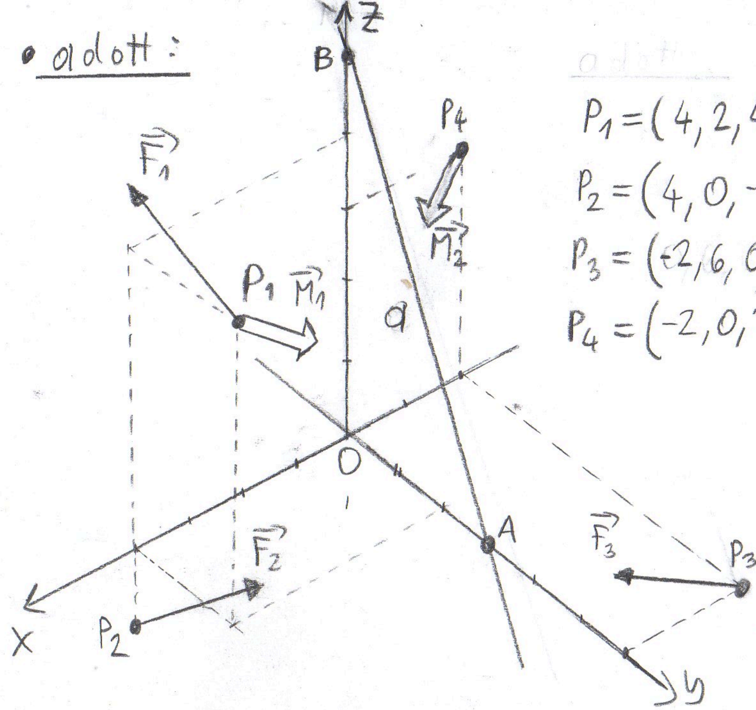
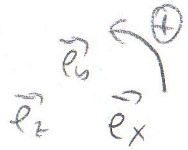
• **adott:**



$P_1 = (4,2,4)\,m$	$A = (0,3,0)\,m$
$P_2 = (4,0,-1)\,m$	$B = (0,0,5)\,m$
$P_3 = (-2,6,0)\,m$	
$P_4 = (-2,0,3)\,m$	



$\vec{F_1} = (2\vec{e_x} - 4\vec{e_y} + 3\vec{e_z})\ kN$	$\vec{M_1} = (-2\vec{e_x} + 3\vec{e_y} - \vec{e_z})\ kNm$
$\vec{F_2} = (-3\vec{e_x} + \vec{e_y} + 2\vec{e_z})\ kN$	$\vec{M_2} = (3\vec{e_x} - 3\vec{e_z})\ kN$
$\vec{F_3} = (2\vec{e_x} - 3\vec{e_y})\ kN$	

• **Feladat:** a) origóba redukált vektorkettős $(\vec{F}, \vec{M_0})$

b) $\vec{M_A}$ és $\vec{M_B}$ nyomatékok

c) "a" tengelyre számított M_a nyomaték

(A és B pont felhasználásával is!)

a) origóba redukált vektorkettős:

• $\vec{F} = \sum_{i=1}^{3} \vec{F_i} = \vec{F_1} + \vec{F_2} + \vec{F_3} = (2\vec{e_x} - 4\vec{e_y} + 3\vec{e_z}) + (-3\vec{e_x} + \vec{e_y} + 2\vec{e_z}) +$

$+ (2\vec{e_x} - 3\vec{e_y}) = (2-3+2)\vec{e_x} + (-4+1-3)\vec{e_y} + (3+2)\vec{e_z} =$

$= (\vec{e_x} - 6\vec{e_y} + 5\vec{e_z})\ kN$

• $\vec{M_0} = \sum_{i=1}^{3} \vec{r_i} \times \vec{F_i} + \sum_{j=1}^{2} \vec{M_j} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{r_3}\times\vec{F_3} + \vec{M_1} + \vec{M_2} =$

$$\vec{r_1}\times\vec{F_1} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 4 & 2 & 4 \\ 2 & -4 & 3 \end{vmatrix} = \vec{e_x}\overbrace{(2\cdot 3 - (-4)\cdot 4)}^{22} - \vec{e_y}\overbrace{(4\cdot 3 - 2\cdot 4)}^{4} + \vec{e_z}\overbrace{(4\cdot(-4) - 2\cdot 2)}^{-20} =$$

$$= (22\vec{e_x} - 4\vec{e_y} - 20\vec{e_z})\ kNm$$

$$\vec{r_2}\times\vec{F_2} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 4 & 0 & -1 \\ -3 & 1 & 2 \end{vmatrix} = \vec{e_x}\overbrace{(0\cdot 2 - 1\cdot(-1))}^{1} - \vec{e_y}\overbrace{(4\cdot 2 - (-3)(-1))}^{5} + \vec{e_z}\overbrace{(4\cdot 1 - (-3)\cdot 0)}^{4} =$$

$$= (\vec{e_x} - 5\vec{e_y} + 4\vec{e_z})\ kNm$$

$$\vec{r_3}\times\vec{F_3} = (-2\vec{e_x} + 6\vec{e_y}) \times (2\vec{e_x} - 3\vec{e_y}) = \underbrace{(-2)(-3)}_{6}\underbrace{\vec{e_x}\times\vec{e_y}}_{\vec{e_z}} + \underbrace{6\cdot 2}_{12}\underbrace{\vec{e_y}\times\vec{e_x}}_{-\vec{e_z}} =$$

$$= 6\vec{e_z} - 12\vec{e_z} = -6\vec{e_z}$$

$= (22\vec{e_x} - 4\vec{e_y} - 20\vec{e_z}) + (\vec{e_x} - 5\vec{e_y} + 4\vec{e_z}) - 6\vec{e_z} + (-2\vec{e_x} + 3\vec{e_y} - \vec{e_z}) +$

$+ (3\vec{e_x} - 3\vec{e_z}) = (22+1-2+3)\vec{e_x} + (-4-5+3)\vec{e_y} +$

$+ (-20+4-6-1-3)\vec{e_z} = (24\vec{e_x} - 6\vec{e_y} - 26\vec{e_z})\ kNm$

b) A és B pontokra számított nyomatékok:

- $\vec{M}_A = \vec{r}_{A0} \times \vec{F} + \vec{M}_0 =$

$$\boxed{\vec{r}_{A0} \times \vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -3 & 0 \\ 1 & -6 & 5 \end{vmatrix} = \vec{e}_x(-3\cdot5-(-6)\cdot0) - \vec{e}_y(0\cdot5-1\cdot0) + \vec{e}_z(0\cdot(-6)-1\cdot(-3)) = -15\vec{e}_x + 0\vec{e}_y + 3\vec{e}_z}$$

$$= -15\vec{e}_x + 0\vec{e}_y + 3\vec{e}_z + 24\vec{e}_x - 6\vec{e}_y - 26\vec{e}_z =$$

$$= (9\vec{e}_x - 6\vec{e}_y - 23\vec{e}_z)\ \text{kNm}$$

- $\vec{M}_B = \vec{r}_{B0} \times \vec{F} + \vec{M}_0 =$

$$\boxed{\vec{r}_{B0} \times \vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -5 \\ 1 & -6 & 5 \end{vmatrix} = \vec{e}_x(0\cdot5-(-6)(-5)) - \vec{e}_y(0\cdot5-1\cdot(-5)) + \vec{e}_z(0\cdot(-6)-1\cdot0) = -30\vec{e}_x - 5\vec{e}_y + 0\vec{e}_z}$$

$$= -30\vec{e}_x - 5\vec{e}_y + 0\vec{e}_z + 24\vec{e}_x - 6\vec{e}_y - 26\vec{e}_z =$$

$$= (-6\vec{e}_x - 11\vec{e}_y - 26\vec{e}_z)\ \text{kNm}$$

c) "a" tengelyre számított M_a nyomaték

- $$\vec{e}_a = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{-3\vec{e}_y + 5\vec{e}_z}{\sqrt{(-3)^2+5^2}} = -\frac{3}{\sqrt{34}}\vec{e}_y + \frac{5}{\sqrt{34}}\vec{e}_z$$

- $$M_a = \vec{M}_A \cdot \vec{e}_a = (9\vec{e}_x - 6\vec{e}_y - 23\vec{e}_z)\cdot\left(0\vec{e}_x - \frac{3}{\sqrt{34}}\vec{e}_y + \frac{5}{\sqrt{34}}\vec{e}_z\right) =$$

$$= 9\cdot0 + (-6)\left(-\frac{3}{\sqrt{34}}\right) + (-23)\frac{5}{\sqrt{34}} =$$

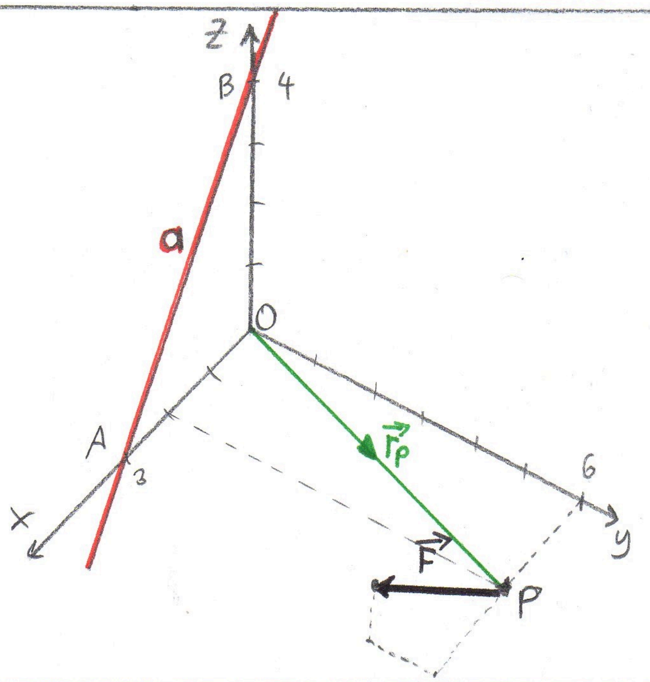
$$= \frac{18}{\sqrt{34}} - \frac{115}{\sqrt{34}} = -\frac{97}{\sqrt{34}} =$$

- $$M_a = \vec{M}_B \cdot \vec{e}_a = (-6\vec{e}_x - 11\vec{e}_y - 26\vec{e}_z)\cdot\left(0\vec{e}_x - \frac{3}{\sqrt{34}}\vec{e}_y + \frac{5}{\sqrt{34}}\vec{e}_z\right) =$$

$$= -6\cdot0 + (-11)\cdot\left(-\frac{3}{\sqrt{34}}\right) + (-26)\frac{5}{\sqrt{34}} =$$

$$= \frac{33}{\sqrt{34}} - \frac{130}{\sqrt{34}} = -\frac{97}{\sqrt{34}} =$$

ERŐ PONTRA ÉS TENGELYRE SZÁMÍTOTT NYOMATÉKA (3D)



• **adott:**

$P(2,6,0)$

$A(3,0,0)$

$B(0,0,4)$

$\vec{F} = (4\vec{e}_x - 3\vec{e}_y + 2\vec{e}_z)\,N$

• **Feladat:**

a) A és B pontra számított $\vec{M}_A$ és $\vec{M}_B$ nyomatékok

b) a tengelyre számított M_a nyomaték

a) $\vec{M}_A$ és $\vec{M}_B$ nyomaték meghatározása

• $\vec{r}_{AP}$ és $\vec{r}_{BP}$ vektorok:

$$\vec{r}_{AP} = \vec{r}_{AO} + \vec{r}_P = -3\vec{e}_x + (2\vec{e}_x + 6\vec{e}_y) = -\vec{e}_x + 6\vec{e}_y$$

$$\vec{r}_{BP} = \vec{r}_{BO} + \vec{r}_P = -4\vec{e}_z + (2\vec{e}_x + 6\vec{e}_y) = 2\vec{e}_x + 6\vec{e}_y - 4\vec{e}_z$$

$$\bullet\ \vec{M}_A = \vec{r}_{AP} \times \vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -1 & 6 & 0 \\ 4 & -3 & 2 \end{vmatrix} = \vec{e}_x\overbrace{(6\cdot 2-(-3)\cdot 0)}^{12} - \vec{e}_y\overbrace{(-1\cdot 2-4\cdot 0)}^{-2} + \vec{e}_z\overbrace{(-1\cdot(-3)-4\cdot 6)}^{-21} =$$

$$= (12\vec{e}_x + 2\vec{e}_y - 21\vec{e}_z)\,Nm$$

$$\bullet\ \vec{M}_B = \vec{r}_{BP} \times \vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 2 & 6 & -4 \\ 4 & -3 & 2 \end{vmatrix} = \vec{e}_x\overbrace{(6\cdot 2-(-4)(-3))}^{0} - \vec{e}_y\overbrace{(2\cdot 2-(-4)\cdot 4)}^{20} + \vec{e}_z\overbrace{(2\cdot(-3)-6\cdot 4)}^{-30} =$$

$$= (-20\vec{e}_y - 30\vec{e}_z)\,Nm$$

b) M_a nyomaték meghatározása

$\vec{a} = -3\vec{e}_x + 4\vec{e}_z \qquad |\vec{a}| = \sqrt{3^2+4^2} = 5\,m$

$$\vec{e}_a = \frac{\vec{a}}{|\vec{a}|} = \frac{-3\vec{e}_x + 4\vec{e}_z}{5} = (-0{,}6\,\vec{e}_x + 0{,}8\,\vec{e}_z)$$

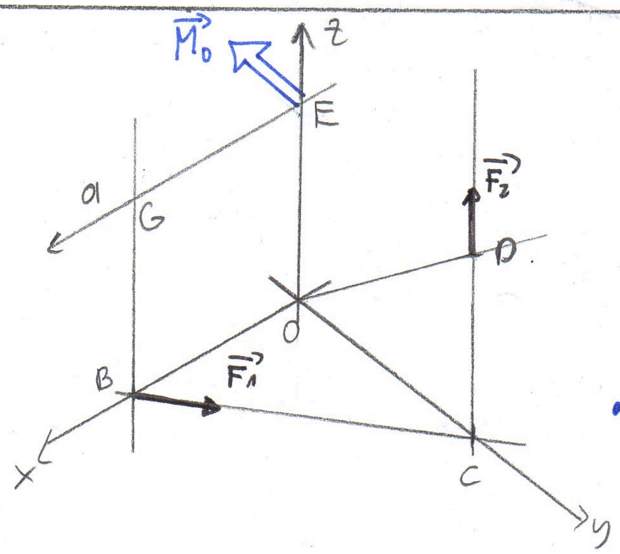
$$M_a = \vec{M}_A \cdot \vec{e}_a = (12\vec{e}_x + 2\vec{e}_y - 21\vec{e}_z)\cdot(-0{,}6\vec{e}_x + 0{,}8\vec{e}_z) = 12\cdot(-0{,}6) + 2\cdot 0 - 21\cdot 0{,}8$$

$$= -24\,Nm$$

$$M_a = \vec{M}_B\,\vec{e}_a = (-20\vec{e}_y - 30\vec{e}_z)\cdot(-0{,}6\vec{e}_x + 0{,}8\vec{e}_z) = -20\cdot 0 - 30\cdot 0{,}8 + 0\cdot(-0{,}6) =$$

$$= -24\,Nm$$

PONTRA ÉS TENGELYRE SZÁMÍTOTT NYOMATÉK (3D)



• adott:

B(3, 0, 0) m $\vec{F}_1 = (-30\vec{e}_x + 40\vec{e}_y)$ N

C(0, 4, 0) m $\vec{F}_2 = (80\vec{e}_z)$ N

D(0, 4, 3) m $\vec{M}_0 = (50\vec{e}_x + 60\vec{e}_z)$ Nm

E(0, 0, 5) m

G(3, 0, 5) m

• Feladat:

a) $\vec{F}, \vec{M}_0$ és $\vec{F}, \vec{M}_E$ eredővektorkettős

b) M_z és M_a

a) • $\vec{F} = \vec{F}_1 + \vec{F}_2 = (-30\vec{e}_x + 40\vec{e}_y) + 80\vec{e}_z = \underline{\underline{(-30\vec{e}_x + 40\vec{e}_y + 80\vec{e}_z)\,N}}$

• $\vec{M}_O = \vec{M}_0 + \vec{r}_B \times \vec{F}_1 + \vec{r}_D \times \vec{F}_2$

$$\vec{r}_B \times \vec{F}_1 = 3\vec{e}_x \times (-30\vec{e}_x + 40\vec{e}_y) = 3\cdot(-30)\overbrace{\vec{e}_x \times \vec{e}_x}^{\vec{0}} + 3\cdot 40\,\overbrace{\vec{e}_x \times \vec{e}_y}^{\vec{e}_z} = (120\vec{e}_z)\,Nm$$

$$\vec{r}_D \times \vec{F}_2 = (4\vec{e}_y + 3\vec{e}_z) \times 80\vec{e}_z = 4\cdot 80\underbrace{\vec{e}_y \times \vec{e}_z}_{\vec{e}_x} + 3\cdot 80\,\underbrace{\vec{e}_z \times \vec{e}_z}_{\vec{0}} = (320\vec{e}_x)\,Nm$$

$$\vec{M}_O = 50\vec{e}_x + 60\vec{e}_z + 120\vec{e}_z + 320\vec{e}_x = \underline{\underline{(370\vec{e}_x + 180\vec{e}_z)\,Nm}}$$

• $\vec{M}_E = \vec{M}_0 + \vec{r}_{EB} \times \vec{F}_1 + \vec{r}_{ED} \times \vec{F}_2 =$

$$\vec{r}_{EB} \times \vec{F}_1 = (3\vec{e}_x - 5\vec{e}_z) \times (-30\vec{e}_x + 40\vec{e}_y) = 3\cdot(-30)\overbrace{\vec{e}_x \times \vec{e}_x}^{\vec{0}} + 3\cdot 40\,\overbrace{\vec{e}_x \times \vec{e}_y}^{\vec{e}_z} +$$

$$+ (-5)\cdot(-30)\underbrace{\vec{e}_z \times \vec{e}_x}_{\vec{e}_y} + (-5)\cdot 40\,\underbrace{\vec{e}_z \times \vec{e}_y}_{-\vec{e}_x} = (200\vec{e}_x + 150\vec{e}_y + 120\vec{e}_z)\,Nm$$

$$\vec{r}_{ED} \times \vec{F}_2 = (4\vec{e}_y - 2\vec{e}_z) \times (80\vec{e}_z) = 4\cdot 80\underbrace{\vec{e}_y \times \vec{e}_z}_{\vec{e}_x} + (-2)\,80\,\underbrace{\vec{e}_z \times \vec{e}_z}_{\vec{0}} = (320\vec{e}_x)\,Nm$$

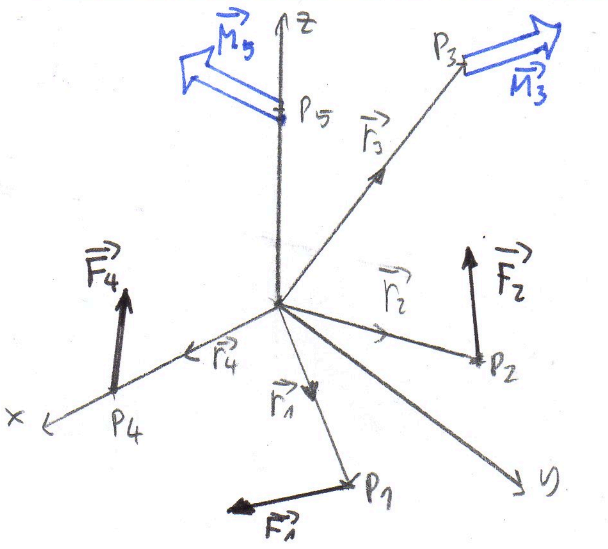
$$M_E = 50\vec{e}_x + 60\vec{e}_z + 200\vec{e}_x + 150\vec{e}_y + 120\vec{e}_z + 320\vec{e}_x = \underline{\underline{(570\vec{e}_x + 150\vec{e}_y + 180\vec{e}_z)\,Nm}}$$

b)

$$M_z = \vec{M}_O \cdot \vec{e}_z = (370\vec{e}_x + 180\vec{e}_z)\cdot\vec{e}_z = \underline{\underline{180\,Nm}}$$

$$M_a = \vec{M}_E \cdot \vec{e}_a = \vec{M}_E \cdot \vec{e}_x = (570\vec{e}_x + 150\vec{e}_y + 180\vec{e}_z)\cdot\vec{e}_x = \underline{\underline{570\,Nm}}$$

ERŐRENDSZER EGYENSÚLYA (3D)



• adott:

$\vec{r}_1 = (2\vec{e}_x + 3\vec{e}_y - \vec{e}_z)\,m$

$\vec{r}_2 = (-\vec{e}_x + 2\vec{e}_y)\,m$

$\vec{r}_3 = (-5\vec{e}_x - 3\vec{e}_y + 6\vec{e}_z)\,m$

$\vec{r}_4 = (3\vec{e}_x)\,m$

$\vec{r}_5 = (5\vec{e}_z)\,m$

$\vec{F}_1 = (2\vec{e}_x - 2\vec{e}_y - \vec{e}_z)\,N$

$\vec{F}_2 = (\vec{e}_x + 4\vec{e}_y + 3\vec{e}_z)\,N$

$\vec{F}_4 = (-3\vec{e}_x - 2\vec{e}_y + 4\vec{e}_z)\,N$

$\vec{M}_3 = (-2\vec{e}_x + 5\vec{e}_y + 2\vec{e}_z)\,Nm$

$\vec{M}_5 = (-3\vec{e}_x - 4\vec{e}_y - 4\vec{e}_z)\,Nm$

• Feladat: egyensúlyi-e az E.R.?

Az O pontra redukált vektorkettős:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_4 = (2\vec{e}_x - 2\vec{e}_y - \vec{e}_z) + (\vec{e}_x + 4\vec{e}_y + 3\vec{e}_z) + (-3\vec{e}_x - 2\vec{e}_y + 4\vec{e}_z) = (6\vec{e}_z)\,N$$

$$\vec{M}_O = \vec{M}_3 + \vec{M}_5 + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_4 \times \vec{F}_4$$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 2 & 3 & -1 \\ 2 & -2 & -1 \end{vmatrix} = \vec{e}_x\overbrace{\left(3\cdot(-1) - (-2)(-1)\right)}^{-5} - \vec{e}_y\overbrace{\left(2\cdot(-1) - 2\cdot(-1)\right)}^{0} + \vec{e}_z\overbrace{\left(2\cdot(-2) - 2\cdot 3\right)}^{-10} =$$
$$= (-5\vec{e}_x - 10\vec{e}_z)\,Nm$$

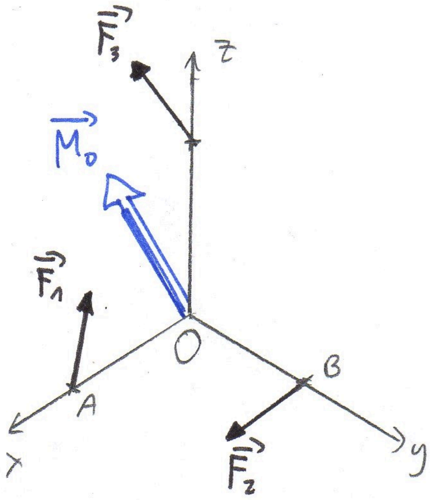
$$\vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -1 & 2 & 0 \\ 1 & 4 & 3 \end{vmatrix} = \vec{e}_x\overbrace{\left(2\cdot 3 - 0\cdot 4\right)}^{6} - \vec{e}_y\overbrace{\left(-1\cdot 3 - 1\cdot 0\right)}^{-3} + \vec{e}_z\overbrace{\left(-1\cdot 4 - 1\cdot 2\right)}^{-6} =$$
$$= (6\vec{e}_x + 3\vec{e}_y - 6\vec{e}_z)\,Nm$$

$$\vec{r}_4 \times \vec{F}_4 = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 3 & 0 & 0 \\ -3 & -2 & 4 \end{vmatrix} = \vec{e}_x\overbrace{\left(0\cdot 4 - (-2)\cdot 0\right)}^{0} - \vec{e}_y\overbrace{\left(3\cdot 4 - (-3)\cdot 0\right)}^{12} + \vec{e}_z\overbrace{\left(3\cdot(-2) - (-3)\cdot 0\right)}^{-6} =$$
$$= (-12\vec{e}_y - 6\vec{e}_z)\,Nm$$

$$\vec{M}_O = -2\vec{e}_x + 5\vec{e}_y + 2\vec{e}_z - 3\vec{e}_x - 4\vec{e}_y - 4\vec{e}_z - 5\vec{e}_x - 10\vec{e}_z + 6\vec{e}_x + 3\vec{e}_y - 6\vec{e}_z -$$
$$- 12\vec{e}_y - 6\vec{e}_z = (-4\vec{e}_x - 8\vec{e}_y - 24\vec{e}_z)\,Nm$$

$\vec{F} \neq 0$; $\vec{M}_O \neq 0$ $\Rightarrow$ Az erőrendszer nem egyensúlyi

ERŐRENDSZER EGYENSÚLYA (3D)



• adott:

$\vec{F}_1 = (-\vec{e}_x - 1{,}5\vec{e}_y)\ kN$

$\vec{F}_2 = (1{,}5\vec{e}_y - 4\vec{e}_z)\ kN$

$\vec{F}_3 = (\vec{e}_x + 4\vec{e}_z)\ kN$

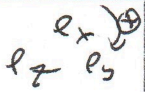
$\vec{M}_0 = (12\vec{e}_x - 4\vec{e}_y + 6\vec{e}_z)\ kNm$

$\vec{r}_A = (4\vec{e}_x)\ m \quad \vec{r}_B = (3\vec{e}_y)\ m \quad \vec{r}_C = (4\vec{e}_z)\ m$

• feladat:

egyensúlyi-e az ER.?

Az O ponti redukált vektorkettős:



$$\vec{F} = F_1 + F_2 + F_3 = (-\vec{e}_x - 1{,}5\vec{e}_y) + (1{,}5\vec{e}_y - 4\vec{e}_z) + (\vec{e}_x + 4\vec{e}_z) = \vec{0}$$

$$\vec{M}_0 = \vec{M}_0 + \sum_{i=1}^{3} \vec{r}_i \times \vec{F}_i = \vec{M}_0 + \vec{r}_A \times \vec{F}_1 + \vec{r}_B \times \vec{F}_2 + \vec{r}_C \times \vec{F}_3$$

$$\vec{r}_A \times \vec{F}_1 = 4\vec{e}_x \times (-\vec{e}_x - 1{,}5\vec{e}_y) = 4\cdot(-1)\underbrace{\vec{e}_x \times \vec{e}_x}_{\vec{0}} + 4\cdot(-1{,}5)\underbrace{\vec{e}_x \times \vec{e}_y}_{\vec{e}_z} = (-6\vec{e}_z)$$

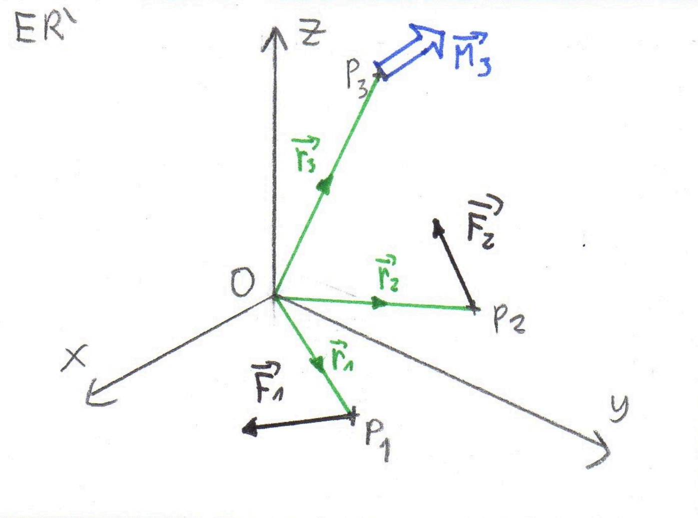
$$\vec{r}_B \times \vec{F}_2 = 3\vec{e}_y \times (1{,}5\vec{e}_y - 4\vec{e}_z) = 3\cdot 1{,}5\underbrace{\vec{e}_y \times \vec{e}_y}_{\vec{0}} + 3\cdot(-4)\underbrace{\vec{e}_y \times \vec{e}_z}_{\vec{e}_x} = (-12\vec{e}_x)$$

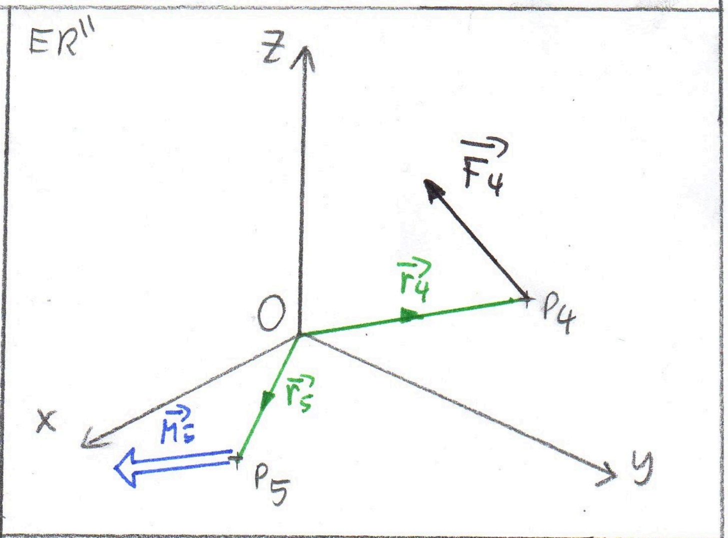
$$\vec{r}_C \times \vec{F}_3 = 4\vec{e}_z \times (\vec{e}_x + 4\vec{e}_z) = 4\cdot 1\underbrace{\vec{e}_z \times \vec{e}_x}_{\vec{e}_y} + 4\cdot 4\cdot\underbrace{\vec{e}_z \times \vec{e}_z}_{\vec{0}} = 4\vec{e}_y$$

$$\vec{M}_0 = 12\vec{e}_x - 4\vec{e}_y + 6\vec{e}_z - 6\vec{e}_z - 12\vec{e}_x + 4\vec{e}_y = \vec{0}$$

Az erőrendszer egyensúlyi

ER. EGYENÉRTÉKŰSÉGE (3D)





$\vec{r}_1 = (\vec{e}_x + 2\vec{e}_y - \vec{e}_z)\ m$

$\vec{r}_2 = (-2\vec{e}_x + \vec{e}_y)\ m$

$\vec{r}_3 = (-5\vec{e}_x - 3\vec{e}_y - \vec{e}_z)\ m$

$\vec{F}_1 = (2\vec{e}_x - 2\vec{e}_y + \vec{e}_z)\ N$

$\vec{F}_2 = (\vec{e}_x + 3\vec{e}_z)\ N$

$\vec{M}_3 = (-2\vec{e}_x + \vec{e}_y + 4\vec{e}_z)\ Nm$

$\vec{r}_4 = (-2\vec{e}_x + \vec{e}_y)\ m$

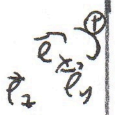
$\vec{r}_5 = (3\vec{e}_x + 2\vec{e}_y - \vec{e}_z)\ m$

$\vec{F}_4 = (3\vec{e}_x - 2\vec{e}_y + 4\vec{e}_z)\ N$

$\vec{M}_5 = (-3\vec{e}_x - 4\vec{e}_y - 4\vec{e}_z)\ Nm$

• **Feladat:** ER' $\stackrel{?}{=}$ ER''

$$\vec{F}' = \vec{F}_1 + \vec{F}_2 = (2\vec{e}_x - 2\vec{e}_y + \vec{e}_z) + (\vec{e}_x + 3\vec{e}_z) = \underline{\underline{(3\vec{e}_x - 2\vec{e}_y + 4\vec{e}_z)\ N}}$$



$$\vec{M}'_0 = \vec{M}_3 + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 2 & -1 \\ 2 & -2 & 1 \end{vmatrix} = \vec{e}_x\left(2\cdot 1 - (-2)(-1)\right) - \vec{e}_y\left(1\cdot 1 - 2\cdot(-1)\right) + \vec{e}_z\left(1\cdot(-2) - 2\cdot 2\right) =$$

$$= (-3\vec{e}_y - 6\vec{e}_z)\ Nm$$

$$\vec{r}_2 \times \vec{F}_2 = (-2\vec{e}_x + \vec{e}_y) \times (\vec{e}_x + 3\vec{e}_z) =$$

$$= -2\cdot 1(\vec{e}_x \times \vec{e}_x) + (-2)\cdot 3\,\underbrace{\vec{e}_x \times \vec{e}_z}_{-\vec{e}_y} + 1\cdot 1\cdot \underbrace{\vec{e}_y \times \vec{e}_x}_{-\vec{e}_z} + 1\cdot 3\,\underbrace{\vec{e}_y \times \vec{e}_z}_{\vec{e}_x} =$$

$$= 3\vec{e}_x + 6\vec{e}_y - \vec{e}_z$$

$$\vec{M}'_0 = -2\vec{e}_x + \vec{e}_y + 4\vec{e}_z - 3\vec{e}_y - 6\vec{e}_z + 3\vec{e}_x + 6\vec{e}_y - \vec{e}_z =$$

$$= \underline{\underline{(\vec{e}_x + 4\vec{e}_y - 3\vec{e}_z)\ Nm}}$$

$$\vec{F}'' = \vec{F}_4 = \underline{\underline{(3\vec{e}_x - 2\vec{e}_y + 4\vec{e}_z)}}\ N$$

$$\vec{M}''_0 = \vec{M}_5 + \vec{r}_4 \times \vec{F}_4$$

$$\vec{r}_4 \times \vec{F}_4 = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -2 & 1 & 0 \\ 3 & -2 & 4 \end{vmatrix} =$$

$$= \vec{e}_x\overbrace{\left(1\cdot 4 - 0\cdot(-2)\right)}^{4} - \vec{e}_y\overbrace{\left(-2\cdot 4 - 3\cdot 0\right)}^{-8} + \vec{e}_z\underbrace{\left((-2)(-2) - 3\cdot 1\right)}_{+1} = (4\vec{e}_x + 8\vec{e}_y + \vec{e}_z)\ Nm$$

$$\vec{M}''_0 = -3\vec{e}_x - 4\vec{e}_y - 4\vec{e}_z + 4\vec{e}_x + 8\vec{e}_y + \vec{e}_z =$$

$$= \underline{\underline{(\vec{e}_x + 4\vec{e}_y - 3\vec{e}_z)\ Nm}}$$

$$\left.\begin{array}{l} \vec{F}' = \vec{F}'' \\ \vec{M}'_0 = \vec{M}''_0 \end{array}\right\} \Rightarrow \text{ER}' = \text{ER}''$$

A 2 erőrendszer egyenértékű