1. $w = z^3, \quad w_{1,2} = \frac{1-i\pm\sqrt{(i-1)^2+4i}}{2} = \frac{1-i\pm\sqrt{2i}}{2} = \frac{1-i\pm(1+i)}{2} \quad \to \quad w_1 = 1,\ w_2 = -i$

$$\boxed{\begin{aligned} z_{1,2,3} &= \sqrt[3]{w_1} = (\cos(0^\circ + k\cdot 120^\circ) + i\sin(0^\circ + k\cdot 120^\circ)), \quad k = 0,1,2 \\ z_{4,5,6} &= \sqrt[3]{w_2} = (\cos(90^\circ + k\cdot 120^\circ) + i\sin(90^\circ + k\cdot 120^\circ)), \quad k = 0,1,2 \end{aligned}}$$

2. $a_{n+1} - a_n = [(n+1)^2 - 2(n+1) + 3] - [n^2 - 2n + 3] = 2n - 1 > 0$

$\to$ $\boxed{\text{A sorozat szigorúan monoton növekvő.}}$

$$\lim_{n\to\infty} n^2 - 2n + 3 = \lim_{n\to\infty} n^2\left(1 - \frac{2}{n} + \frac{3}{n^2}\right) = \boxed{\infty}$$

3. (a) $$\lim_{x\to\infty} \frac{2^{3x-1} + 6^x}{7^x - 8^{x+1}} = \lim_{x\to\infty} \frac{\frac{1}{2}\cdot 8^x + 6^x}{7^x - 8\cdot 8^x} = \lim_{x\to\infty} \frac{8^x}{8^x}\cdot\frac{\frac{1}{2} + \left(\frac{6}{8}\right)^x}{\left(\frac{7}{8}\right)^x - 8} = \boxed{-\frac{1}{16}}$$

(b) $$\lim_{x\to 1} \frac{5 - 5x}{\cos(2x-2)} = \frac{0}{\cos(0)} = \boxed{0}$$

4. $f(x_0) = e^{-4}, \quad f'(x) = e^{-x^2}(-2x) \quad \to \quad f'(x_0) = 4e^{-4}$

$\to$ Az érintő egyenlete: $\boxed{y - e^{-4} = 4e^{-4}(x+2)}$

5. $$D_f = \mathbb{R}\setminus\{-2\}, \quad f'(x) = \frac{1}{(x+2)^2}2(x+2) + 5 + x = \frac{2}{x+2} + \frac{(x+5)(x+2)}{x+2} =$$

$$\frac{x^2+7x+12}{x+2} = \frac{(x+3)(x+4)}{x+2} = 0 \quad \to \quad x_1 = -3,\ x_2 = -4$$

	$x<-4$	$x=-4$	$-4<x<-3$	$x=-3$	$-3<x<-2$	$x=-2$	$x>-2$
f'	$-$	0	$+$	0	$-$	n. é.	$+$
f	$\searrow$	lok. min. $f(-4) = \ln(4) - 4$	$\nearrow$	lok. max. $f(-3) = -\frac{21}{2}$	$\searrow$	n. é.	$\nearrow$

6. (a) $$\int_0^{\pi/12} \underbrace{x}_{f}\, \underbrace{\cos(3x)}_{g'}\, dx = \left[x\frac{\sin(3x)}{3}\right]_0^{\pi/12} - \int_0^{\pi/12} \frac{\sin(3x)}{3}dx =$$

$$\frac{\pi}{12}\frac{\sin\left(\frac{\pi}{4}\right)}{3} + \left[\frac{\cos(3x)}{9}\right]_0^{\pi/12} = \boxed{\frac{\pi}{36\sqrt{2}} + \frac{1}{9\sqrt{2}} - \frac{1}{9}} \approx 0,02916$$

(b) $$V = \pi\int_1^{\sqrt{2}} x^3 dx = \pi\left[\frac{x^4}{4}\right]_1^{\sqrt{2}} = \pi\left(\frac{4}{4} - \frac{1}{4}\right) = \boxed{\frac{3\pi}{4}}$$

7. (a) $$y_a(x) = \int \frac{2x^2 - 3x^3}{x}dx = \int 2x - 3x^2 dx = \boxed{x^2 - x^3 + C}$$

$y_a(2) = 5 \quad \to \quad 2^2 - 2^3 + C = 5 \quad \to \quad C = 9 \quad \to \quad \boxed{y_p(x) = x^2 - x^3 + 9}$

(b) $y_{(h,a)}:\ \lambda^2 - \lambda - 2 = 0 \quad \to \quad \lambda_1 = 2,\ \lambda_2 = -1$

$y_{(i,p)}:\ y = Ax + B,\ y' = A,\ y'' = 0 \quad \to \quad 0 - 2A - 4(Ax+B) = 1 - x \quad \to \quad A = \frac{1}{4},\ B = -\frac{3}{8}$

$$\boxed{y(x) = C_1 e^{2x} + C_2 e^{-x} + \frac{1}{4}x - \frac{3}{8}}$$