Integrálszámítás

2014. november 21.

Határozza meg a következő határozatlan integrálokat!

1. **Feladat:**

$$\boxed{\int (5x^4 + 3x^3 - 14x - 4)\mathrm{d}x =}$$

Megoldás:

$$\int (5x^4+3x^3-14x-4)\mathrm{d}x = 5\frac{x^5}{5}+3\frac{x^4}{4}-14\frac{x^2}{2}-4x+c = x^5+3\frac{x^4}{4}-7x^2-4x+c$$

2. **Feladat:**

$$\boxed{\int (7\sqrt[5]{x} + \sqrt{x^3} - \sqrt{14})\mathrm{d}x =}$$

Megoldás:

$$\int (7\sqrt[5]{x}+\sqrt{x^3}-\sqrt{14})\mathrm{d}x = \int (7x^{\frac{1}{5}}+x^{\frac{3}{2}}-\sqrt{14})\mathrm{d}x = 7\frac{x^{\frac{6}{5}}}{\frac{6}{5}}+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}-\sqrt{14}x+c =$$

$$= 7\frac{5}{6}\sqrt[5]{x^6} + \frac{2}{5}\sqrt{x^5} - \sqrt{14}x + c = \frac{35}{6}\sqrt[5]{x^6} + \frac{2}{5}\sqrt{x^5} - \sqrt{14}x + c$$

3. **Feladat:**

$$\boxed{\int \left(\frac{11}{x} - \frac{\sqrt{3}}{x^4} + \frac{7}{\sqrt{x}} - \frac{2}{\sqrt[7]{x^4}}\right)\mathrm{d}x =}$$

Megoldás:

$$\int \left(\frac{11}{x} - \frac{\sqrt{3}}{x^4} + \frac{7}{\sqrt{x}} - \frac{2}{\sqrt[7]{x^4}}\right)\mathrm{d}x = \int \left(11\frac{1}{x} - \sqrt{3}x^{-4} + 7x^{-\frac{1}{2}} - 2x^{-\frac{4}{7}}\right)\mathrm{d}x =$$

$$= 11\ln|x|-\sqrt{3}\frac{x^{-3}}{-3}+7\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-2\frac{x^{\frac{3}{7}}}{\frac{3}{7}}+c = 11\ln|x|+\sqrt{3}\frac{1}{3x^3}+14\sqrt{x}-\frac{14}{3}\sqrt[7]{x^3}+c$$

4. **Feladat:**

$$\boxed{\int \frac{x^2+3x-5}{4x^3}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{x^2+3x-5}{4x^3}\mathrm{d}x = \frac{1}{4}\int \left(\frac{1}{x} + 3\cdot\frac{1}{x^2} - 5\frac{1}{x^3}\right)\mathrm{d}x =$$

$$= \frac{1}{4}\int \left(\frac{1}{x} + 3\cdot x^{-2} - 5x^{-3}\right)\mathrm{d}x = \frac{1}{4}\left(\ln|x| + 3\cdot\frac{x^{-1}}{-1} - 5\frac{x^{-2}}{-2}\right) + c =$$

$$= \frac{1}{4}\left(\ln|x| - \frac{3}{x} + \frac{5}{2}\frac{1}{x^2}\right) + c$$

5. **Feladat:**

$$\boxed{\int (x^2+3)^2(4-x)\mathrm{d}x =}$$

Megoldás:

$$\int (x^2+3)^2(4-x)\mathrm{d}x = \int (x^4+6x^2+9)(4-x)\mathrm{d}x =$$

$$\int (-x^5+4x^4-6x^3+24x^2-9x+36)\mathrm{d}x = -\frac{x^6}{6}+4\frac{x^5}{5}-6\frac{x^4}{4}+24\frac{x^3}{3}-9\frac{x^2}{2}+36x+c$$

6. **Feladat:**

$$\boxed{\int \frac{(4x+1)(5-x)}{\sqrt[3]{x}}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{(4x+1)(5-x)}{\sqrt[3]{x}}\mathrm{d}x = \int \frac{-4x^2+19x+5}{x^{\frac{1}{3}}}\mathrm{d}x =$$

$$\int \left(-4x^{\frac{5}{3}} + 19x^{\frac{2}{3}} + 5x^{-\frac{1}{3}}\right)\mathrm{d}x = -\frac{3}{2}\sqrt[3]{x^8} + \frac{57}{5}\sqrt[3]{x^5} + \frac{15}{2}\sqrt[3]{x^2} + c$$

7. **Feladat:**

$$\boxed{\int \frac{\sqrt{x\sqrt[3]{x\sqrt[5]{x}}}}{x^2}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{\sqrt{x\sqrt[3]{x\sqrt[5]{x}}}}{x^2}\mathrm{d}x = \int \frac{\sqrt{\sqrt[3]{\sqrt[5]{x^{21}}}}}{x^2}\mathrm{d}x = \int \frac{\sqrt[30]{x^{21}}}{x^2}\mathrm{d}x = \int x^{-\frac{13}{10}}\mathrm{d}x = -\frac{10}{3\sqrt[10]{x^3}}+c$$

8. **Feladat:**

$$\boxed{\int \left(\frac{x}{5}+\frac{5}{x}\right)^2 \mathrm{d}x =}$$

Megoldás:

$$\int \left(\frac{x}{5}+\frac{5}{x}\right)^2 \mathrm{d}x = \int \left(\frac{x^2}{25}+2\frac{x}{5}\cdot\frac{5}{x}+\frac{25}{x^2}\right)\mathrm{d}x = \int \left(\frac{1}{25}x^2+2+25\frac{1}{x^2}\right)\mathrm{d}x =$$

$$= \frac{1}{25}\frac{x^3}{3}+2x-25\frac{1}{x}+c = \frac{x^3}{75}-\frac{25}{x}+2x+c$$

9. **Feladat:**

$$\boxed{\int \frac{(2x+3)^7}{3}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{(2x+3)^7}{3}\mathrm{d}x = \frac{1}{3}\int (2x+3)^7\mathrm{d}x = \frac{1}{3}\frac{(2x+3)^8}{8\cdot 2}+c = \frac{1}{48}(2x+3)^8+c$$

10. **Feladat:**

$$\boxed{\int e^{6x-5}\mathrm{d}x = \cdots = \frac{1}{3}\ln(9x^2+2)+c}$$

Megoldás:

$$\int e^{6x-5}\mathrm{d}x = \frac{1}{6}\int \underbrace{6}_{g'}\cdot\underbrace{e^{6x-5}}_{f(g)}\mathrm{d}x = \frac{1}{6}e^{6x-5}+c$$

11. **Feladat:**

$$\boxed{\int \frac{\sin(\frac{x}{3})}{4}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{\sin(\frac{x}{3})}{4}\mathrm{d}x = \frac{3}{4}\int \underbrace{\frac{1}{3}}_{g'}\cdot\underbrace{\sin\left(\frac{x}{3}\right)}_{f(g)}\mathrm{d}x = -\frac{3}{4}\underbrace{\cos\left(\frac{x}{3}\right)}_{F(g)}+c$$

12. **Feladat:**

$$\boxed{\int \frac{\sin 5x}{\cos 5x}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{\sin 5x}{\cos 5x}\mathrm{d}x = -\frac{1}{5}\int \frac{-5\sin 5x}{\cos 5x}\mathrm{d}x \overset{\frac{f'}{f}}{=} -\frac{1}{5}\ln|\cos 5x|+c$$

13. **Feladat:**

$$\boxed{\int \frac{2}{(x+4)^3}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{2}{(x+4)^3}\mathrm{d}x = 2\int (x+4)^{-3}\mathrm{d}x = 2\frac{(x+4)^{-2}}{-2} = -\frac{1}{(x+4)^2} + c$$

14. **Feladat:**

$$\boxed{\int \frac{-x}{\sqrt{x^2-5}}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{-x}{\sqrt{x^2-5}}\mathrm{d}x = -\frac{1}{2}\int \underbrace{2x}_{g'}\underbrace{(x^2-5)^{-\frac{1}{2}}}_{f(g)}\mathrm{d}x = -\frac{1}{2}\frac{(x^2-5)^{\frac{1}{2}}}{\frac{1}{2}} = -\sqrt{x^2-5}+c$$

15. **Feladat:**

$$\boxed{\int \frac{\mathrm{tg}\ ^5(2x)}{\cos^2(2x)}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{\mathrm{tg}\ ^5(2x)}{\cos^2(2x)}\mathrm{d}x = \int \underbrace{(\mathrm{tg}\ (2x))^5}_{f(g)}\underbrace{\frac{1}{\cos^2(2x)}}_{g'}\mathrm{d}x = \frac{1}{12}\mathrm{tg}\ ^6(2x) + c$$

16. **Feladat:**

$$\boxed{\int \frac{7}{\sqrt{7-9x}}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{7}{\sqrt{7-9x}}\mathrm{d}x = -\frac{7}{9}\int \underbrace{(-9)}_{g'}\underbrace{(7-9x)^{-\frac{1}{2}}}_{f(g)}\mathrm{d}x =$$

$$= -\frac{7}{9}\frac{(7-9x)^{\frac{1}{2}}}{\frac{1}{2}} + c = -\frac{14}{9}\sqrt{7-9x} + c$$

17. **Feladat:**

$$\boxed{\int (x^3+6x+10)^{10}(x^2+2)\mathrm{d}x =}$$

Megoldás:

$$\int (x^3+6x+10)^{10}(x^2+2)\mathrm{d}x = \frac{1}{3}\int \underbrace{(x^3+6x+10)^{10}}_{f(g)}\underbrace{(3x^2+6)}_{g'}\mathrm{d}x =$$

$$= \frac{1}{3}\frac{(x^3+6x+10)^{11}}{11} + c = \frac{1}{33}(x^3+6x+10)^{11} + c$$

18. **Feladat:**

$$\boxed{\int \sqrt[5]{(11+3x)^3}\mathrm{d}x =}$$

Megoldás:

$$\int \sqrt[5]{(11+3x)^3}\mathrm{d}x = \int (11+3x)^{\frac{3}{5}}\mathrm{d}x = \frac{1}{3}\int \underbrace{3}_{g'}\cdot\underbrace{(11+3x)^{\frac{3}{5}}}_{f(g)}\mathrm{d}x =$$

$$= \frac{1}{3}\frac{(11+3x)^{\frac{8}{5}}}{\frac{8}{5}} + c = \frac{5}{24}(11+3x)^{\frac{8}{5}} + c$$

19. **Feladat:**

$$\boxed{\int \sin(3x^2+6x)(x+1)\mathrm{d}x =}$$

Megoldás:

$$\int \sin(3x^2+6x)(x+1)\mathrm{d}x = \frac{1}{6}\int \underbrace{\sin(3x^2+6x)}_{f(g)}\underbrace{(6x+6)}_{g'}\mathrm{d}x =$$

$$= -\frac{1}{6}\cos(3x^2+6x) + c$$

20. **Feladat:**

$$\boxed{\int \frac{5}{7x-3}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{5}{7x-3}\mathrm{d}x = 5\int \frac{1}{7x-3}\mathrm{d}x = \frac{5}{7}\int \frac{7}{7x-3}\mathrm{d}x \overset{\frac{f'}{f}}{=} \frac{5}{7}\ln|7x-3| + c$$

21. **Feladat:**

$$\boxed{\int \frac{x^2-1}{3x-x^3}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{x^2-1}{3x-x^3}\mathrm{d}x = -\frac{1}{3}\int \frac{3-3x^2}{3x-x^3}\mathrm{d}x \overset{\frac{f'}{f}}{=} -\frac{1}{3}\ln|3x-x^3| + c$$

22. **Feladat:**

$$\boxed{\int \frac{6x-2e^{-x}}{9x^2+3e^{-x}}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{6x-2e^{-x}}{9x^2+3e^{-x}}\mathrm{d}x = 2\int \frac{3x-e^{-x}}{9x^2+3e^{-x}}\mathrm{d}x =$$

$$= \frac{2}{3}\int \frac{3(3x-e^{-x})}{9x^2+3e^{-x}}\mathrm{d}x \overset{\frac{f'}{f}}{=} \frac{2}{3}\ln|9x^2+3e^{-x}|+c$$

23. **Feladat:**

$$\boxed{\int (5x+3)e^x\mathrm{d}x =}$$

Megoldás:

$$\int (5x+3)e^x\mathrm{d}x = (5x+3)e^x - \int 5e^x\mathrm{d}x = (5x+3)e^x - 5e^x + c = (5x-2)e^x + c$$

$$f' = e^x \qquad f = e^x$$
$$g = 5x+3 \qquad g' = 5$$

24. **Feladat:**

$$\boxed{\int x^2\sin x\mathrm{d}x =}$$

Megoldás:

$$\int x^2\sin x\mathrm{d}x = (-\cos x)\,x^2 - \int (-\cos x)\,2x\mathrm{d}x =$$

$$f' = \sin x \qquad f = -\cos x$$
$$g = x^2 \qquad g' = 2x$$

$$= -x^2\cos x + \int 2x\cos x\mathrm{d}x = -x^2\cos x + (2x\sin x - \int 2\sin x\mathrm{d}x) =$$

$$f' = \cos x \qquad f = \sin x$$
$$g = 2x \qquad g' = 2$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + c$$

25. **Feladat:**

$$\boxed{\int (3x+7)\cos x \mathrm{d}x =}$$

Megoldás:

$$\int (3x+7)\cos x \mathrm{d}x = (3x+7)\sin x - \int 3\sin x \mathrm{d}x = (3x+7)\sin x + 3\cos x + c$$

$$f' = \cos x \qquad f = \sin x$$
$$g = 3x+7 \qquad g' = 3$$

26. **Feladat:**

$$\boxed{\int x^4 \ln x \mathrm{d}x =}$$

Megoldás:

$$\int x^4 \ln x \mathrm{d}x = \frac{x^5}{5}\ln x - \int \frac{x^5}{5}\cdot\frac{1}{x}\mathrm{d}x = \frac{x^5}{5}\ln x - \frac{1}{5}\int x^4 \mathrm{d}x =$$

$$f' = x^4 \qquad f = \frac{x^5}{5}$$
$$g = \ln x \qquad g' = \frac{1}{x}$$

$$= \frac{x^5}{5}\ln x - \frac{1}{5}\cdot\frac{x^5}{5} + c = \frac{x^5}{5}\ln x - \frac{x^5}{25} + c$$

27. **Feladat:**

$$\boxed{\int 5\log_3 x \mathrm{d}x =}$$

Megoldás:

$$\int 5\log_3 x \mathrm{d}x = 5x\cdot\log_3 x - \int \underbrace{5x\cdot\frac{1}{x\ln 3}}_{\frac{5}{\ln 3}} \mathrm{d}x = 5x\cdot\log_3 x - \frac{5}{\ln 3}x + c$$

$$f' = 5 \qquad f = 5x$$
$$g = \log_3 x \qquad g' = \frac{1}{x\ln 3}$$

28. **Vizsgafeladat:** Végezze el a kijelölt műveleteket!

(a)

$$\boxed{\int \left(\sqrt{4x-11}+(6x-5)^7\right) \mathrm{d}x =}$$

Megoldás:

$$\int \left(\sqrt{4x-11}+(6x-5)^7\right) \mathrm{d}x = \int \sqrt{4x-11}\mathrm{d}x + \int (6x-5)^7 \mathrm{d}x =$$

$$= \int (4x-11)^{\frac{1}{2}}\mathrm{d}x + \int (6x-5)^7\mathrm{d}x = \frac{1}{4}\int \underbrace{4}_{g'}\underbrace{(4x-11)^{\frac{1}{2}}}_{f(g)}\mathrm{d}x + \frac{1}{6}\int \underbrace{6}_{g'}\underbrace{(6x-5)^7}_{f(g)}\mathrm{d}x =$$

$$= \frac{1}{4}\frac{(4x-11)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{6}\frac{(6x-5)^8}{8} + c = \frac{1}{6}(4x-11)^{\frac{3}{2}} + \frac{1}{48}(6x-5)^8 + c$$

(b)

$$\boxed{\int \left(\frac{2}{4-x}+\frac{x}{2x^2+7}-\frac{\cos x}{\sin^5 x}\right) \mathrm{d}x =}$$

Megoldás:

$$\int \left(\frac{2}{4-x}+\frac{x}{2x^2+7}-\frac{\cos x}{\sin^5 x}\right) \mathrm{d}x = \int \frac{2}{4-x}\mathrm{d}x + \int \frac{x}{2x^2+7}\mathrm{d}x - \int \frac{\cos x}{\sin^5 x}\mathrm{d}x =$$

$$= -2\int \frac{\overbrace{-1}^{f'}}{\underbrace{4-x}_{f}}\mathrm{d}x + \frac{1}{4}\int \frac{\overbrace{4x}^{f'}}{\underbrace{2x^2+7}_{f}}\mathrm{d}x - \int \underbrace{\sin^{-5} x}_{f(g)}\underbrace{\cos x}_{g'}\mathrm{d}x =$$

$$= -2\ln|4-x| + \frac{1}{4}\ln|2x^2+7| + \frac{\sin^{-4} x}{4} + c$$

(c)

$$\boxed{\int (x+2)^2 \cos\left(\frac{x}{2}\right) \mathrm{d}x =}$$

Megoldás:

$$\int (x+2)^2 \cos\left(\frac{x}{2}\right) \mathrm{d}x = 2(x+2)^2 \sin\left(\frac{x}{2}\right) - \int 4(x+2)\sin\left(\frac{x}{2}\right) \mathrm{d}x =$$

$$f' = \cos\left(\frac{x}{2}\right) \qquad f = \frac{\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} = 2\sin\left(\frac{x}{2}\right)$$

$$g = (x+2)^2 \qquad g' = 2(x+2)$$

$$= 2(x+2)^2 \sin\left(\frac{x}{2}\right) - \left[-8(x+2)\cos\left(\frac{x}{2}\right) + \int 8\cos\left(\frac{x}{2}\right)\right] =$$

$$f' = \sin\left(\frac{x}{2}\right) \qquad f = -2\cos\left(\frac{x}{2}\right)$$
$$g = 4(x+2) \qquad g' = 4$$

$$= 2(x+2)^2 \sin\left(\frac{x}{2}\right) + 8(x+2)\cos\left(\frac{x}{2}\right) - \int 8\cos\left(\frac{x}{2}\right) =$$

$$= 2(x+2)^2 \sin\left(\frac{x}{2}\right) + 8(x+2)\cos\left(\frac{x}{2}\right) - 16\sin\left(\frac{x}{2}\right) + c$$

(d)

$$\boxed{\int (3x^2+x)\sin(4x)\mathrm{d}x =}$$

Megoldás:

$$\int (3x^2+x)\sin(4x)\mathrm{d}x = -(3x^2+x)\frac{\cos(4x)}{4} + \int (6x+1)\frac{\cos(4x)}{4}\mathrm{d}x =$$

$$f' = \sin(4x) \qquad f = \frac{-\cos(4x)}{4}$$
$$g = 3x^2 + x \qquad g' = 6x+1$$

$$= -(3x^2+x)\frac{\cos(4x)}{4} + \frac{1}{4}\int (6x+1)\cos(4x)\mathrm{d}x =$$

$$= -(3x^2+x)\frac{\cos(4x)}{4} + \frac{1}{4}\left((6x+1)\frac{\sin(4x)}{4} - \int 6\frac{\sin(4x)}{4}\mathrm{d}x\right) =$$

$$f' = \cos(4x) \qquad f = \frac{\sin(4x)}{4}$$
$$g = 6x+1 \qquad g' = 6$$

$$= -(3x^2+x)\frac{\cos(4x)}{4} + \frac{1}{16}(6x+1)\sin(4x) + \frac{3}{8}\frac{\cos(4x)}{4} + c$$

(e)

$$\boxed{\int \frac{x}{3}5^{3-x}\mathrm{d}x =}$$

Megoldás:

$$\int \frac{x}{3}5^{3-x}\mathrm{d}x = \int \frac{x}{3}5^{3-x}\mathrm{d}x = -\frac{x}{3}\cdot\frac{5^{3-x}}{\ln 5} + \int \frac{1}{3}\cdot\frac{5^{3-x}}{\ln 5}\mathrm{d}x =$$

$$f' = 5^{3-x} \qquad f = \frac{5^{3-x}}{(-1)\ln 5} = -\frac{5^{3-x}}{\ln 5}$$

$$g = \frac{x}{3} = \frac{1}{3}x \qquad g' = \frac{1}{3}$$

$$= -\frac{x}{3} \cdot \frac{5^{3-x}}{\ln 5} + \int \frac{1}{3} \cdot \frac{5^{3-x}}{\ln 5} \mathrm{d}x = -\frac{x}{3\ln 5} 5^{3-x} + \frac{1}{3\ln 5} \cdot \frac{5^{3-x}}{-\ln 5} + c =$$

$$= -\frac{x}{3\ln 5} 5^{3-x} - \frac{1}{3\ln^2 5} 5^{3-x} + c$$

(f)

$$\boxed{\int (7x+13)e^{-2x} \mathrm{d}x =}$$

Megoldás:

$$\int (7x+13)e^{-2x} \mathrm{d}x = -(7x+13)\frac{e^{-2x}}{2} + \int 7\frac{e^{-2x}}{2} \mathrm{d}x =$$

$$= -(7x+13)\frac{e^{-2x}}{2} + \frac{7}{2}\frac{e^{-2x}}{-2} + c == -(7x+13)\frac{e^{-2x}}{2} - \frac{7}{4}e^{-2x} + c$$

$$f' = e^{-2x} \qquad f = \frac{e^{-2x}}{-2}$$

$$g = 7x + 13 \qquad g' = 7$$

(g)

$$\boxed{\int (x^4 + 7x^2) \ln(x) \mathrm{d}x =}$$

Megoldás:

$$\int (x^4+7x^2) \ln(x) \mathrm{d}x = \left(\frac{x^5}{5} + 7\frac{x^3}{3}\right) \ln x - \int \left(\frac{x^5}{5} + 7\frac{x^3}{3}\right) \frac{1}{x} \mathrm{d}x =$$

$$f' = x^4 + 7x^2 \qquad f = \frac{x^5}{5} + 7\frac{x^3}{3}$$

$$g = \ln x \qquad g' = \frac{1}{x}$$

$$= \left(\frac{x^5}{5} + 7\frac{x^3}{3}\right) \ln x - \int \left(\frac{x^4}{5} + 7\frac{x^2}{3}\right) \mathrm{d}x = \left(\frac{x^5}{5} + 7\frac{x^3}{3}\right) \ln x - \frac{x^5}{25} - 7\frac{x^3}{9} + c$$

(h)

$$\boxed{\int \ln^2 x \mathrm{d}x =}$$

Megoldás:

$$\int \ln^2 x \mathrm{d}x = x\ln^2 x - \int x \cdot 2\ln x \cdot \frac{1}{x}\mathrm{d}x = x\ln^2 x - \int 2\ln x \mathrm{d}x =$$

$$f' = 1 \qquad f = x$$
$$g = \ln^2 x \qquad g' = 2\ln x \cdot \frac{1}{x}$$

$$= x\ln^2 x - \left(2x\ln x - \int \underbrace{2x \cdot \frac{1}{x}}_{2}\mathrm{d}x\right) = x\ln^2 x - 2x\ln x + 2x + c$$

$$f' = 2 \qquad f = 2x$$
$$g = \ln x \qquad g' = \frac{1}{x}$$

(i)

$$\boxed{\int \frac{2}{x^2}\ln(5x)\mathrm{d}x =}$$

Megoldás:

$$\int \frac{2}{x^2}\ln(5x)\mathrm{d}x = -\frac{2}{x}\ln(5x) + \int \underbrace{\frac{2}{x} \cdot \frac{1}{x}}_{2x^{-2}}\mathrm{d}x = -\frac{2}{x}\ln(5x) + 2\frac{x^{-1}}{-1} + c = -\frac{2}{x}\ln(5x) - \frac{2}{x} + c$$

$$f' = \frac{2}{x^2} = 2x^{-2} \qquad f = 2\frac{x^{-1}}{-1} = -\frac{2}{x}$$
$$g = \ln(5x) \qquad g' = \frac{1}{5x} \cdot 5 = \frac{1}{x}$$