Signals and systems

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Definitions, Representations, Primitives, Combinations

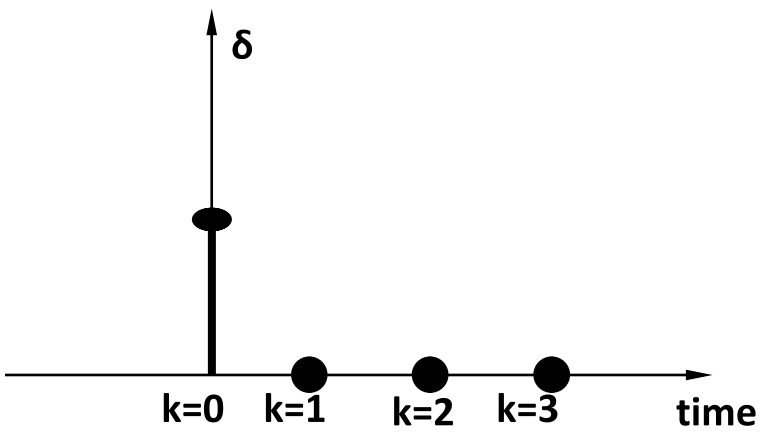
SIGNALS


I'M GETTING MIXED SIGNALS!

Primitives

Pulse function
Kroneker delta function

$$\delta[k] = \begin{cases} 1, if\ k = 0 \\ 0, if\ k \neq 0 \end{cases}$$



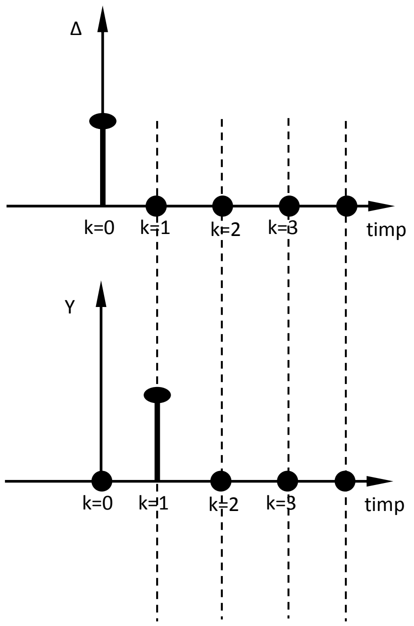
Combinations

$$y[n] = a \cdot \delta[n]$$

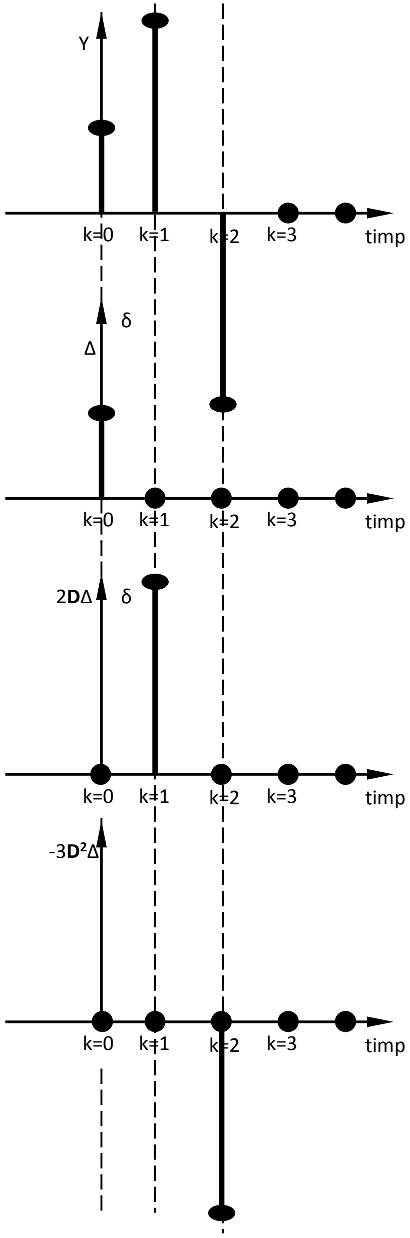
$$y[n] = x[n-1] \vee n \Leftrightarrow Y = \boldsymbol{D}X$$

$$y[n] = x[n] + z[n]$$

Examples



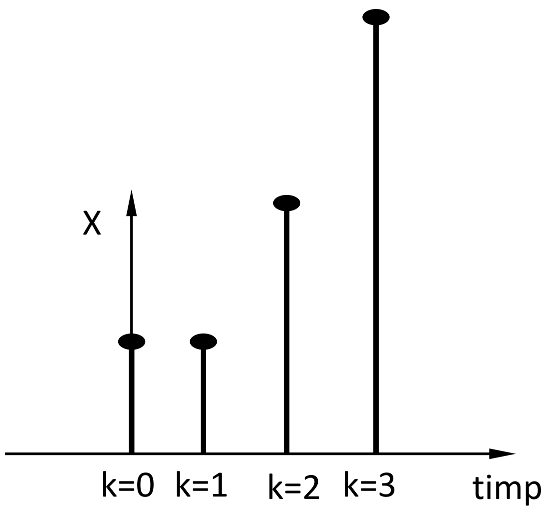
$$Y = D\Delta$$



$$Y = \Delta + 2D\Delta - 3D^2\Delta$$

Examples

$$x_k = x_{k-1} + x_{k-2}$$



x_0	1
x_1	1
x_2	2
x_3	3
x_4	5
x_5	8
x_6	13
……	

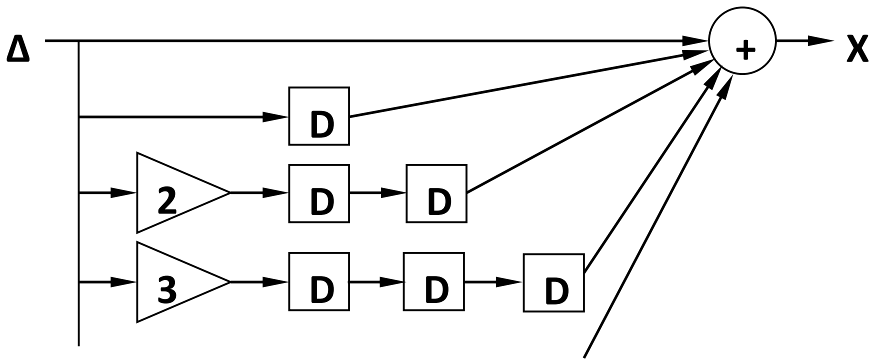
$$x_k = x_{k-1} + x_{k-2}$$

$$s_k=(x_{k-1},x_{k-2}).$$

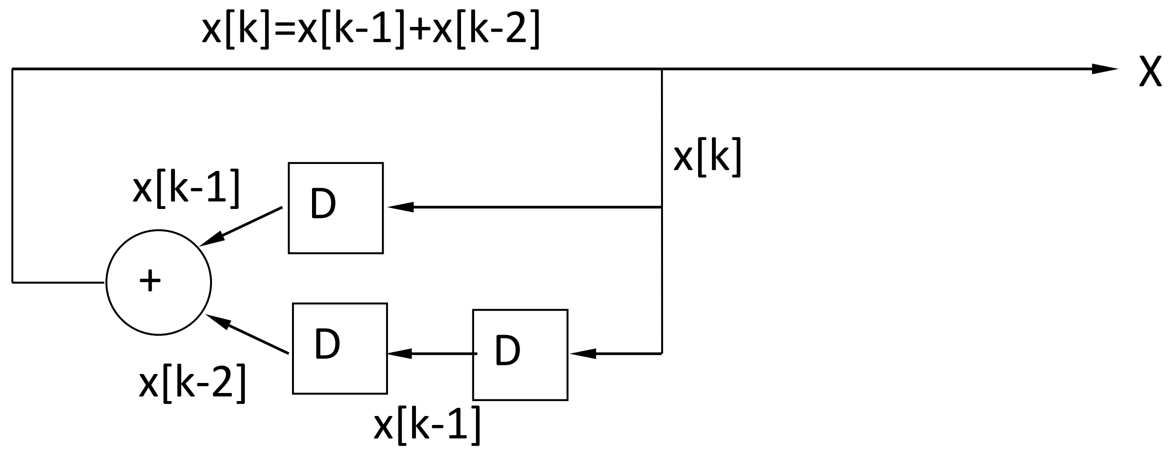
Time	0	1	2	3	4	5	6
i	0	0	0	0	0	0	0
s_k	1	2	3	5	8	13	21
	1	1	2	3	5	8	13
o	2	3	5	8	13	21	34

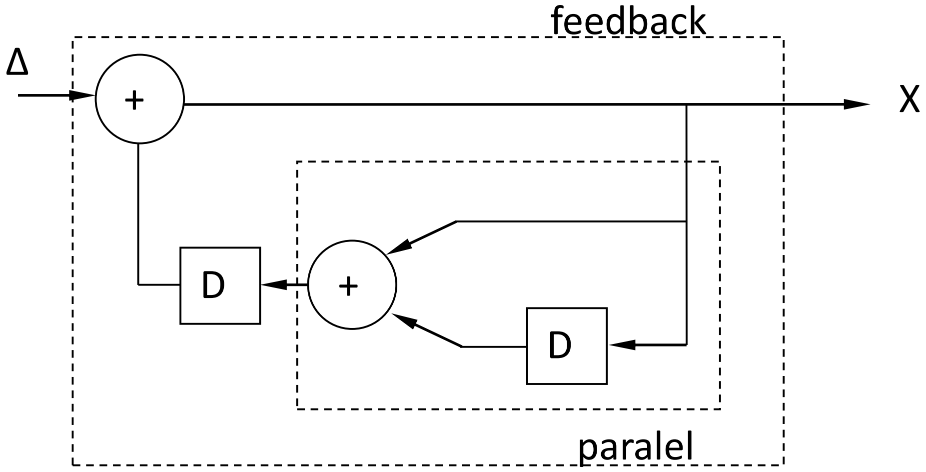
$$x_k = x_{k-1} + x_{k-2}, \forall\, k \Leftrightarrow X = \delta[0] + \delta[1] + 2\delta[2] + 3\delta[2] + 5\delta[2] + \cdots$$

$$\Leftrightarrow X = (1 + \boldsymbol{D} + 2\boldsymbol{D}^2 + 3\boldsymbol{D}^3 + 5\boldsymbol{D}^4 + \cdots)\Delta$$



$$x_k = x_{k-1} + x_{k-2}$$





$$x_k - x_{k-1} - x_{k-2} = 0$$

$$X(1 - \boldsymbol{D} - \boldsymbol{D}^2) = 0$$

$$F = \frac{1}{1 - \boldsymbol{D} - \boldsymbol{D}^2} \qquad f(x) = \frac{1}{1 - x - x^2}$$

$$f(x) = 1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + \cdots$$

Definitions, Representations, Primitives, Combinations, Examples

SYSTEMS

Definitions, representations

$$y[k] = a_1 y[k-1] + a_2 y[k-2] + \cdots + a_m y[k-m] + b_0 x[k] + b_1 x[k-1] + \cdots + b_n x[k-n]$$

$$y[k] = a_1 y[k-1] + a_2 y[k-2] + \cdots + a_m y[k-m] +$$

$$m \geq n$$

$$y[k] = a_1 y[k-1] + a_2 y[k-2] + \cdots + a_m y[k-m] + b_0 x[k] + b_1 x[k-1] + \cdots + b_n x[k-n]$$

$$y[k] = a_1 y[k-1] + a_2 y[k-2] + \cdots + a_m y[k-m] +$$

$$+ b_0 x[k] + b_1 x[k-1] + \cdots + b_n x[k-n]$$

$$m \geq n$$

$$y[k] = a_1 y[k-1] + a_2 y[k-2] + \cdots + a_m y[k-m] + b_0 x[k] + b_1 x[k-1] + \cdots + b_n x[k-n]$$

$$m \geq n$$

$$Y = a_1 \boldsymbol{D} Y + a_2 \boldsymbol{D}^{\boldsymbol{2}} Y + \cdots + a_m \boldsymbol{D}^m Y + b_0 X + b_1 \boldsymbol{D} X + \cdots + b_n \boldsymbol{D}^n X$$

$$Y = \frac{b_0 + b_1 \boldsymbol{D} + \cdots + b_n \boldsymbol{D}^n}{1 - a_1 \boldsymbol{D} - a_2 \boldsymbol{D}^{\boldsymbol{2}} - \cdots - a_m \boldsymbol{D}^m} X$$

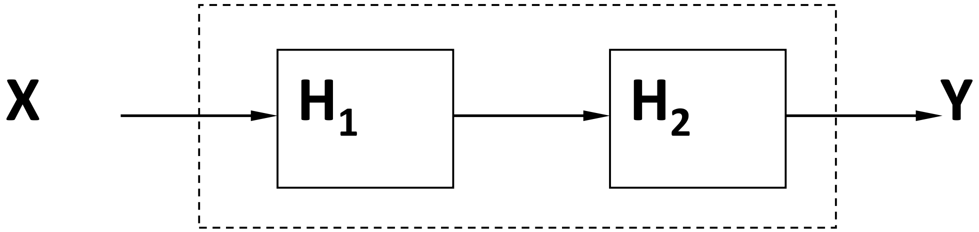
Primitives

$$H = \frac{Y}{X} = \frac{N(\mathbf{D})}{M(\mathbf{D})}$$

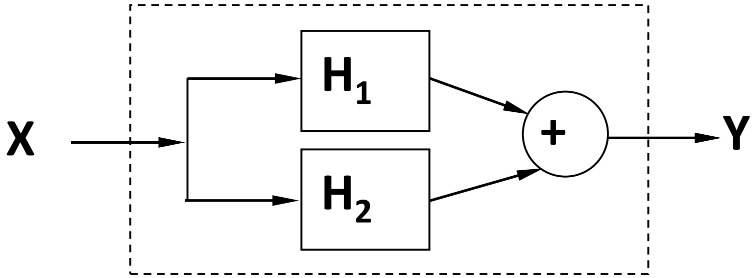
$$H = \frac{Y}{X} = \mathbf{D}$$

Combinations

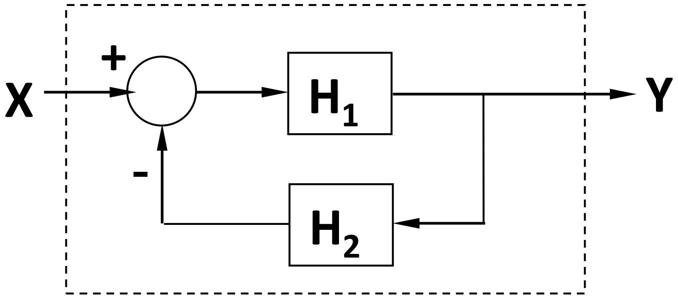
$$H = \frac{Y}{X} = D$$



$$H = H_1 \cdot H_2$$



$$H = H_1 + H_2$$

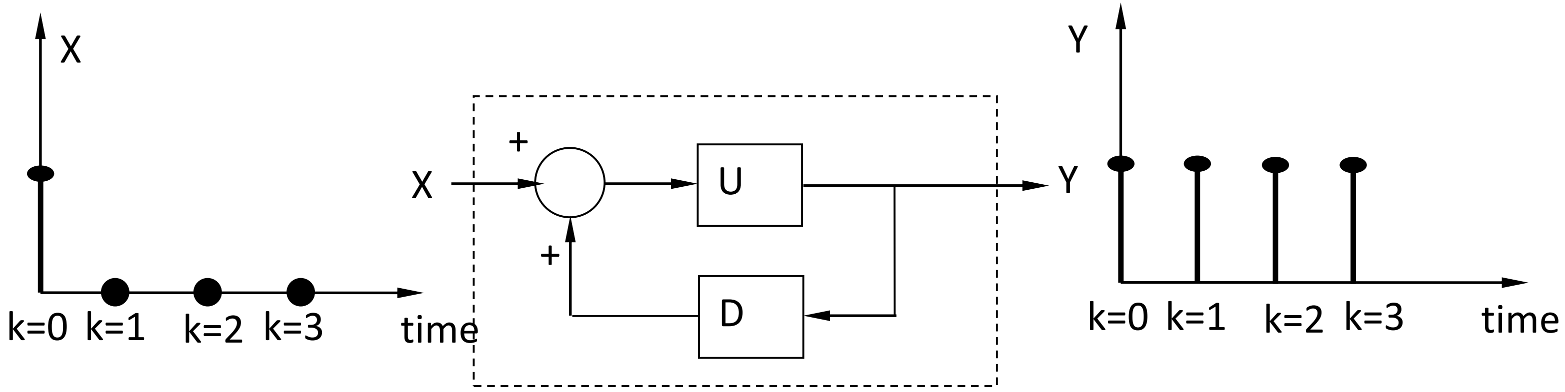


$$H = \frac{H_1}{1 + H_2} \cdot$$

Examples

$$y[k] = y[k-1] + x[k]$$

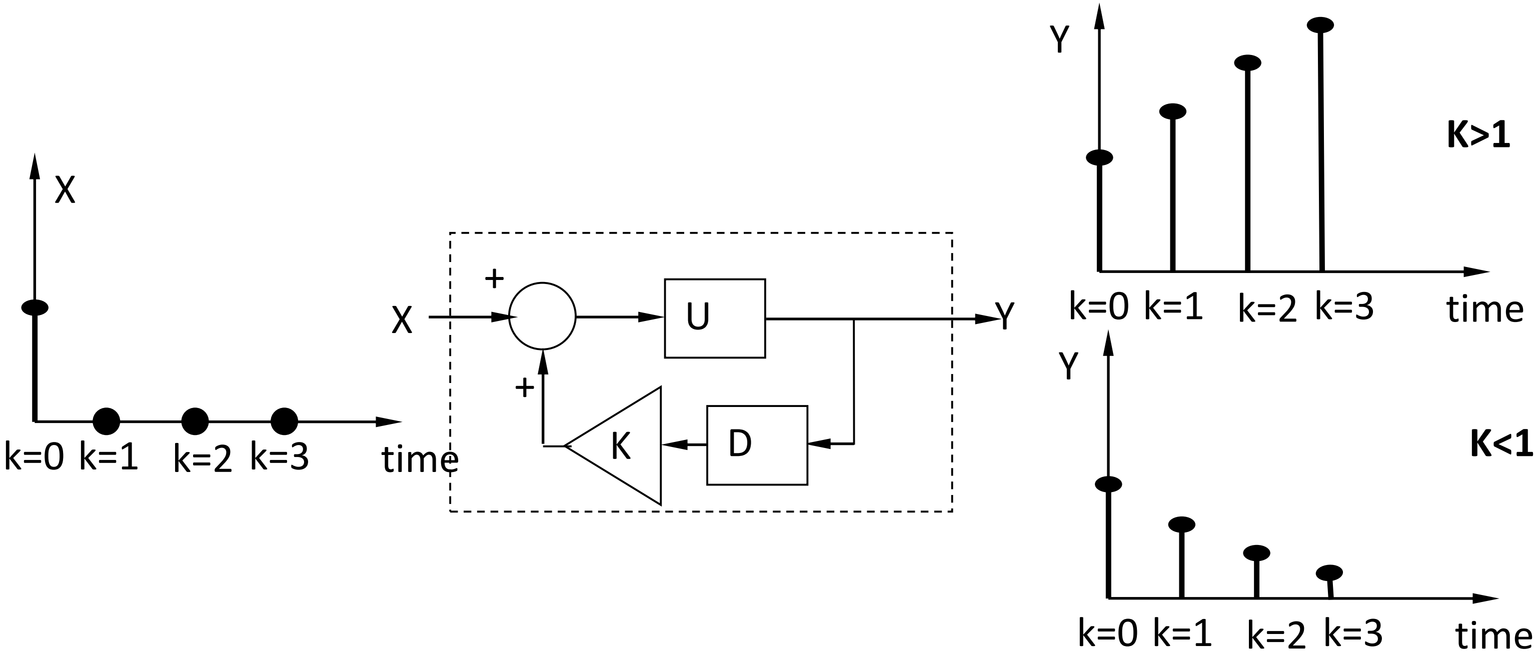
$$Y = Y\boldsymbol{D} + X \Leftrightarrow H = \frac{Y}{X} = \frac{1}{1-\boldsymbol{D}}$$


X
k=0
k=1
k=2
k=3
time
+
X
U
Y
+
D
Y
k=0
k=1
k=2
k=3
time

```
>>> my_sistem=Fback_p_1_v2(Unitar(0))
>>> my_sistem.succpas([1,0,0,0,0,0,0,0,0])
[1, 1, 1, 1, 1, 1, 1, 1, 1]
```

```
>>> my_sistem_1=Fback_p_2_v2(Unitar(0),Gain(0.5))
>>> my_sistem_1.succpas([1,0,0,0,0,0,0,0,0])
[1, 0.5, 0.25, 0.125, 0.0625, 0.03125, 0.015625, 0.007812
>>> my_sistem_2=Fback_p_2_v2(Unitar(0),Gain(1.5))
>>> my_sistem_2.succpas([1,0,0,0,0,0,0,0,0])
[1, 1.5, 2.25, 3.375, 5.0625, 7.59375, 11.390625, 17.0859
```

$$Y = KYD + X \Leftrightarrow H = \frac{Y}{X} = \frac{1}{1 - KD}$$



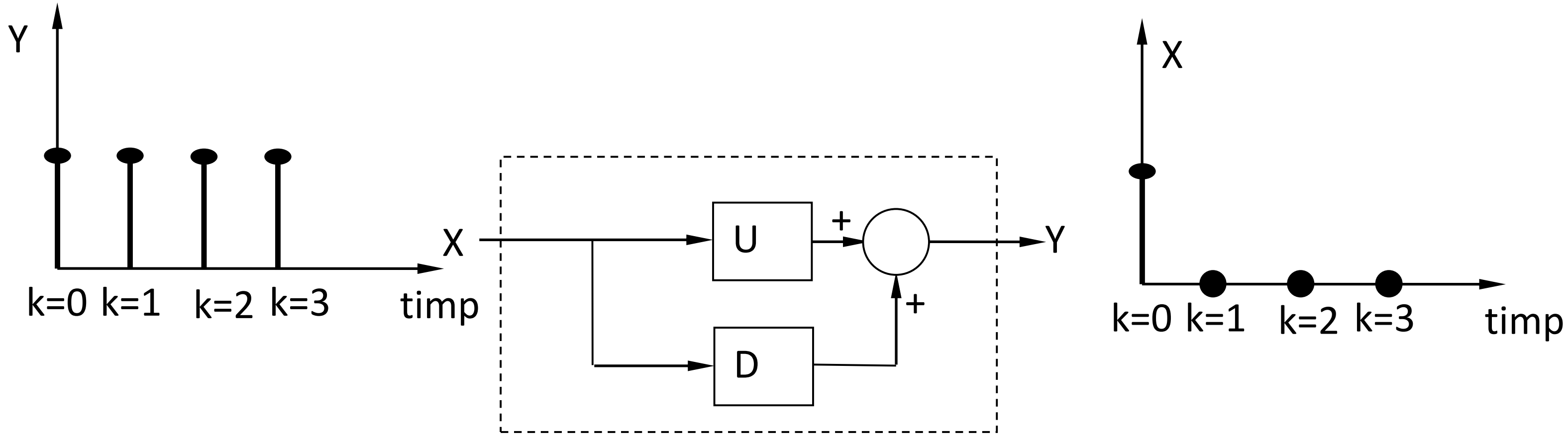
Examples

$$y[k] = x[k] - x[k-1]$$

$$y[k] = x[k] - x[k-1] \Leftrightarrow Y = X - \boldsymbol{D}X \Rightarrow H = (1 - \boldsymbol{D})$$

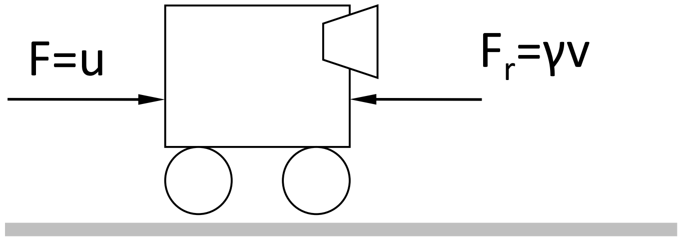
Examples

$$y[k] = x[k] - x[k-1] \Leftrightarrow Y = X - \boldsymbol{D}X \Rightarrow H = (1 - \boldsymbol{D})$$




```
>>> my_sistem=Paralel_m_1(Unitar(0),Intarziere(0))
>>> my_sistem.succpas([1,1,1,1,1,1])
[1, 0, 0, 0, 0, 0]
```

CASE STUDY



$$m\ddot{x} = u - \gamma\dot{x} \Leftrightarrow m\dot{v} + \gamma v = u \Leftrightarrow m\frac{dv}{dt} + \gamma v = u$$

$$m\ddot{x} = u - \gamma\dot{x} \Leftrightarrow m\dot{v} + \gamma v = u \Leftrightarrow m\frac{dv}{dt} + \gamma v = u$$

$$m\frac{\Delta v}{\Delta t} + \gamma v = u \Rightarrow m(v_k - v_{k-1}) + \Delta t\gamma v_k = \Delta t u_k$$

$$m(v_k - v_{k-1}) + \Delta t\gamma v_k = \Delta t u_k \Leftrightarrow$$

$$m(V - \boldsymbol{D}V) + \Delta t\gamma V = \Delta t U \Rightarrow$$

$$\Rightarrow H = \frac{V}{U} = \frac{\Delta t}{(m + \Delta t\gamma) - m\boldsymbol{D}}$$

$$H = \frac{V}{U} = \frac{\Delta t}{(m + \Delta t\gamma)} \cdot \frac{1}{1 - \frac{m}{(m + \Delta t\gamma)}\boldsymbol{D}}$$

```
>>> robi=Fback_p_2_v2(Unitar(0),Gain(0.9))
>>> output=[0.02*rez for rez in robi.succpas([1,0,0,0,
>>> output
[0.02,
 0.018000000000000002,
 0.016200000000000003,
 0.014580000000000003,
 0.013122000000000003,
 0.011809800000000004]
```

CONCLUSIONS

Great Moments in Computer History:
Charles Lindbergh Jr. completes
the first simulated transatlantic flight.

Home work

Design a simulation environment

- Design the UML diagram
  - The primitive
  - The combination functions
  - The interface
- Choose a programming language and construct your programs
- Write the documentation and show 3 examples