5}{4}x\right]_2^{10} = 22\right]$$

17. Határozza meg a $\int_{-\infty}^{\infty} f(x)\,dx$ improprius integrált, ha

$$f(x) = \begin{cases} 0 & \text{ha } x \leq -1 \\ \mathrm{e}^{2-2x} & \text{ha } x > -1 \end{cases}$$

$$\left[\int_{-\infty}^{-1} 0\,dx + \int_{-1}^{\infty} \mathrm{e}^{2-2x}\,dx = \lim_{b\to\infty}\int_{-1}^{b} \mathrm{e}^{2-2x}\,dx = \lim_{b\to\infty}\left[-\frac{1}{2}\mathrm{e}^{2-2x}\right]_{-1}^{b} = \frac{1}{2}\mathrm{e}^4 = 27,299\right]$$

18. $\displaystyle\int_0^{\infty} x\cdot \mathrm{e}^{-x^2}\,dx$

$$\left[\lim_{b\to\infty}\int_0^b x\cdot\mathrm{e}^{-x^2}\,dx = \lim_{b\to\infty}\left[-\frac{1}{2}\cdot\mathrm{e}^{-x^2}\right]_0^b = \frac{1}{2}\right]$$

19. $\displaystyle\int_2^{\infty} \frac{1}{x\cdot\ln^3 x}\,dx$

$$\left[\lim_{b\to\infty}\int_2^b \frac{1}{x}\cdot(\ln x)^{-3}\,dx = \lim_{b\to\infty}\left[\frac{(\ln x)^{-2}}{-2}\right]_2^b = \lim_{b\to\infty}\left[-\frac{1}{2}\cdot\frac{1}{(\ln x)^2}\right]_2^b = \frac{1}{2}\cdot\frac{1}{(\ln 2)^2} = 1,0407\right]$$

20. $\displaystyle\int_{-\infty}^{0} \frac{\mathrm{e}^{3x}}{1+\mathrm{e}^{3x}}\,dx$

$$\left[\lim_{a\to-\infty}\int_a^0 \frac{\mathrm{e}^{3x}}{1+\mathrm{e}^{3x}}\,dx = \lim_{a\to-\infty}\left[\frac{1}{3}\cdot\ln|1+\mathrm{e}^{3x}|\right]_a^0 = \frac{1}{3}\cdot\ln 2 = 0,23105\right]$$

21. $\displaystyle\int_{-\infty}^{-2} \frac{x}{\sqrt[5]{(1+x^2)^6}}\,dx$

$$\left[\lim_{a\to-\infty}\int_a^{-2} x\cdot(1+x^2)^{-\frac{6}{5}}\,dx = \lim_{a\to-\infty}\left[\frac{1}{2}\cdot\frac{(1+x^2)^{-\frac{1}{5}}}{-\frac{1}{5}}\right]_a^{-2} = \lim_{a\to-\infty}\left[-\frac{5}{2}\cdot\frac{1}{\sqrt[5]{(1+x^2)}}\right]_a^{-2} - 1,8119\right]$$