

$$1. w = z^2, \quad w_{1,2} = \frac{-3i \pm \sqrt{(-3i)^2 + 8}}{2} = \frac{-3i \pm \sqrt{-1}}{2} = \frac{-3i \pm i}{2} \rightarrow w_1 = -i, w_2 = -2i$$

$$z_{1,2} = \sqrt{w_1} = (\cos(-45^\circ + k \cdot 180^\circ) + i \sin(-45^\circ + k \cdot 180^\circ)), \quad k = 0, 1$$

$$z_{3,4} = \sqrt{w_2} = \sqrt{2}(\cos(-45^\circ + k \cdot 180^\circ) + i \sin(-45^\circ + k \cdot 180^\circ)), \quad k = 0, 1$$

$$2. a_{n+1} - a_n = [1 + e^{-2(n+1)+1}] - [1 + e^{-2n+1}] = e^{(-2n+1)-2} - e^{-2n+1} = e^{-2n+1} \cdot \overbrace{(e^{-2} - 1)}^{<0} < 0$$

→ A sorozat szigorúan monoton csökkenő.

$$\lim_{n \rightarrow \infty} 1 + e^{-2n+1} = 1$$

$$3. (a) \lim_{x \rightarrow \infty} \frac{\sqrt{x+3x^4}}{\sqrt[3]{x^6+1}} = \lim_{x \rightarrow \infty} \frac{x^2 \sqrt{\frac{1}{x^3}+3}}{x^2 \sqrt[3]{1+\frac{1}{x^6}}} = \sqrt{3}$$

$$(b) \lim_{x \rightarrow 0} \frac{\ln(3x^2+1)}{\ln(1+x^2)} = \lim_{x \rightarrow 0} \frac{\frac{6x}{3x^2+1}}{\frac{2x}{1+x^2}} = \lim_{x \rightarrow 0} \frac{6x}{2x} \cdot \frac{1+x^2}{3x^2+1} = 3$$

„0/0”

$$4. f(x_0) = \frac{3}{4}, \quad f'(x) = \frac{1 \cdot (x+1) - x \cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2} \rightarrow f'(x_0) = \frac{1}{16}$$

→ Az érintő egyenlete:  $y - \frac{3}{4} = \frac{1}{16}(x - 3)$

$$5. D_f = \mathbb{R}, \quad f'(x) = e^x(x^2 - 5x + 6) + e^x(2x - 5) = e^x(x^2 - 3x + 1)$$

$$f''(x) = e^x(x^2 - 3x + 1) + e^x(2x - 3) = e^x(x^2 - x - 2) = e^x(x+1)(x-2) \rightarrow x_1 = -1, x_2 = 2$$

	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$f''$	+	0	-	0	+
$f$	konvex	infl. pont. $f(-1) = \frac{12}{e}$	konkáv	infl. pont. $f(2) = 0$	konvex

$$6. (a) \int \frac{5x}{\sqrt[3]{x^2+1}} dx = \frac{5}{2} \int (2x)(x^2+1)^{-1/3} dx = \frac{5}{2} \frac{(x^2+1)^{2/3}}{\frac{2}{3}} + C = \frac{15}{4}(x^2+1)^{2/3} + C$$

$$(b) \int_1^e \underbrace{x^2}_{g'} \underbrace{\ln(x)}_f dx = \left[ \ln(x) \frac{x^3}{3} \right]_1^e - \int_1^e \frac{1}{x} \cdot \frac{x^3}{3} dx = \frac{e^3}{3} - \left[ \frac{x^3}{9} \right]_1^e = \frac{2e^3 + 1}{9}$$

$$7. (a) y_a(x) = \int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{2x}{x^2+1} dx = \frac{1}{2} \ln(x^2+1) + C$$

$$y_a(3) = 0 \rightarrow \frac{1}{2} \ln(10) + C = 0 \rightarrow C = -\frac{\ln(10)}{2} \rightarrow y_p(x) = \frac{1}{2} \ln(x^2+1) - \frac{\ln(10)}{2}$$

$$(b) y_{(h,a)} : \lambda^2 + 2\lambda + 2 = 0 \rightarrow \lambda_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$$

$$y_{(i,p)} : y = Ax + B, y' = A, y'' = 0 \rightarrow 0 + 4A + 4(Ax + B) = x \rightarrow A = \frac{1}{4}, B = -\frac{1}{4}$$

$$y(x) = e^{-x} (C_1 \cos(x) + C_2 \sin(x)) + \frac{1}{4}x - \frac{1}{4}$$