

$$1. w = z^3, w = a + bi \rightarrow 3(a + bi) - i(a - bi) = 2 + 2i \rightarrow (3a - b) + (3b - a)i = 2 + 2i$$

$$\begin{array}{l} 3a - b = 2 \\ 3b - a = 2 \end{array} \rightarrow \begin{array}{l} a = 1 \\ b = 1 \end{array} \rightarrow w = 1 + i = \sqrt{2}(\cos(45^\circ) + i \sin(45^\circ))$$

$$z_{1,2,3} = \sqrt[3]{w} = \sqrt[6]{2}(\cos(15^\circ + k \cdot 120^\circ) + i \sin(15^\circ + k \cdot 120^\circ)), \quad k = 0, 1, 2$$

$$2. a_{n+1} - a_n = \frac{\overbrace{5 - 2(n+1)}^{3-2n}}{\underbrace{3 - 4(n+1)}_{-1-4n}} - \frac{5 - 2n}{3 - 4n} = \frac{(3 - 2n)(3 - 4n) - (5 - 2n)(-1 - 4n)}{(-1 - 4n)(3 - 4n)}$$

$$= \frac{14}{\underbrace{(-1 - 4n)}_{<0} \underbrace{(3 - 4n)}_{<0}} > 0 \rightarrow \text{A sorozat szigorúan monoton növekvő.}$$

$$\lim_{n \rightarrow \infty} \frac{5 - 2n}{3 - 4n} = \lim_{n \rightarrow \infty} \frac{\frac{5}{n} - 2}{\frac{3}{n} - 4} = \frac{1}{2} = A$$

$$|a_n - A| < \varepsilon \rightarrow \left| \frac{5 - 2n}{3 - 4n} - \frac{1}{2} \right| < 0,001 \rightarrow \frac{|7|}{|3 - 4n| \cdot |2|} < 0,001 \rightarrow$$

$$\frac{7}{(4n - 3) \cdot 2} < 0,001 \rightarrow \frac{7000}{2} < 4n - 3 \rightarrow n > 875,75 \rightarrow N = 875$$

$$3. (a) \lim_{x \rightarrow \infty} \frac{4 \cdot 5^{x-1} + 9^{x+1}}{6^{x+2} - 3^{2x-1}} = \lim_{x \rightarrow \infty} \frac{\frac{4}{5} \cdot 5^x + 9 \cdot 9^x}{36 \cdot 6^x - \frac{1}{3} \cdot 9^x} = \lim_{x \rightarrow \infty} \frac{\frac{4}{5} \cdot \left(\frac{5}{9}\right)^x + 9}{36 \cdot \left(\frac{6}{9}\right)^x - \frac{1}{3}} = \frac{9}{-\frac{1}{3}} = -27$$

$$(b) \lim_{x \rightarrow \infty} \left(\frac{2x^2 - 3}{2x^2 + 1} \right)^{3x^2} = \lim_{x \rightarrow \infty} \underbrace{\left[\left(1 + \frac{-4}{2x^2 + 1} \right)^{2x^2 + 1} \right]}_{\rightarrow e^{-4}} \underbrace{\left(\frac{3x^2}{2x^2 + 1} \right)^{-\frac{3}{2}}}_{\rightarrow e^{-6}} = (e^{-4})^{\frac{3}{2}} = e^{-6}$$

$$4. f'(x) = \frac{(x-2)^2 - (x+1)2(x-2)}{(x-2)^4} + 3 = \frac{-x-4}{(x-2)^3} + 3 = 3 \rightarrow x_0 = -4$$

$$y - f(x_0) = f'(x_0)(x - x_0), \quad f(-4) = -\frac{145}{12} \rightarrow \text{Az érintő egyenlete: } y + \frac{145}{12} = 3(x + 4)$$

$$5. -x^2 + x + 12 = -(x+3)(x-4) > 0 \rightarrow D_f = (-3, 4), \quad f'(x) = \frac{-2x+1}{-(x+3)(x-4)} \rightarrow x = \frac{1}{2}$$

	$x \leq -3$	$-3 < x < \frac{1}{2}$	$x = \frac{1}{2}$	$\frac{1}{2} < x < 4$	$x \geq 4$
f'	n. é.	+	0	-	n. é.
f	n. é.	\nearrow	lok. max. $f(\frac{1}{2}) = \ln(\frac{49}{4})$	\searrow	n. é.

$$6. (a) \int \frac{5x}{\sqrt{1+16x^2}} dx = \frac{5}{32} \int 32x (1+16x^2)^{-\frac{1}{2}} dx = \frac{5}{32} \frac{(1+16x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{5}{16} \sqrt{1+16x^2} + C$$

$$(b) \int_0^\pi \underbrace{x^2}_f \underbrace{\cos\left(\frac{1}{2}x\right)}_{g'} dx = \left[2x^2 \sin\left(\frac{1}{2}x\right) \right]_0^\pi - \int_0^\pi \underbrace{4x}_f \underbrace{\sin\left(\frac{1}{2}x\right)}_{g'} dx =$$

$$2\pi^2 + \left[8x \cos\left(\frac{1}{2}x\right) \right]_0^\pi + \int_0^\pi 8 \cos\left(\frac{1}{2}x\right) dx = 2\pi^2 + 0 - \left[16 \sin\left(\frac{1}{2}x\right) \right]_0^\pi = 2\pi^2 - 16 \approx 3,74$$

$$7. (a) \underbrace{y'}_{a(x)} + \underbrace{\frac{6x}{3x^2+2}}_{b(x)} y = \sqrt{x} \rightarrow A(x) = \ln(3x^2 + 2), \quad y(x) = c(x)e^{-A(x)} = c(x) \frac{1}{3x^2 + 2}$$

$$c(x) = \int e^{A(x)} b(x) dx = \int (3x^2 + 2) \sqrt{x} dx = \int 3x^{\frac{5}{2}} + 2x^{\frac{1}{2}} dx = \frac{6}{7} x^{\frac{7}{2}} + \frac{4}{3} x^{\frac{3}{2}} + K, \quad y(x) = \frac{\frac{6}{7} x^{\frac{7}{2}} + \frac{4}{3} x^{\frac{3}{2}} + K}{3x^2 + 2}$$

$$(b) \ y_{(h,a)} : \lambda^2 + 4\lambda + 13 = 0 \quad \rightarrow \quad \lambda_{1,2} = \frac{-4 \pm \sqrt{16 - 52}}{2} = -2 \pm 3i$$

$$y_{(i,p)} : y = Ax + B, \ y' = A, \ y'' = 0 \quad \rightarrow \quad 0 + 4A + 13(Ax + B) = 26x \quad \rightarrow \quad A = 2, \ B = -\frac{8}{13}$$

$$y(x) = e^{-2x} (C_1 \cos(3x) + C_2 \sin(3x)) + 2x - \frac{8}{13}$$