1. $w = z^3, \quad w_{1,2} = \frac{i \pm \sqrt{(-i)^2 + 24i^2}}{2i} = \frac{i \pm \sqrt{-25}}{2i} = \frac{i \pm 5i}{2i} \quad \rightarrow \quad w_1 = 3, \; w_2 = -2$

$z_{1,2,3} = \sqrt[3]{w_1} = \sqrt[3]{3}(\cos(k \cdot 120°) + i \sin(k \cdot 120°)), \quad k = 0, 1, 2$
$z_{4,5,6} = \sqrt[3]{w_2} = \sqrt[3]{2}(\cos(60° + k \cdot 120°) + i \sin(60° + k \cdot 120°)), \quad k = 0, 1, 2$

2. $a_{n+1} - a_n = \frac{n}{n+2} - \frac{n-1}{n+1} = \frac{n(n+1) - (n-1)(n+2)}{(n+2)(n+1)} = \underbrace{\frac{2}{(n+2)}}_{>0}\underbrace{\frac{}{(n+1)}}_{>0} > 0$

$\rightarrow$ A sorozat szigorúan monoton növekvő.

$\lim_{n\to\infty} \left(2 + \frac{n-1}{n+1}\right) = 2 + \lim_{n\to\infty} \frac{n}{n} \cdot \frac{1 - \frac{1}{n}}{1 + \frac{1}{n}} = 3$

3. (a) $\lim_{x\to\infty} \underbrace{\frac{2^x + 1}{1 - x^2}}_{„\infty/-\infty''} = \lim_{x\to\infty} \underbrace{\frac{2^x \ln(2)}{-2x}}_{„\infty/-\infty''} = \lim_{x\to\infty} \frac{2^x \ln^2(2)}{-2} = \frac{\infty}{-2} = -\infty$

(b) $\lim_{x\to 0} \frac{2^{3x-1} + 6^x}{7^{x+1} - 8^x} = \frac{2^{-1} + 6^0}{7^1 - 8^0} = \frac{\frac{3}{2}}{6} = \frac{1}{4}$

4. $f(x_0) = \frac{\sin(\pi)}{\cos 2\pi} = 0, \quad f'(x) = \frac{\cos(x)\cos(2x) + \sin(x)\sin(2x)}{\cos^2(2x)} \quad \rightarrow \quad f'(x_0) = \frac{-1 + 0}{1} = -1$

$\rightarrow$ Az érintő egyenlete: $y = -(x - \pi)$

5. $D_f = \mathbb{R}, \quad f'(x) = e^{2x+1}2(x^2 - 2x - 1) + e^{2x+1}(2x - 2) = e^{2x+1}(2x^2 - 2x - 4) = 2e^{2x+1}(x + 1)(x - 2)$
$\rightarrow \quad x_1 = -1, \; x_2 = 2$

	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
f'	$+$	0	$-$	0	$+$
f	$\nearrow$	lok. max. $f(-1) = 2e^{-1}$	$\searrow$	lok. min. $f(2) = -e^5$	$\nearrow$

6. (a) $\int_0^{\pi/6} \underbrace{x}_{f}\underbrace{\sin(3x)}_{g'} dx = \left[x\frac{-\cos(3x)}{3}\right]_0^{\pi/6} + \int_0^{\pi/6} \frac{\cos(3x)}{3} dx = 0 + \left[\frac{\sin(3x)}{9}\right]_0^{\pi/6} = \frac{1}{9}$

(b) $V = \pi \int_1^{\sqrt{3}} \left(\frac{1}{\sqrt{x^3}}\right)^2 dx = \pi \int_1^{\sqrt{3}} x^{-3} dx = \pi \left[\frac{x^{-2}}{-2}\right]_1^{\sqrt{3}} = \frac{\pi}{3}$

7. (a) $y_a(x) = \int \frac{2 + x}{\sqrt{x}} dx = \int 2x^{-\frac{1}{2}} + x^{\frac{1}{2}} dx = 4x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$

$y_a(1) = 2 \quad \rightarrow \quad 4 + \frac{2}{3} + C = 2 \quad \rightarrow \quad C = -\frac{8}{3} \quad \rightarrow \quad y_p(x) = 4x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} - \frac{8}{3}$

(b) $y_{(h,a)}: \; \lambda^2 - \lambda - 20 = 0 \quad \rightarrow \quad \lambda_{1,2} = \frac{1 \pm \sqrt{1 + 80}}{2} \quad \rightarrow \quad \lambda_1 = 5, \; \lambda_2 = -4$

$y_{(i,p)}: \; y = Ax + B, \; y' = A, \; y'' = 0 \quad \rightarrow \quad 0 - A - 20(Ax + B) = x \quad \rightarrow \quad A = -\frac{1}{20}, \; B = \frac{1}{400}$

$y(x) = C_1 e^{5x} + C_2 e^{-4x} - \frac{1}{20}x + \frac{1}{400}$