1. $w = z^2, \quad w_{1,2} = \frac{2\pm\sqrt{(-2)^2-8}}{2} = \frac{2\pm 2i}{2} = 1 \pm i$

$z_{1,2} = \sqrt{w_1} = \sqrt[4]{2}(\cos(22,5^\circ + k\cdot 180^\circ) + i\sin(22,5^\circ + k\cdot 180^\circ)), \quad k = 0,1$

$z_{3,4} = \sqrt{w_2} = \sqrt[4]{2}(\cos(-22,5^\circ + k\cdot 180^\circ) + i\sin(-22,5^\circ + k\cdot 180^\circ)), \quad k = 0,1$

2. $a_{n+1} - a_n = \frac{n-1}{2n+5} - \frac{n-2}{2n+3} = \frac{7}{\underbrace{(2n+5)}_{>0}\underbrace{(2n+3)}_{>0}} > 0 \quad \rightarrow$ A sorozat szig. mon. növekvő.

$\lim_{n\to\infty} \frac{n-2}{2n+3} = \lim_{n\to\infty} \frac{n}{n} \cdot \frac{1-\frac{2}{n}}{2+\frac{3}{n}} = \frac{1}{2}$

3. (a) $\lim_{x\to\infty} \underbrace{\frac{e^x - 1}{x}}_{„\infty/\infty''} = \lim_{x\to\infty} \frac{e^x}{1} = \frac{\infty}{1} = \infty$

(b) $\lim_{x\to\infty} \left(\sqrt{2x^2+3x} - \sqrt{2x^2+1}\right) = \lim_{x\to\infty} \frac{(2x^2+3x)-(2x^2+1)}{\sqrt{2x^2+3x}+\sqrt{2x^2+1}}$

$= \lim_{x\to\infty} \frac{x}{x} \cdot \frac{3-\frac{1}{x}}{\sqrt{2+\frac{3}{x}}+\sqrt{2+\frac{1}{x^2}}} = \frac{3}{2\sqrt{2}}$

4. $f(x_0) = \frac{\ln(2)}{2}, \quad f'(x) = \frac{1}{2}\frac{1}{1+x^2}2x = \quad \rightarrow \quad f'(x_0) = \frac{1}{2}$

$\rightarrow$ Az érintő egyenlete: $y - \frac{\ln(2)}{2} = \frac{1}{2}(x-1)$

5. $D_f = \mathbb{R}, \quad f'(x) = -12x + x^2 + x^3 = x(x-3)(x+4) \quad \rightarrow \quad x_1 = 0,\ x_2 = 3,\ x_3 = -4$

	$x < -4$	$x = -4$	$-4 < x < 0$	$x = 0$	$0 < x < 3$	$x = 3$	$x > 3$
f'	$-$	0	$+$	0	$-$	0	$+$
f	$\searrow$	lok. min. $f(-4) = \frac{32}{3}$	$\nearrow$	lok. max. $f(0) = 0$	$\searrow$	lok. min. $f(3) = -\frac{99}{4}$	$\nearrow$

6. (a) $\int \frac{2}{3+4x^2}dx = \frac{2}{3}\int \frac{1}{1+\frac{4}{3}x^2}dx = \frac{2}{3}\int \frac{1}{1+\left(\frac{2}{\sqrt{3}}x\right)^2}dx = \frac{2}{3}\cdot\frac{\operatorname{arctg}\left(\frac{2}{\sqrt{3}}x\right)}{\frac{2}{\sqrt{3}}} + C$

(b) $\int_2^e \underbrace{x^3}_{g'}\underbrace{\ln(x)}_{f}\,dx = \left[\ln(x)\frac{x^4}{4}\right]_2^e - \int_2^e \frac{1}{x}\cdot\frac{x^4}{4}dx = \left(\frac{e^4}{4} - \ln(2)\frac{16}{4}\right) - \left[\frac{x^4}{16}\right]_2^e = \frac{3e^4}{16} - 4\ln(2) + 1$

7. (a) $y_a(x) = \int \underbrace{x}_{f}\underbrace{e^x}_{g'}dx = xe^x - \int e^x dx = e^x(x-1) + C$

$y_a(0) = 1 \quad \rightarrow \quad e^0(0-1) + C = 1 \quad \rightarrow \quad C = 2 \quad \rightarrow \quad y_p(x) = e^x(x-1) + 2$

(b) $y_{(h,a)}:\ \lambda^2 + \sqrt{12}\lambda - 1 = 0 \quad \rightarrow \quad \lambda_{1,2} = \frac{-\sqrt{12}\pm\sqrt{12+4}}{2} = -\sqrt{3}\pm 2$

$y_{(i,p)}:\ y = Ax + B,\ y' = A,\ y'' = 0 \quad \rightarrow \quad 0 + \sqrt{12}A - (Ax+B) = x \quad \rightarrow \quad A = -1,\ B = -\sqrt{12}$

$y(x) = C_1 e^{(-\sqrt{3}+2)x} + C_2 e^{(-\sqrt{3}-2)x} - x - \sqrt{12}$