Simulationstechnik

# Fachhochschule Heilbronn

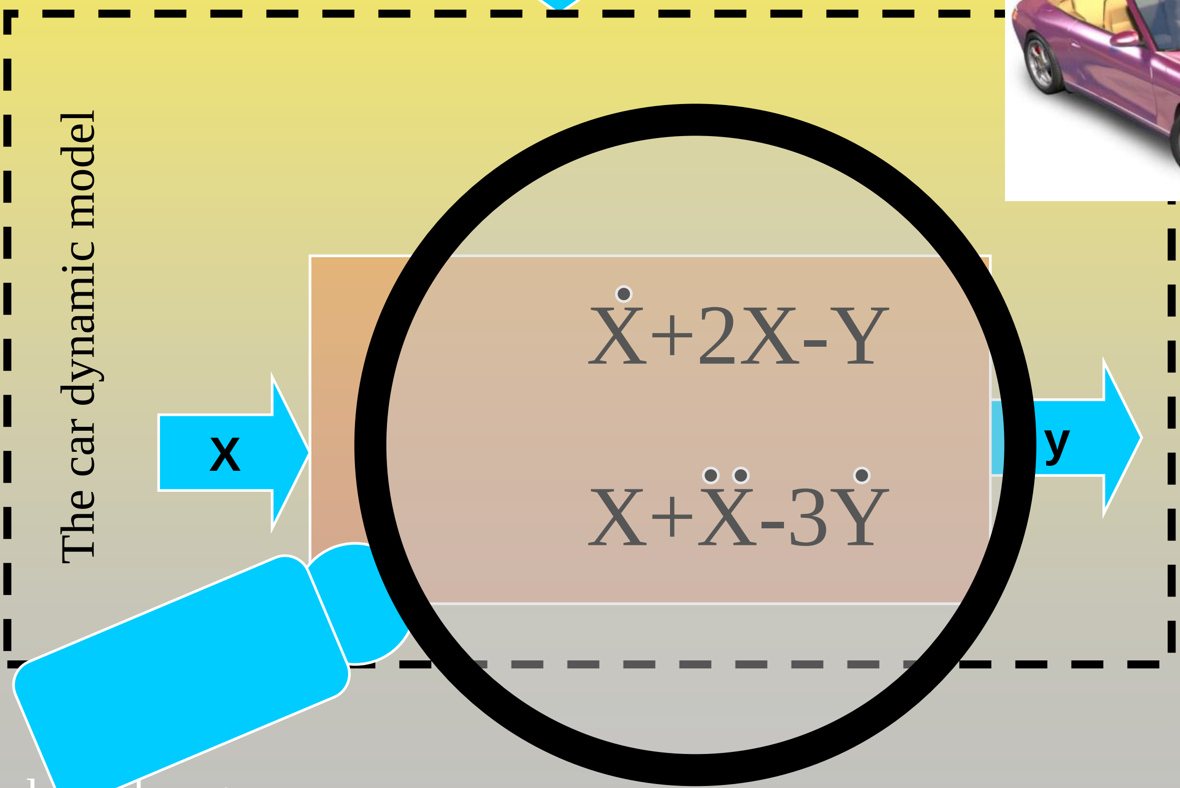
## Automotive System Engineering

# Simulationstechnik

## L-4 Matlab DGL‘s

Dr. Tröster
ASE 5

# Way differential equations in the simulation ?



How can Matlab help me to solve differential equations?

Matlab command window functions

.m file

Simulink

Diff equations

- **A little mathematic about solving diff. equations**

**Diff equations**

**Analytical solutions**

**Only for linear diff equations**

**Integrations**

**Laplace Transform**

**....**

**Many methods**

**Numerical solutions**

**Taylor series**

**For linear and Nonlinear Diff equations**

**Euler**

**Runge-Kutta 23**

**Runge-Kutta 45**

**....**

- **The Matlab command window functions:**

ode23(…) ode45(…) ode113(…) ode15s(…) ode23s(…)

ode23t(…) ode23tb(…)

**Syntax**

**[T,Y] = solver(odefun,tspan,y0)**

**where solver is one of ode45, ode23, ode113, ode15s, ode23s, ode23t, or ode23tb.**

| | |
|---|---|
| **odefun** | **A function that evaluates the right-hand side of the differential equations.** |
| **tspan** | **A vector specifying the interval of integration, [t0,tf].** |
| **y0** | **A vector of initial conditions.** |

# Examples of first order diff. equations

$$y' = \frac{dy}{dt}$$

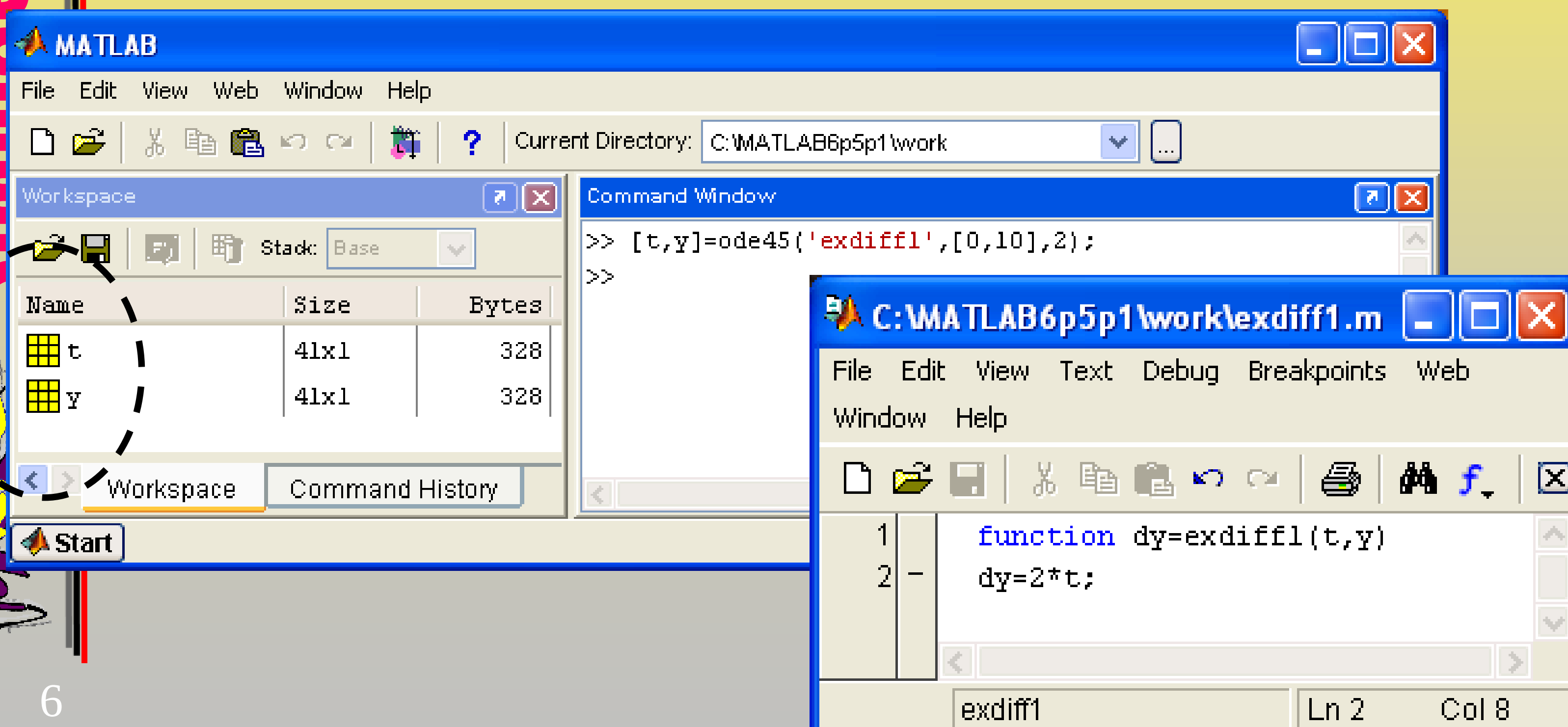
**1°** $y' = 2t$

analytical solution: $y = \int 2t\,dt = t^2 + C$

where: $y(0) = 0^2 + C$

so $y = t^2 + y(0)$; $t = [To, Tf]$

Matlab solution: ode23('exdiff1',[To,Tf],y(0))



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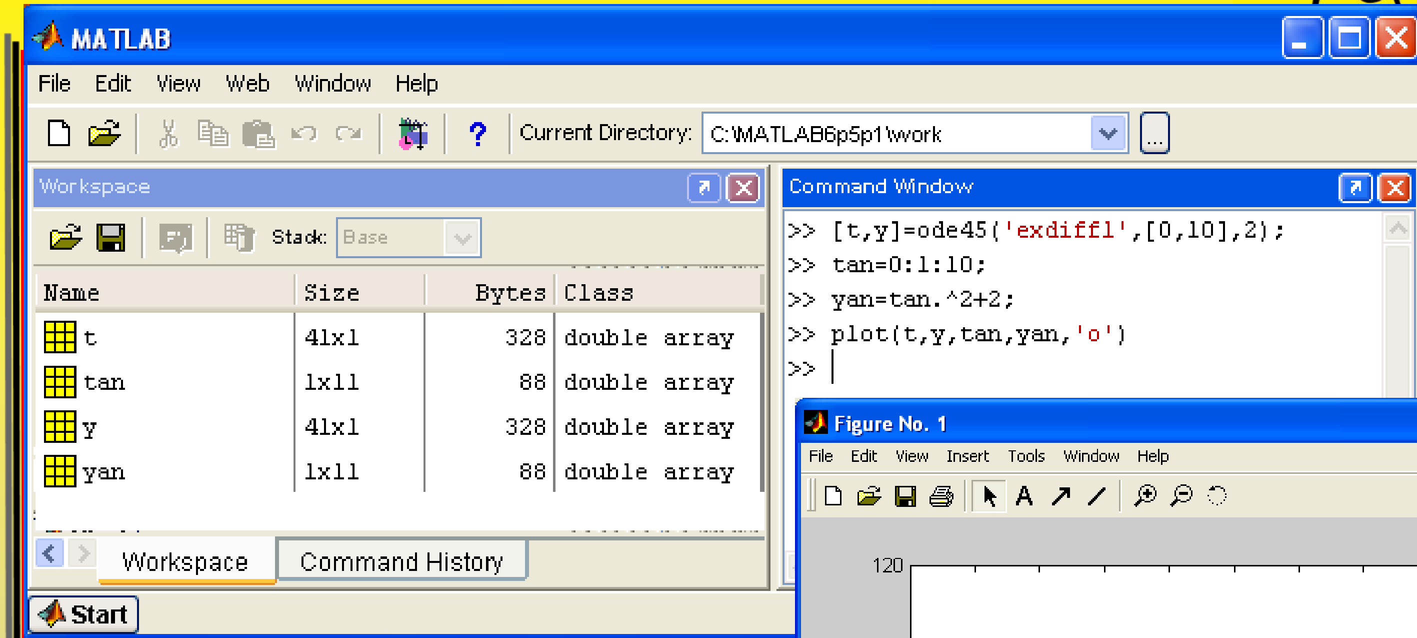


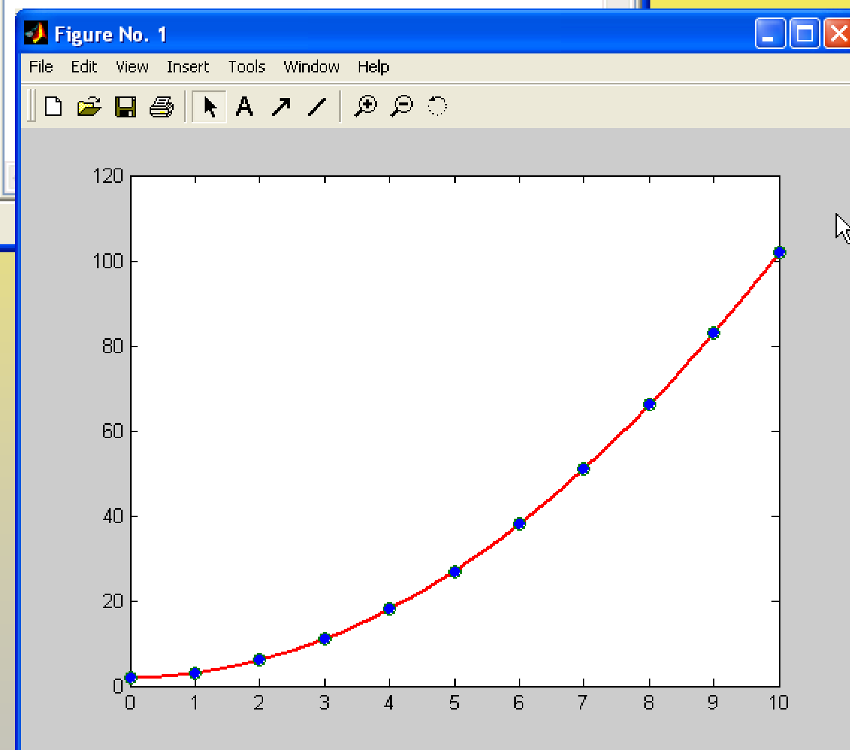


**comparison of the two solutions**

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$y' = \frac{dy}{dt}$

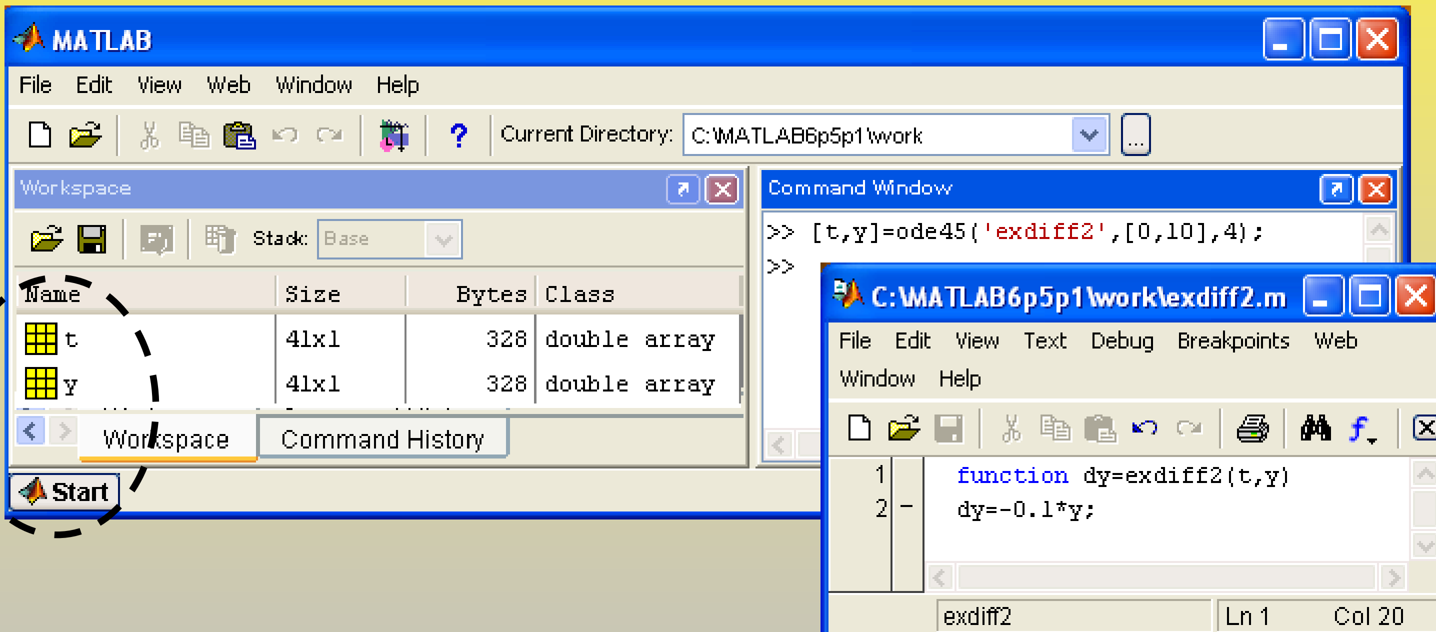
**2°** $y' = -0.1y$

analytical solution: $y = 4e^{-0.1t}$ t=[To,Tf]

Matlab solution: ode23('exdiff2',[To,Tf],4)

$y(0)=4$

$t=[0,10]$



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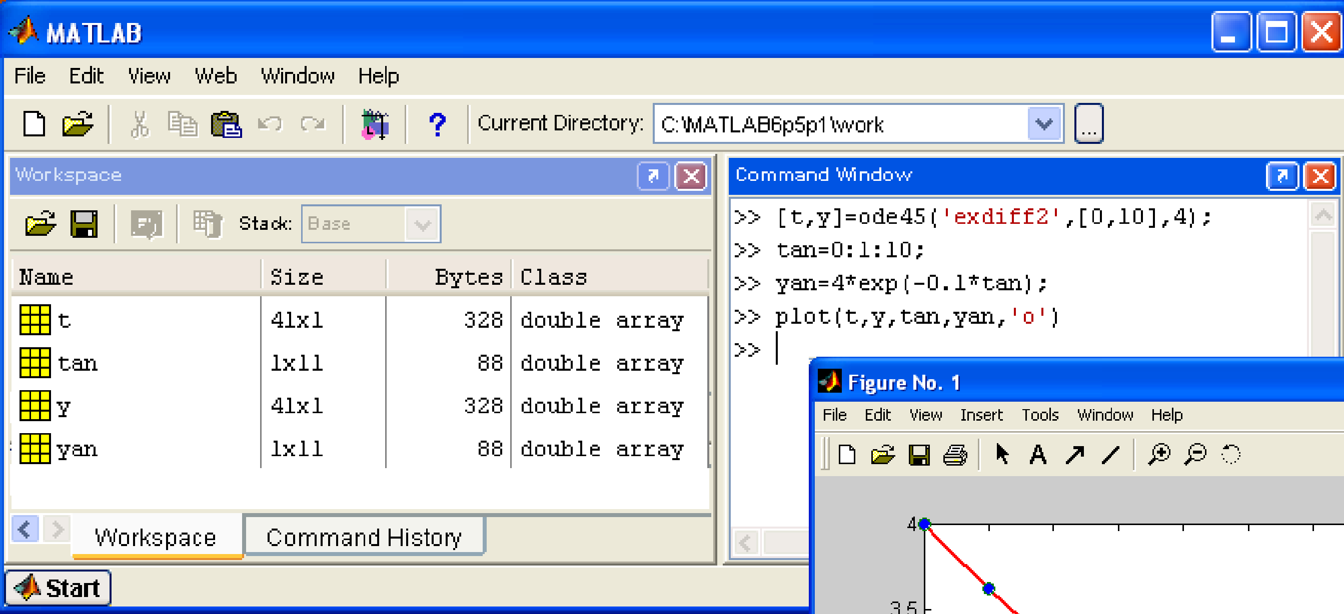


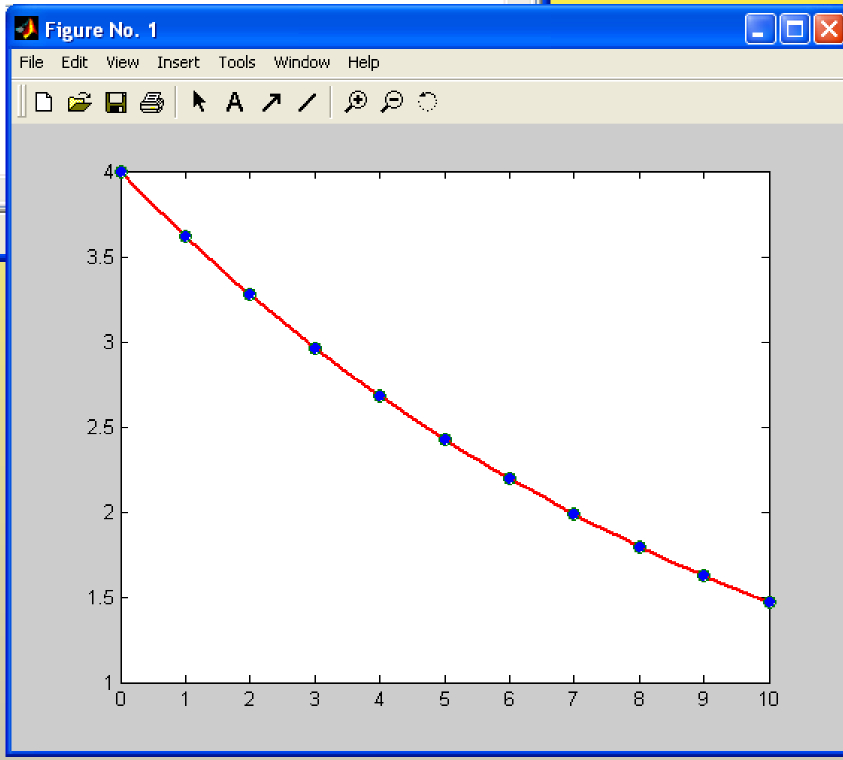


**comparison of the two solutions**

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$y' = \frac{dy}{dt}$

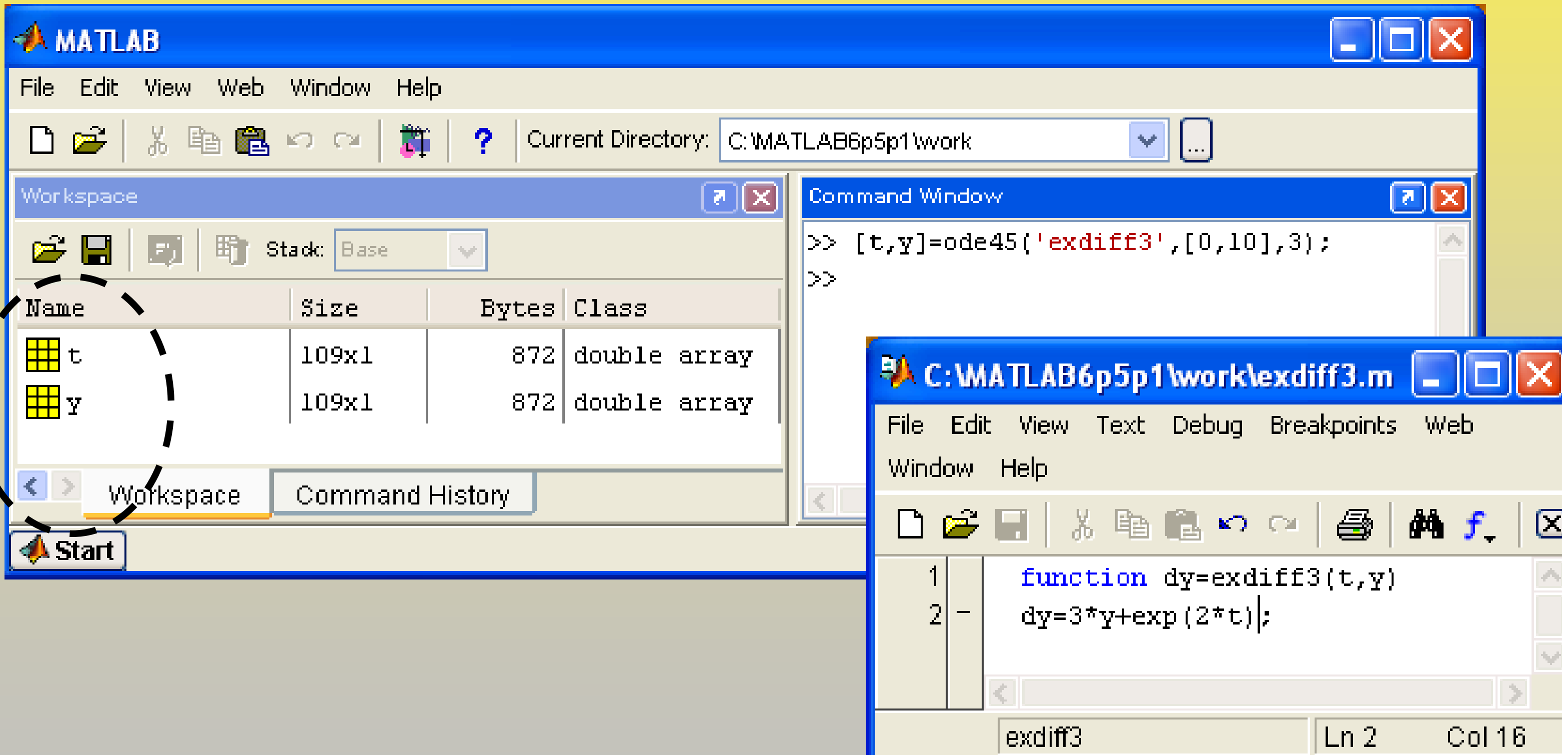
**3°** $y' = 3y + e^{2t}$

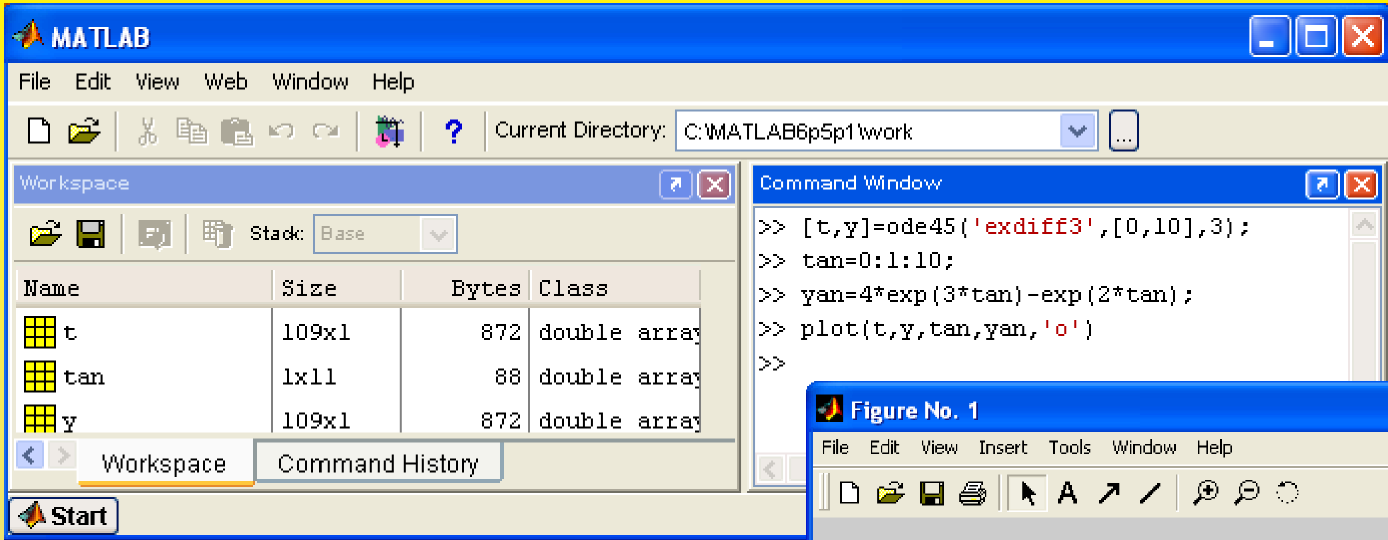
$y(0) = 3$

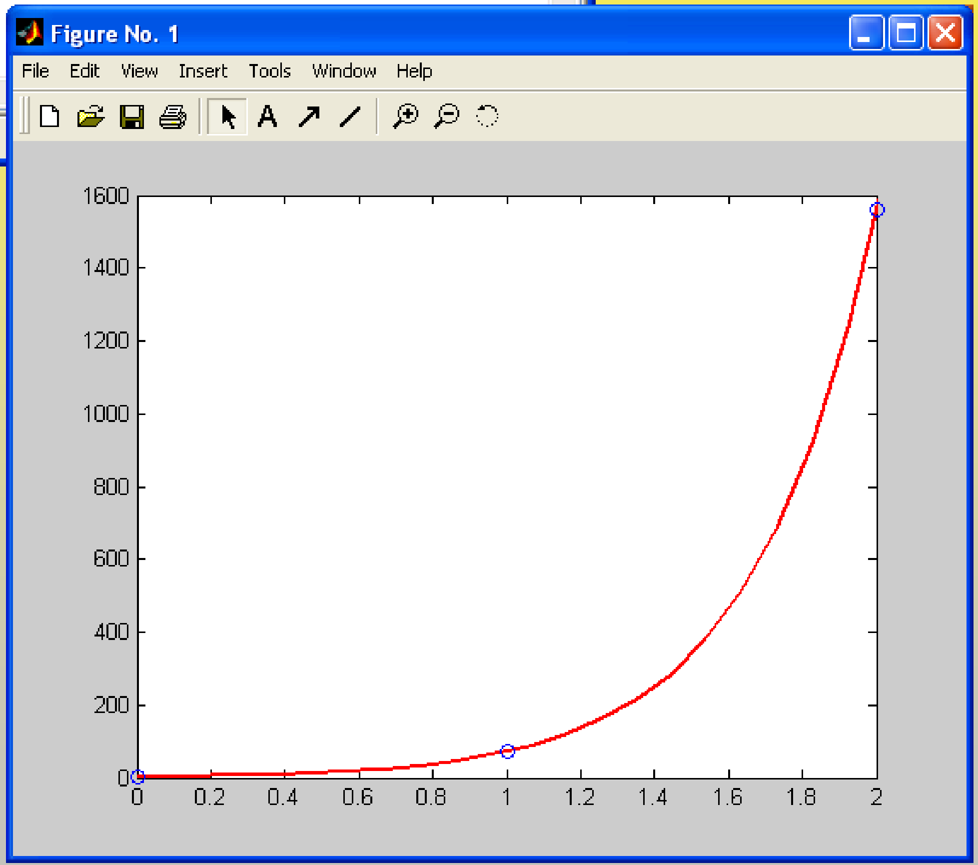
$t = [0,10]$

**analytical solution:** $y = 4e^{3t} - e^{2t}$ $t = [0,10]$

**Matlab solution:** ode23('exdiff3',[0,10],3)



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**comparison of the two solutions**

- Exercises

$y'=5y+e^{2t}$ $y'=2t$ $y'=y+yt$

**"Bad" news: we can use this solvers only for first order diff equations**

**Example:** $y'=y+yt$ $y''=y+yt$

**first order** **second order**

"Good" news: all the diff. equations ca be write like the first order diff. equations

Example: $y''' + ay'' + by' + cy = f(t)$

$$\begin{cases} y1 = y \\ y2 = y' \\ y3 = y'' \end{cases} \qquad \begin{cases} y'1 = y' = y2 \\ y'2 = y'' = y3 \\ y'3 = y''' = f(t) - ay'' - by' - cy = f(t) - ay3 - by2 - cy1 \end{cases}$$

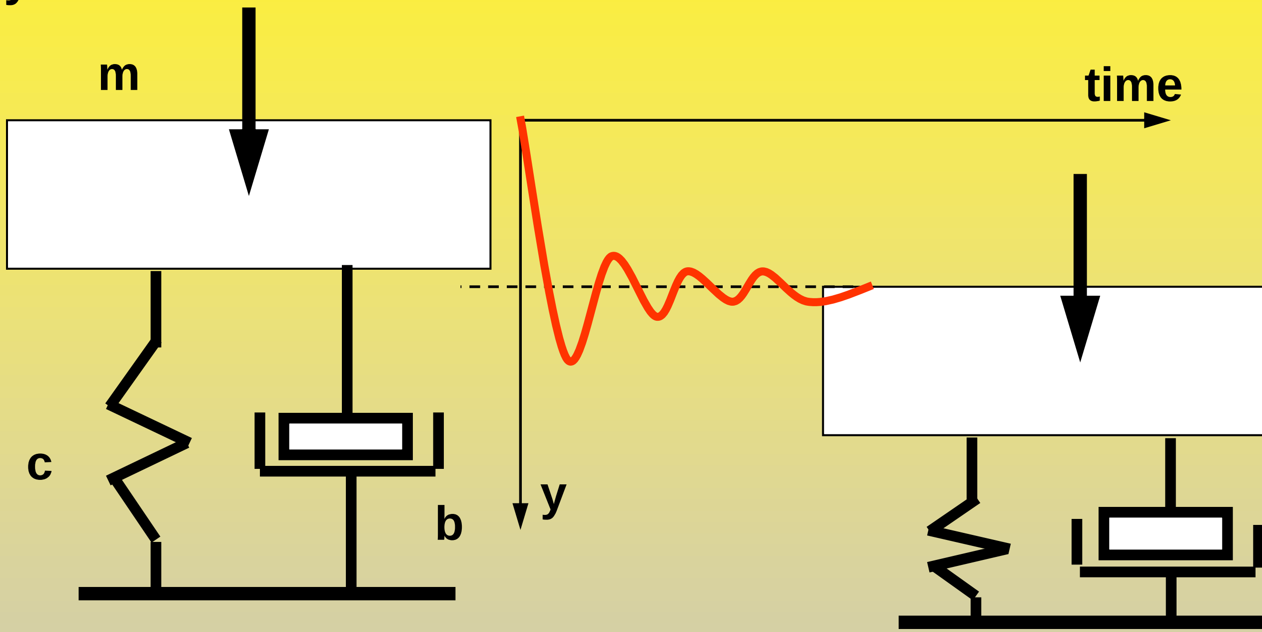
$$\begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix}' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c & -b & -a \end{pmatrix} \begin{pmatrix} y1 \\ y2 \\ y3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ f(t) \end{pmatrix}$$

**$Y' = AY + B$**

first order

# Drill problem

**Obtain the dynamical behavior of the chair**




$$my'' + by' + cy = F$$

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m=10 b=2.5 c=25 F=20

$$10y''+2.5y'+25y=20$$

$$\left\{\begin{array}{l} y1=y \\ y2=y' \end{array}\right. \quad \left\{\begin{array}{l} y1'=y2 \\ y2'=-2.5y1-0.25y2+2 \end{array}\right.$$

$$\begin{pmatrix} y1' \\ y2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2.5 & -0.25 \end{pmatrix} \begin{pmatrix} y1 \\ y2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

Y'=AY+B

Simulationstechnik

m=10 b=2.5 c=25 F=20

$$10y''+2.5y'+25y=20$$

$$\left\{\begin{array}{l} y1=y \\ y2=y' \end{array}\right. \qquad \left\{\begin{array}{l} y1'=y2 \\ y2'=-2.5y1-0.25y2+2 \end{array}\right.$$

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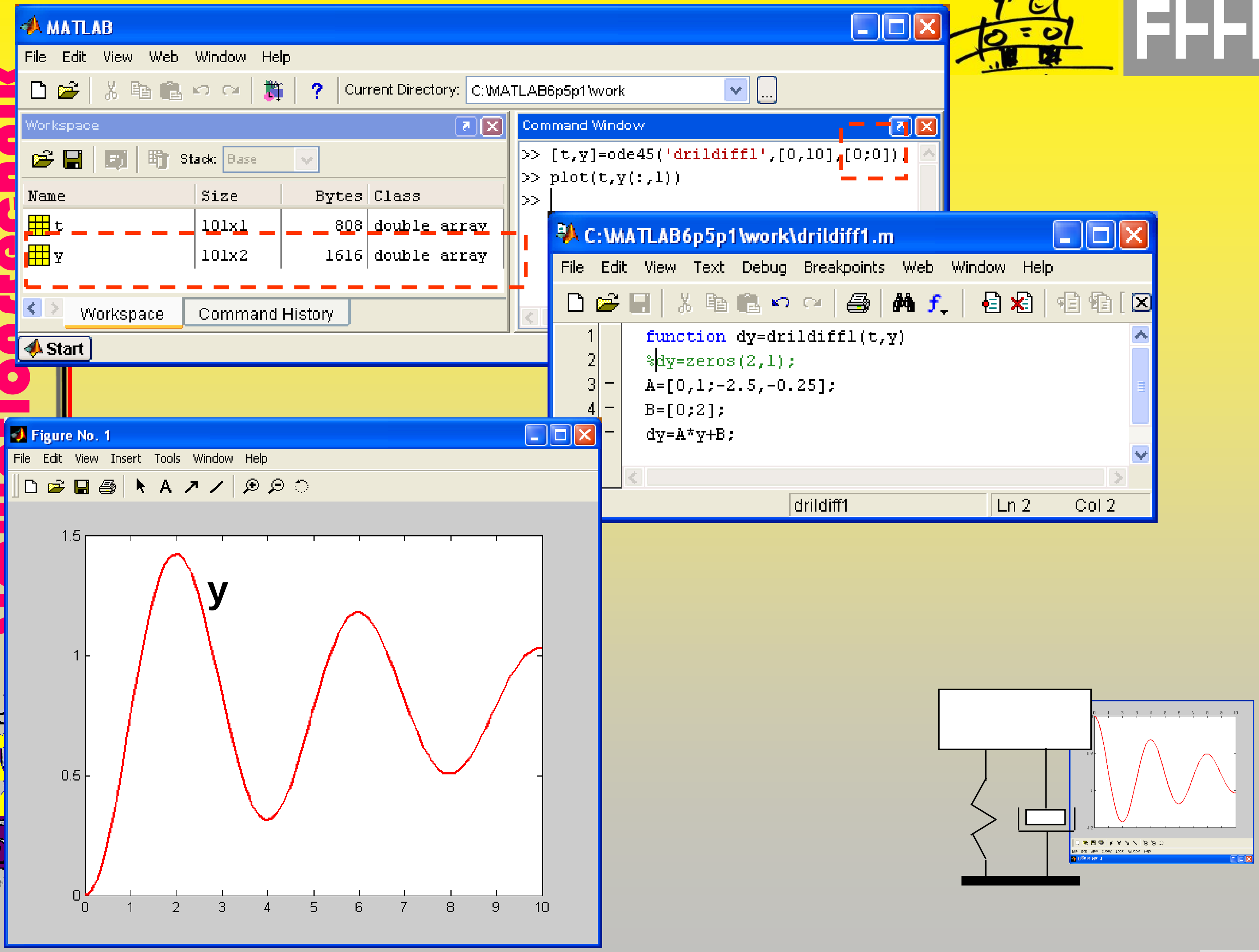
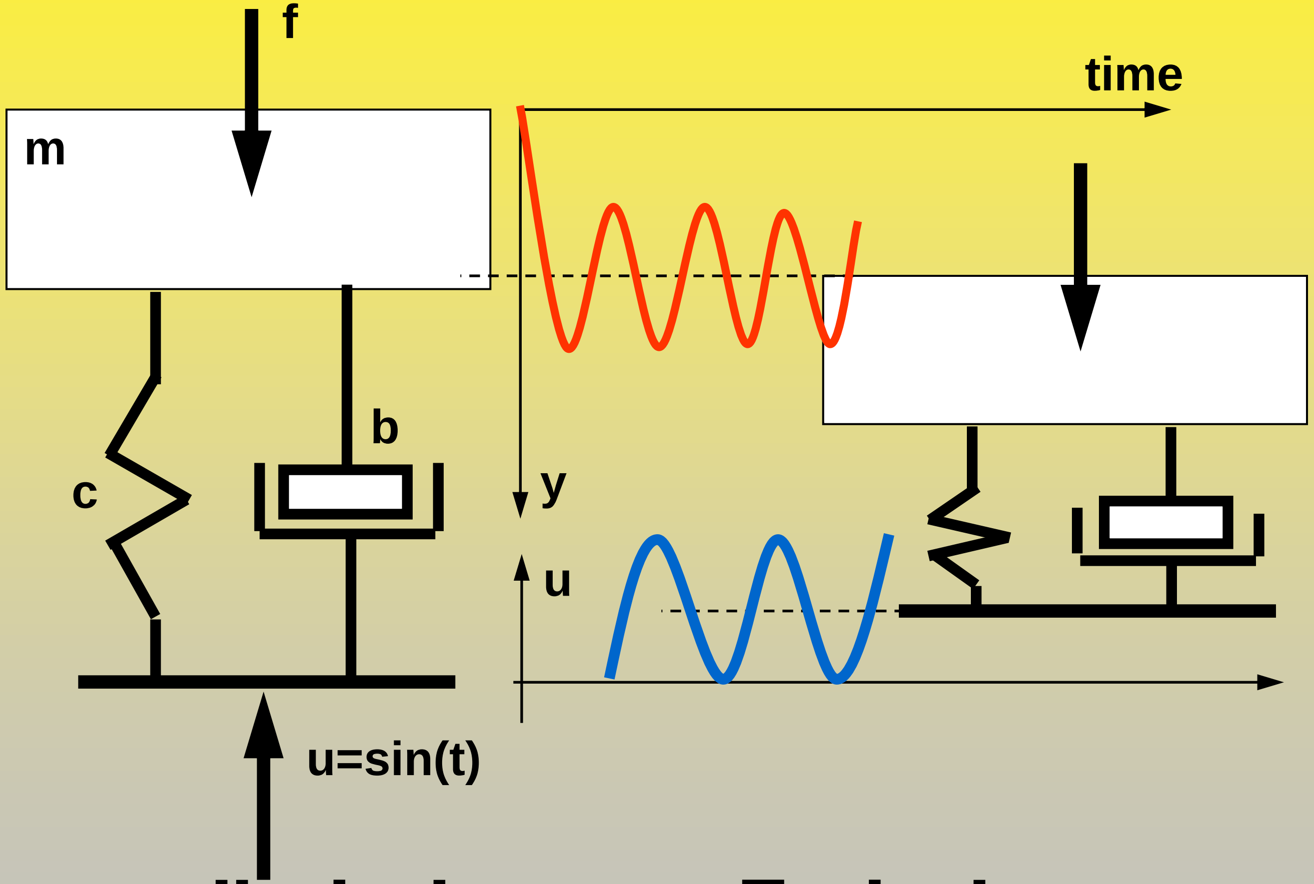
Y'=AY+B


MATLAB
File Edit View Web Window Help
Current Directory: C:\MATLAB6p5p1\work
Workspace
Stack: Base
Name Size Bytes Class
t 101x1 808 double array
y 101x2 1616 double array
Workspace Command History
Start
Command Window
>> [t,y]=ode45('drildiff1',[0,10],[0;0]);
>> plot(t,y(:,1))
>>
C:\MATLAB6p5p1\work\drildiff1.m
File Edit View Text Debug Breakpoints Web Window Help
function dy=drildiff1(t,y)
%dy=zeros(2,1);
A=[0,1;-2.5,-0.25];
B=[0;2];
dy=A*y+B;
drildiff1 Ln 2 Col 2
Figure No. 1
File Edit View Insert Tools Window Help
y
1.5
1
0.5
0
0 1 2 3 4 5 6 7 8 9 10

# Drill problem

Let complicate a little bit our problem and impose a vertical movement for the chair




$$my'' + by' + cy = F - bu' - cu$$

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$$\begin{pmatrix} y1' \\ y2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2.5 & -0.25 \end{pmatrix} \begin{pmatrix} y1 \\ y2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

the last

$$\begin{pmatrix} y1' \\ y2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2.5 & -0.25 \end{pmatrix} \begin{pmatrix} y1 \\ y2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2-0.25\cos(t)-2.5\sin(t) \end{pmatrix}$$

the actual

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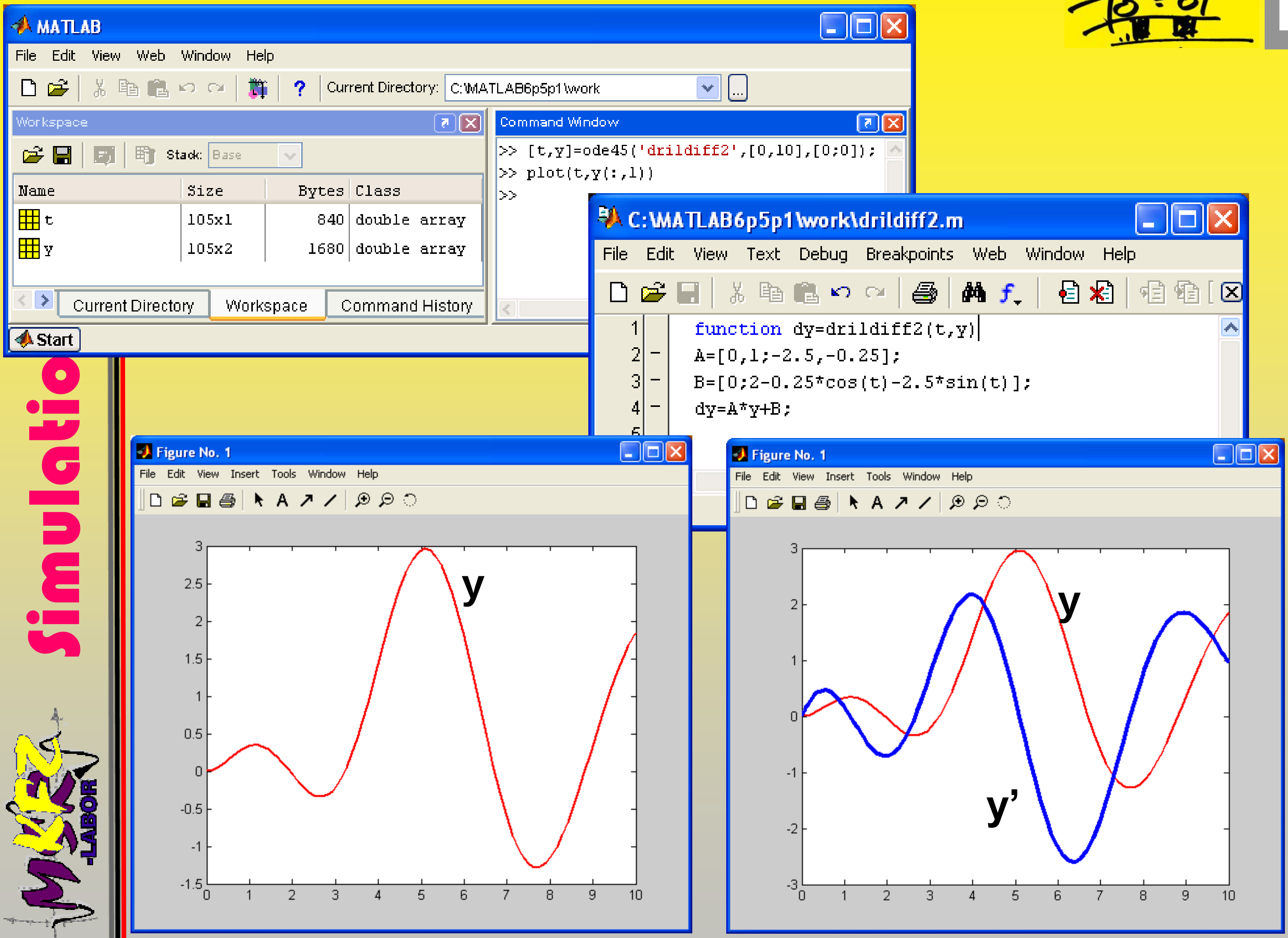
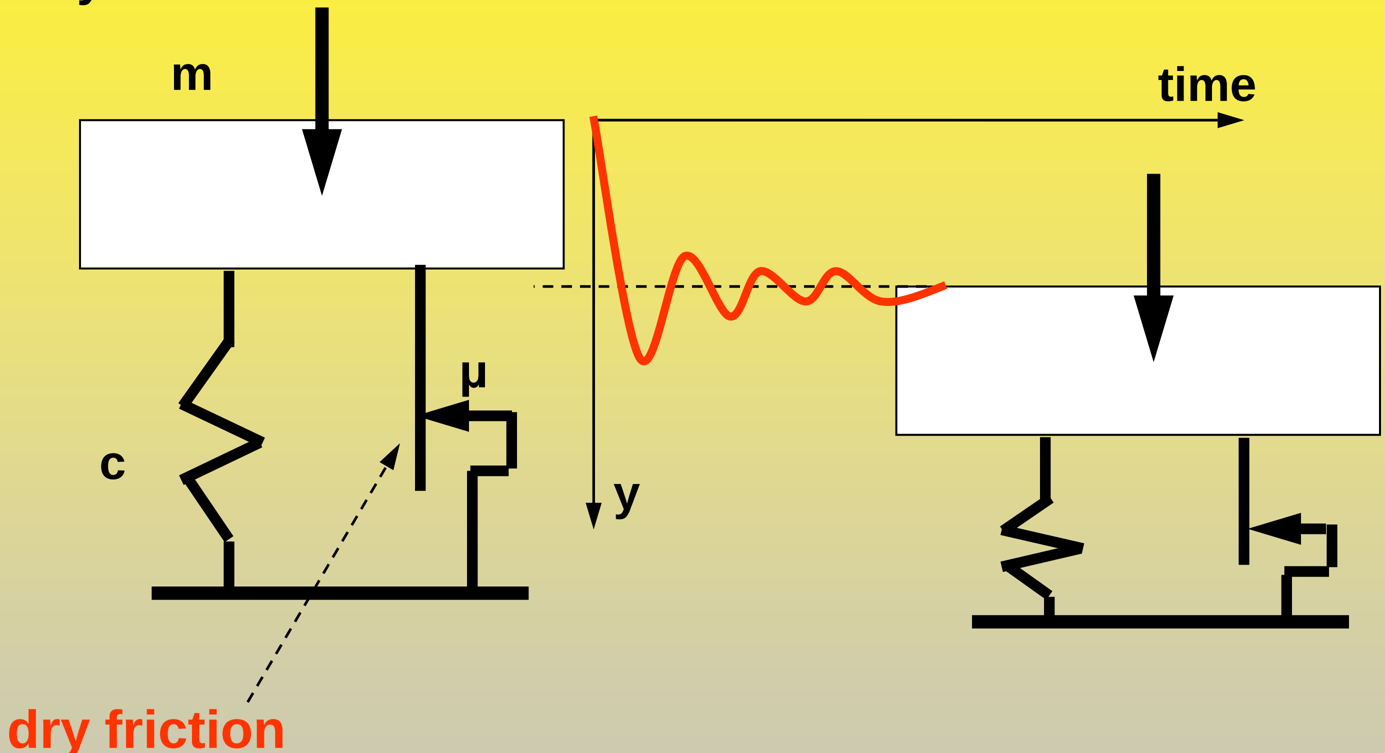

MATLAB
File Edit View Web Window Help
Current Directory: C:\MATLAB6p5p1\work
Workspace
Stack: Base
Name Size Bytes Class
t 105x1 840 double array
y 105x2 1680 double array
Current Directory Workspace Command History
Start
Command Window
>> [t,y]=ode45('drildiff2',[0,10],[0;0]);
>> plot(t,y(:,1))
>>
C:\MATLAB6p5p1\work\drildiff2.m
File Edit View Text Debug Breakpoints Web Window Help
function dy=drildiff2(t,y)
A=[0,1;-2.5,-0.25];
B=[0;2-0.25*cos(t)-2.5*sin(t)];
dy=A*y+B;
Figure No. 1
File Edit View Insert Tools Window Help
y
Figure No. 1
File Edit View Insert Tools Window Help
y
y'

# Drill problem

**Obtain the dynamical behavior of the chair**




$$my'' + cy = F - \mu N$$

$N = mg$

**the first**

$$\begin{pmatrix} y1' \\ y2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2.5 & -0.25 \end{pmatrix} \begin{pmatrix} y1 \\ y2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

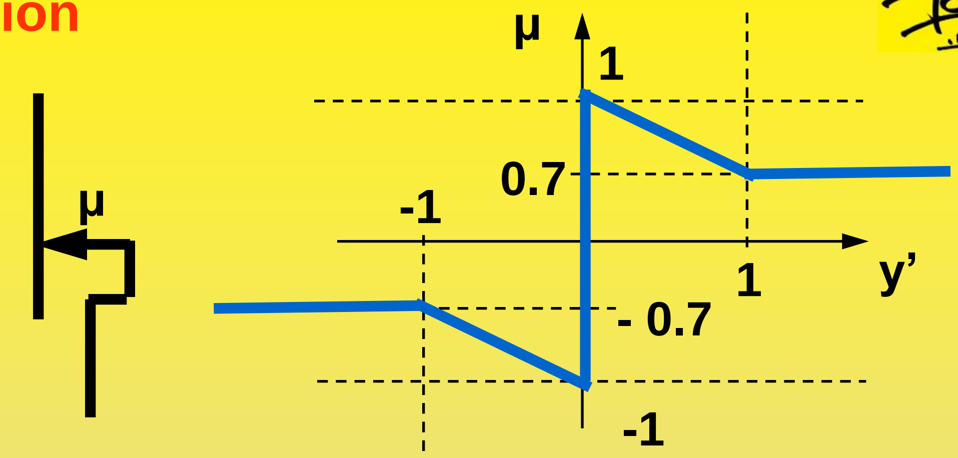
b=0

- μg

**the actual**

$$\begin{pmatrix} y1' \\ y2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2.5 & 0 \end{pmatrix} \begin{pmatrix} y1 \\ y2 \end{pmatrix} + \begin{pmatrix} 0 \\ 2-9.81\mu \end{pmatrix}$$

# the dry friction

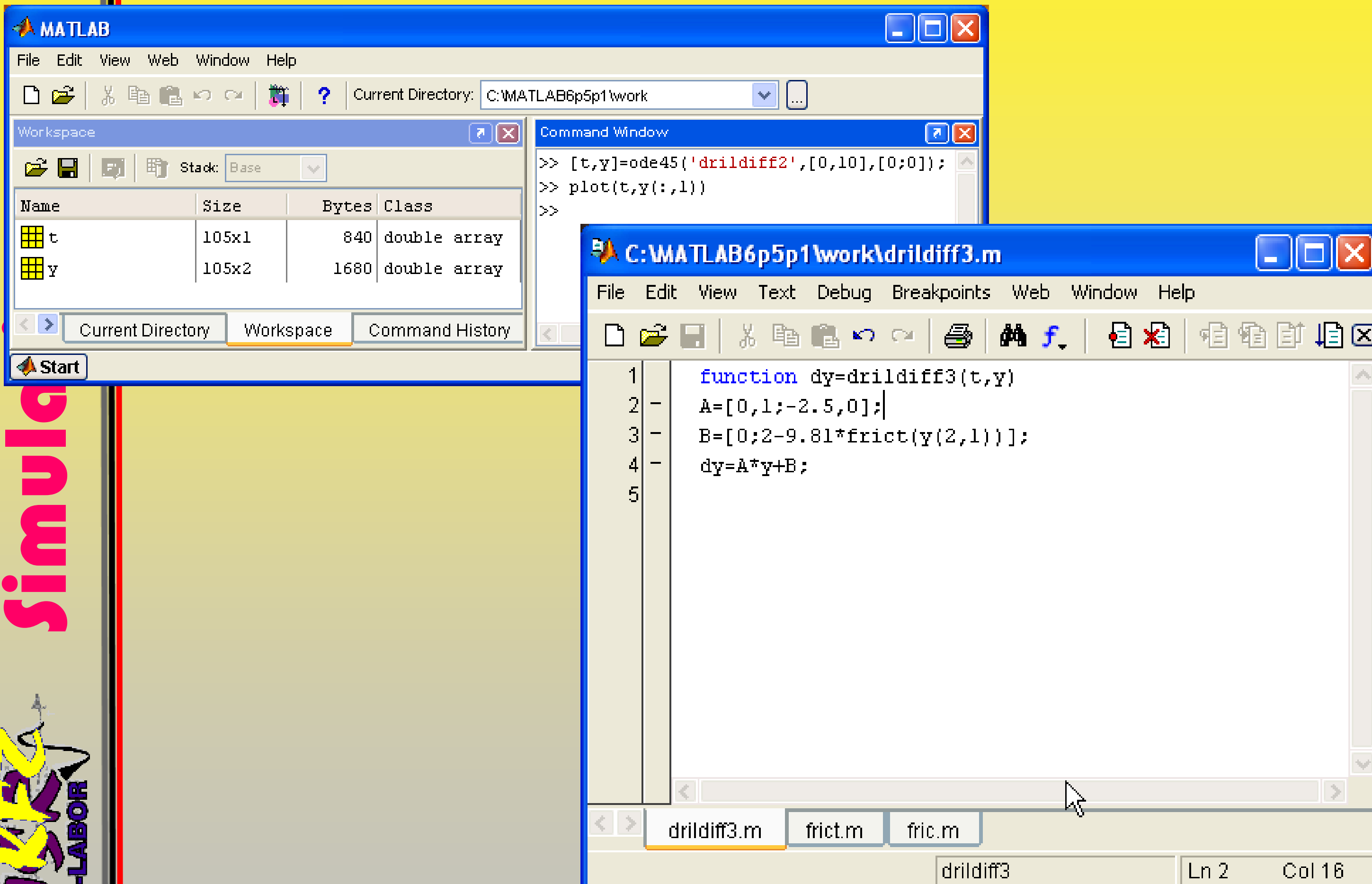



$$\mu = \begin{cases} -0.7 & y' = -\text{NaN} ; -1 \\ -0.3y'-1 & y' = -1 ; 0 \\ 0 & y' = 0 \\ -0.3y'+1 & y' = 0; 1 \\ 0.7 & y' = 1;\text{NaN} \end{cases}$$

C:\MATLAB6p5p1\work\frict.m

```
function [coef]=frict(y2)
if y2<=-1
    coef=-0.7;
elseif y2<0
    coef=-0.3*y2-1;
elseif y2==0
    coef=0;
elseif y2<=1
    coef=-0.3*y2+1;
else
    coef=0.7;
end
```

# the simulation



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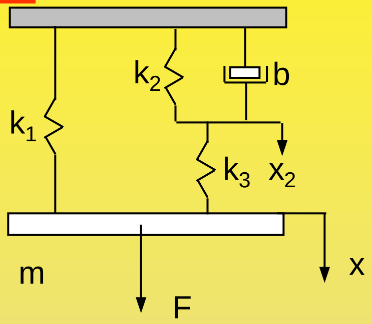
# Exercises

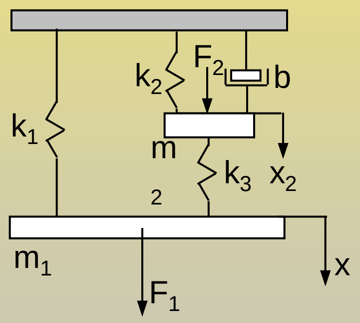
**1.**




$$\begin{cases} k_2x_2 + b\,x'_2 = k_3\,(x-x_2) \\ mx'' + k_3(x-x_2) + k_1\,x = F \end{cases}$$

**1 input 2 output**

**2.**




$$\begin{cases} mx'' + k_2x_2 + b\,x'_2 = k_3\,(x-x_2) + F_2 \\ mx'' + k_3(x-x_2) + k_1\,x = F_1 \end{cases}$$

**2 input 2 output**

# Conclusions