1. $w=z^4,\ w=a+bi \quad\rightarrow\quad i(a+bi)-2(a-bi)=1-2i \quad\rightarrow\quad (-b-2a)+(a+2b)i=1-2i$

$$\begin{matrix}-b-2a=1\\ a+2b=-2\end{matrix} \quad\rightarrow\quad \begin{matrix}a=0\\ b=-1\end{matrix} \quad\rightarrow\quad w=-i=1(\cos(-90°)+i\sin(-90°))$$

$$\boxed{z_{1,2,3,4}=\sqrt[4]{w}=1(\cos(-22,5°+k\cdot 90°)+i\sin(-22,5°+k\cdot 90°)),\quad k=0,1,2,3}$$

2. $$a_{n+1}-a_n=\frac{\overbrace{(n+1)-1}^{n}}{\underbrace{2-3(n+1)}_{-1-3n}}-\frac{n-1}{2-3n}=\frac{n(2-3n)-(n-1)(-1-3n)}{(-1-3n)(2-3n)}$$

$$=\frac{-1}{\underbrace{(-1-3n)}_{<0}\underbrace{(2-3n)}_{<0}}<0 \quad\rightarrow\quad \boxed{\text{A sorozat szigorúan monoton csökkenő.}}$$

$$\lim_{n\to\infty}\frac{n-1}{2-3n}=\lim_{n\to\infty}\frac{1-\frac{1}{n}}{\frac{2}{n}-3}=\boxed{-\frac{1}{3}=A}$$

$$|a_n-A|<\varepsilon \quad\rightarrow\quad \left|\frac{n-1}{2-3n}+\frac{1}{3}\right|<\frac{1}{20} \quad\rightarrow\quad \frac{|-1|}{|2-3n|\cdot|3|}<\frac{1}{20} \quad\rightarrow$$

$$\frac{1}{(3n-2)\cdot 3}<\frac{1}{20} \quad\rightarrow\quad \frac{20}{3}<3n-2 \quad\rightarrow\quad n>\frac{26}{9} \quad\rightarrow\quad \boxed{N=2}$$

3. (a) $$\lim_{x\to-1}\underbrace{\frac{e^{x+1}-1}{-3x-3}}_{„0/0"}=\lim_{x\to-1}\frac{e^{x+1}}{-3}=\boxed{-\tfrac{1}{3}}$$

(b) $$\lim_{x\to\infty}\frac{3\cdot 8^{x+1}-3^{x-2}}{5^{x+1}-2^{3x+1}}=\lim_{x\to\infty}\frac{24\cdot 8^x-\frac{1}{9}\cdot 3^x}{5\cdot 5^x-2\cdot 8^x}=\lim_{x\to\infty}\frac{24-\frac{1}{9}\cdot\frac{3}{8}^x}{5\cdot\frac{5}{8}^x-2}=\frac{24}{-2}=\boxed{-12}$$

4. (a) $$t=\int_0^{\pi/2}\sin(x)dx=[-\cos(x)]_0^{\pi/2}=-\cos(\pi/2)+\cos(0)=\boxed{1}$$

(b) $$T_3(\sin,0,x)=\frac{\sin(0)}{0!}x^0+\frac{\cos(0)}{1!}x^1+\frac{-\sin(0)}{2!}x^2+\frac{-\cos(0)}{3!}x^3=x-\frac{1}{6}x^3$$

$$t=\int_0^{\pi/2}x-\frac{1}{6}x^3dx=\left[\frac{x^2}{2}-\frac{x^4}{24}\right]_0^{\pi/2}=\boxed{\frac{\pi^2}{8}-\frac{\pi^4}{24\cdot 16}}\approx 0,98$$

5. $D_f=\mathbb{R},\ f(x)=(x^2+1)e^{-x},\ f'(x)=2xe^{-x}+(x^2+1)e^{-x}(-1)=(2x-x^2-1)e^{-x}$

$f''(x)=(2-2x)e^{-x}+(2x-x^2-1)e^{-x}(-1)=(x^2-4x+3)e^{-x}=0 \quad\rightarrow\quad x_1=1,\ x_2=3$

	$x<1$	$x=1$	$1<x<3$	$x=3$	$x>3$
f''	+	0	−	0	+
f	konvex	infl. pont $f(1)=\frac{2}{e}$	konkáv	infl. pont $f(3)=\frac{10}{e^3}$	konvex

6. (a) $$\int_0^1 1+\frac{x^2}{\sqrt{x^3+2}}dx=[x]_0^1+\frac{1}{3}\int_0^1 3x^2(x^3+2)^{-1/2}dx=1+\frac{1}{3}\left[\frac{(x^3+2)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1=\boxed{1+\tfrac{2}{3}\left(\sqrt{3}-\sqrt{2}\right)}\approx 1,21$$

(b) $$V=\pi\int_0^{\frac{\pi}{2}}\underbrace{\cos(x)}_{g'}\cdot\underbrace{x}_{f}dx=\pi\left([x\sin(x)]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\sin(x)dx\right)=\pi\left(\frac{\pi}{2}-[-\cos(x)]_0^{\frac{\pi}{2}}\right)=\boxed{\frac{\pi^2}{2}-\pi}\approx 1,79$$

7. (a) $$y'\underbrace{+2}_{a(x)}y=\underbrace{\frac{x}{2}}_{b(x)} \quad\rightarrow\quad A(x)=2x,\ y(x)=c(x)e^{-A(x)}=c(x)e^{-2x}$$

$$c(x)=\int e^{A(x)}b(x)dx=\frac{1}{2}\int\underbrace{e^{2x}}_{g'}\cdot\underbrace{x}_{f}dx=\frac{1}{2}\left(x\frac{e^{2x}}{2}-\int\frac{e^{2x}}{2}dx\right)=\frac{1}{2}\left(x\frac{e^{2x}}{2}-\frac{e^{2x}}{4}+K\right)$$

$$\boxed{y(x)=\frac{x}{4}-\frac{1}{8}+\frac{K}{2}e^{-2x}}$$

(b) $y_{(h,a)}: \; 9\lambda^2 - 6\lambda + 1 = 0 \quad \rightarrow \quad \lambda_1 = \dfrac{1}{3}$

$y_{(i,p)}: \; y = Ax + B, \; y' = A, \; y'' = 0 \quad \rightarrow \quad 0 - 6A + (Ax + B) = 2x \quad \rightarrow \quad A = 2, \; B = 12$

$$\boxed{y(x) = C_1 e^{\frac{1}{3}x} + C_2 x e^{\frac{1}{3}x} + 2x + 12}$$