1. $w = z^4 \quad \to \quad w^2 + 3w + 9 = 0$

$w_{1,2} = \frac{-3 \pm \sqrt{9-36}}{2} = -\frac{3}{2} \pm \frac{3\sqrt{3}}{2} i = 3(\cos(\pm 120°) + i \sin(\pm 120°))$

$$\boxed{\begin{aligned} z_{1,2,3,4} &= \sqrt[4]{w_1} = \sqrt[4]{3}(\cos(30° + k \cdot 90°) + i \sin(30° + k \cdot 90°)), \quad k = 0, 1, 2, 3 \\ z_{5,6,7,8} &= \sqrt[4]{w_2} = \sqrt[4]{3}(\cos(-30° + k \cdot 90°) + i \sin(-30° + k \cdot 90°)), \quad k = 0, 1, 2, 3 \end{aligned}}$$

2. $$a_{n+1} - a_n = \frac{\overbrace{(n+1)+6}^{n+7}}{\underbrace{3-7(n+1)}_{-4-7n}} - \frac{n+6}{3-7n} = \frac{(n+7)(3-7n)-(n+6)(-4-7n)}{(-4-7n)(3-7n)}$$

$$= \frac{45}{\underbrace{(-4-7n)}_{<0}\underbrace{(3-7n)}_{<0}} > 0 \quad \to \quad \boxed{\text{A sorozat szigorúan monoton növekvő.}}$$

$$\lim_{n\to\infty} \frac{n+6}{3-7n} = \lim_{n\to\infty} \frac{1+\frac{6}{n}}{\frac{3}{n}-7} = \boxed{-\frac{1}{7} = A}$$

$$|a_n - A| < \varepsilon \quad \to \quad \left|\frac{n+6}{3-7n} + \frac{1}{7}\right| < 0,02 \quad \to \quad \frac{|45|}{|3-7n| \cdot |7|} < 0,02 \quad \to$$

$$\frac{45}{(7n-3)\cdot 7} < 0,02 \quad \to \quad \frac{2250}{7} < 7n - 3 \quad \to \quad n > \frac{2271}{49} \quad \to \quad \boxed{N = 46}$$

3. (a) $$\lim_{x\to\infty} \left(\sqrt{2x+1} - \sqrt{x+2}\right) = \underbrace{\lim_{x\to\infty} \sqrt{x}}_{\infty} \cdot \underbrace{\lim_{x\to\infty} \left(\sqrt{2+\frac{1}{x}} - \sqrt{1+\frac{2}{x}}\right)}_{\sqrt{2}-1} = \boxed{\infty}$$

(b) $$\lim_{x\to\infty} \frac{x^2+4}{\sqrt{3x^4+2x^2+1}} = \lim_{x\to\infty} \frac{x^2}{x^2} \cdot \frac{1+\frac{4}{x^2}}{\sqrt{3+\frac{2}{x^2}+\frac{1}{x^4}}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

4. (a) $$t = \int_{-\pi/2}^{\pi/2} \cos(x)dx = [\sin(x)]_{-\pi/2}^{\pi/2} = \sin(\pi/2) - \sin(-\pi/2) = \boxed{2}$$

(b) $$T_2(\cos, 0, x) = \frac{\cos(0)}{0!}x^0 + \frac{-\sin(0)}{1!}x^1 + \frac{-\cos(0)}{2!}x^2 = 1 - \frac{1}{2}x^2$$

$$t = \int_{-\pi/2}^{\pi/2} 1 - \frac{1}{2}x^2 dx = \left[x - \frac{x^3}{6}\right]_{pi/2}^{\pi/2} = \boxed{\pi - \frac{\pi^3}{24}} \approx 1,85$$

5. $$D_f = \mathbb{R} \setminus \{0\},\ f'(x) = 2x + 2 - \left(\ln(x^2) + x\frac{2x}{x^2}\right) = 2x - \ln(x^2),\ f''(x) = 2 - \frac{2}{x} \quad \to \quad x_0 = 1$$

	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$x > 1$
f''	+	n. é.	−	0	+
f	konvex	n. é.	konkáv	infl. pont $f(1) = 3$	konvex

6. (a) $$\int \underbrace{(3x-2)}_{g'} \cdot \underbrace{\ln(x^2)}_{f} dx = \ln(x^2)\left(\frac{3x^2}{2} - 2x\right) - \int \left(\frac{3x^2}{2} - 2x\right) \underbrace{\frac{2x}{x^2}}_{\frac{2}{x}} dx =$$

$$\boxed{\ln(x^2)\left(\frac{3x^2}{2} - 2x\right) - \left(\frac{3x^2}{2} - 4x\right) + C}$$

(b) $$\int_1^2 \frac{x^3-1}{x^4-4x+4}dx = \frac{1}{4}\int_1^2 \frac{\overbrace{4x^3-4}^{f'}}{\underbrace{x^4-4x+4}_{f}} = \frac{1}{4}\left[\ln(x^4-4x+4)\right]_1^2 = \boxed{\frac{1}{4}\ln(12)} \approx 0,62$$

7. (a) $\dfrac{y'}{3} + x^2 y = x^2 \quad \rightarrow \quad y' + \underbrace{3x^2}_{a(x)} y = \underbrace{3x^2}_{b(x)} \quad \rightarrow \quad A(x) = x^3,\ y(x) = c(x)e^{-A(x)} = c(x)e^{-x^3}$

$c(x) = \displaystyle\int e^{A(x)} b(x) dx = \int e^{x^3} 3x^2 dx = e^{x^3} + K, \quad \boxed{y(x) = 1 + Ke^{-x^3}}$

(b) $y_{(h,a)}:\ \lambda^2 + 4\lambda + 4 = 0 \quad \rightarrow \quad \lambda_1 = -2$

$y_{(i,p)}:\ y = Ax + B,\ y' = A,\ y'' = 0 \quad \rightarrow \quad 0 + 4A + 4(Ax + B) = 2x - 1 \quad \rightarrow \quad A = \dfrac{1}{2},\ B = -\dfrac{3}{4}$

$\boxed{y(x) = C_1 e^{-2x} + C_2 x e^{-2x} + \dfrac{1}{2}x - \dfrac{3}{4}}$