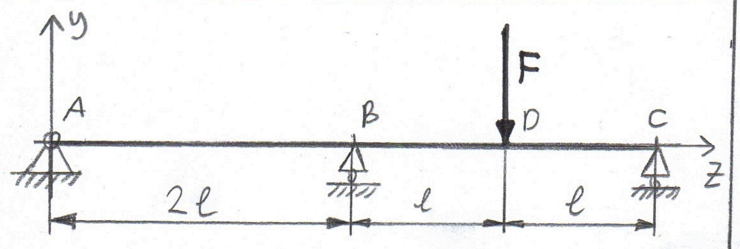
STATIKAILAG HATÁROZATLAN EGYENES RÚD



adott:
$\ell = 2m$
$F = 32\,kN$
$I_x = 50000\,mm^4$
$E = 2{,}1 \cdot 10^5\,MPa$

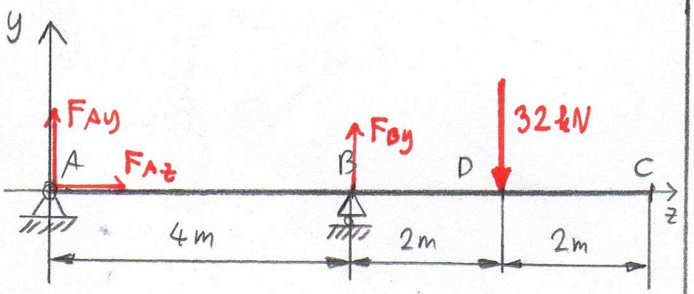
Feladat:
- támasztóerők
- igénybevételi ábrák

- ismeretlen támasztóerők: $F_{Ay}, F_{Az}, F_{By}, F_{Cy} \Rightarrow 4db$
- statikai egyenletek: $F_y = 0;\ F_z = 0;\ M_a = 0 \Rightarrow 3db$

$\Rightarrow$ A tartó statikailag egyszeresen határozatlan

A határozottá tétel egy lehetséges módja: C görgő elhagyása

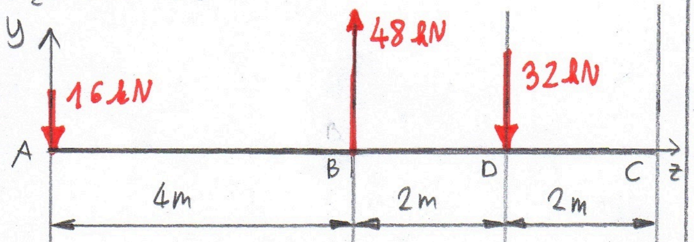
- eredeti terhelés a határozottá tett tartón

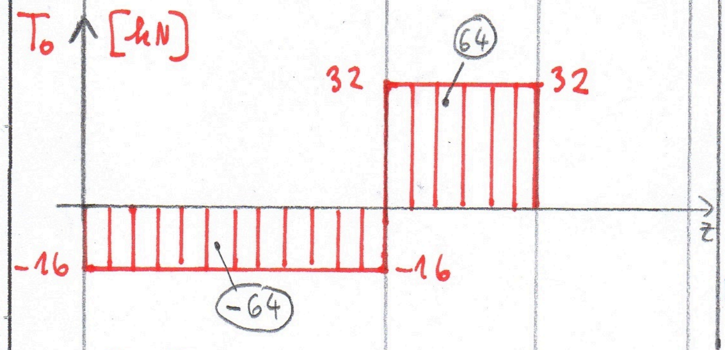


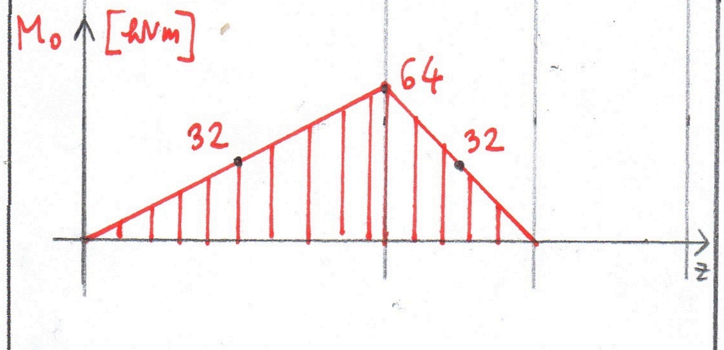
$M_a = 0 = 6 \cdot 32 - 4F_{By} \Rightarrow F_{By} = 48\,kN\ (\uparrow)$

$M_b = 0 = 2 \cdot 32 + 4F_{Ay} \Rightarrow F_{Ay} = -16\,kN\ (\downarrow)$

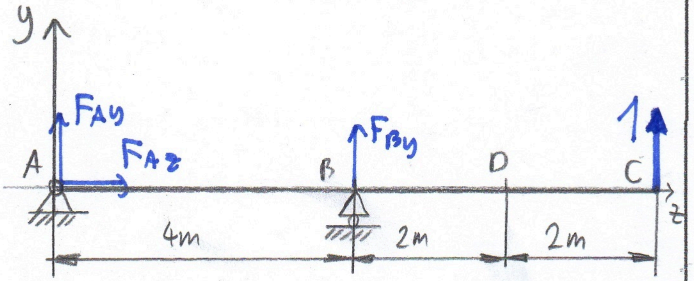
$F_z = 0 = F_{Az}$







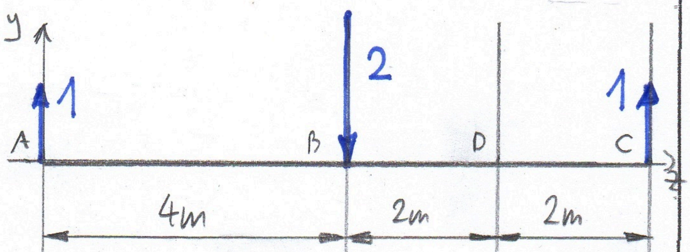
- az elhagyott támasztóerőnek megfelelő egységnyi nagyságú terhelés

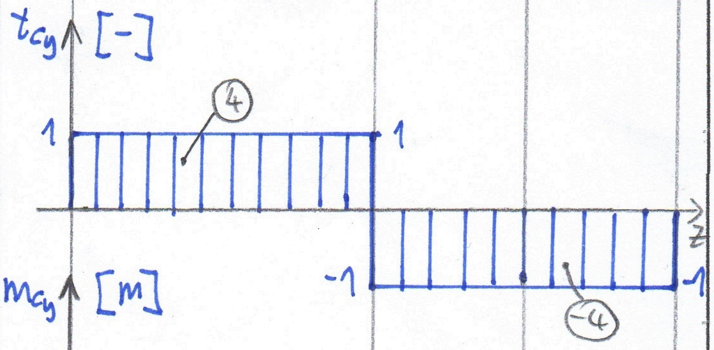


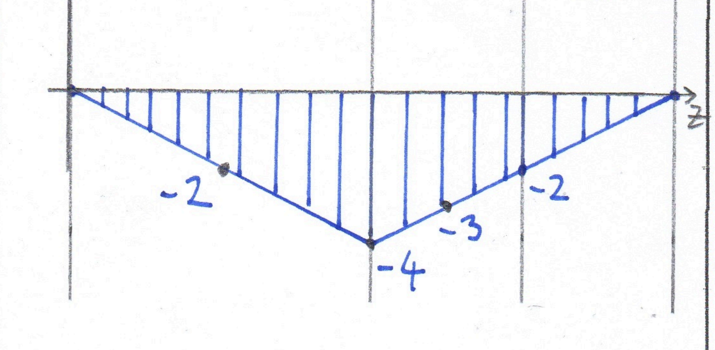
$M_a = 0 = -8 \cdot 1 - 4F_{By} \Rightarrow F_{By} = -2\ (\downarrow)$

$M_b = 0 = -4 \cdot 1 + 4F_{Ay} \Rightarrow F_{Ay} = 1\ (\uparrow)$

$F_z = 0 = F_{Az}$







• az alakváltozási energia:

$$U \approx U_H = \frac{1}{2}\int_0^{4\ell} \frac{M_{hx}^2}{I_x E}\,dz = \frac{1}{2}\int_0^{4\ell} \frac{(M_0 + F_{cy} m_{cy})^2}{I_x E}\,dz$$

• Castigliano-tétel + kinematikai peremfeltétel:

$$v_c = 0 = \frac{\partial U}{\partial F_{cy}} = \int_0^{4\ell} \frac{(M_0 + F_{cy} m_{cy})\, m_{cy}}{I_x E}\,dz \quad / \cdot I_x E$$

$$0 = \int_0^{4\ell} M_0 m_{cy} + F_{cy} m_{cy}^2\, dz$$

$$0 = \int_0^{4\ell} M_0 m_{cy}\, dz + F_{cy}\int_0^{4\ell} m_{cy}^2\, dz$$

$$F_{cy} = -\frac{\int_0^{4\ell} M_0 m_{cy}\, dz}{\int_0^{4\ell} m_{cy}^2\, dz}$$

• integrálok kiszámítása Simpson formulával:

$$\int_0^{4\ell} M_0 m_{cy}\, dz = \int_0^{2\ell} M_0 m_{cy}\, dz + \int_{2\ell}^{3\ell} M_0 m_{cy}\, dz + \underbrace{\int_{3\ell}^{4\ell} M_0 m_{cy}\, dz}_{=0} =$$

$$= \frac{4}{6}\left[0 \cdot 0 + 4 \cdot 32 \cdot (-2) + 64 \cdot (-4)\right] + \frac{2}{6}\left[64 \cdot (-4) + 4 \cdot 32\,(-3) + 0 \cdot (-2)\right] =$$

$$= \frac{4}{6}\left[-512\right] + \frac{2}{6}\left[-640\right] = -\frac{2048}{6} - \frac{1280}{6} = -\frac{3328}{6}\ \mathrm{kNm^3}$$

$$\int_0^{4\ell} m_{cy}^2\, dz = \int_0^{2\ell} m_{cy}^2\, dz + \int_{2\ell}^{4\ell} m_{cy}^2\, dz =$$

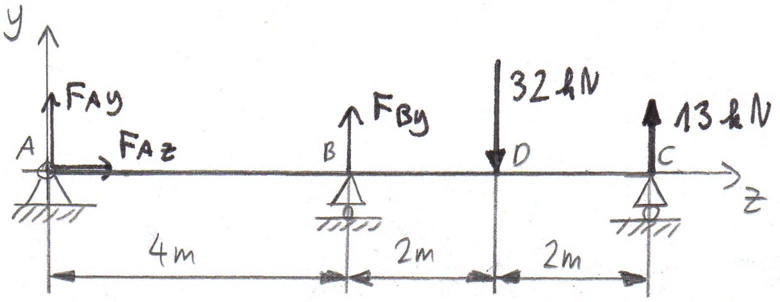
$$= \frac{4}{6}\left[0^2 + 4 \cdot (-2)^2 + (-4)^2\right] + \frac{4}{6}\left[(-4)^2 + 4 \cdot (-2)^2 + 0^2\right] =$$

$$= \frac{4}{6}\left[32\right] + \frac{4}{6}\left[32\right] = \frac{128}{6} + \frac{128}{6} = \frac{256}{6}\ \mathrm{m^3}$$

• visszahelyettesítve F_{cy} képletébe:

$$F_{cy} = -\frac{-\frac{3328}{6}}{\frac{256}{6}} = -\frac{-3328}{256} = 13\,kN\ (\uparrow)$$

• többi támasztóerő → statikai egyenletekből:



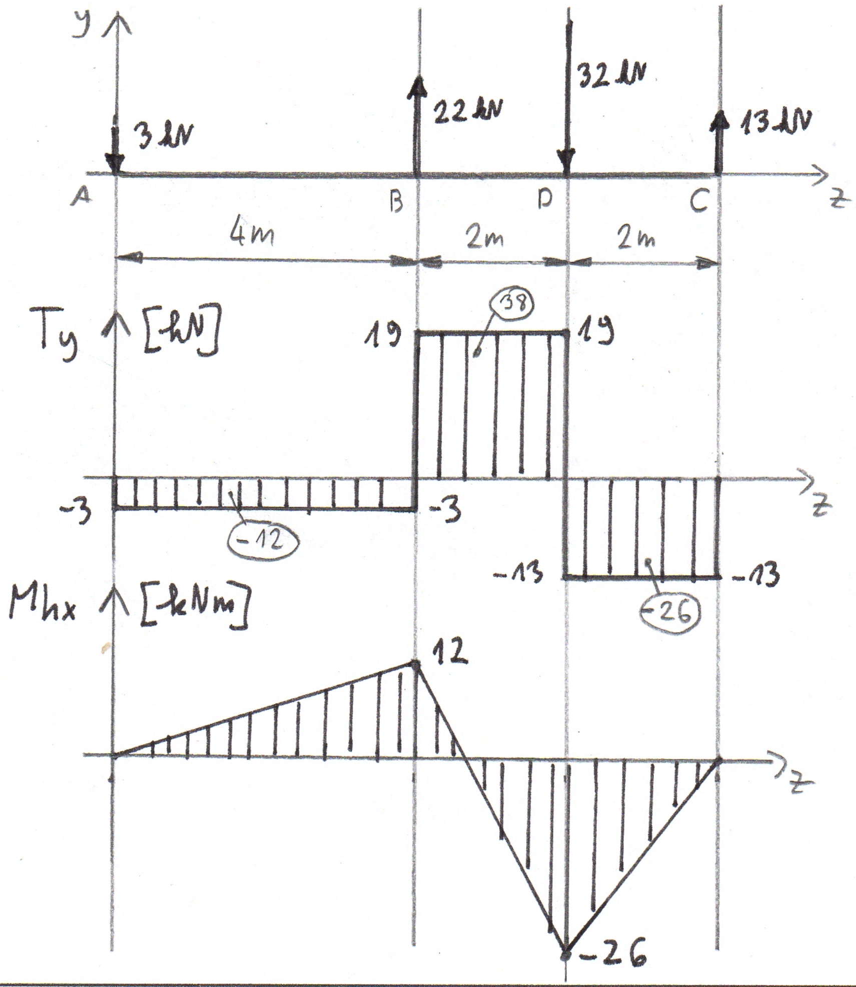
$$M_a = 0 = -4F_{By} + 6\cdot 32 - 8\cdot 13 \Rightarrow F_{By} = \frac{8\cdot 13 - 6\cdot 32}{-4} = 22\,kN\ (\uparrow)$$

$$M_b = 0 = 4F_{Ay} + 2\cdot 32 - 4\cdot 13 \Rightarrow F_{Ay} = \frac{4\cdot 13 - 2\cdot 32}{4} = -3\,kN\ (\downarrow)$$

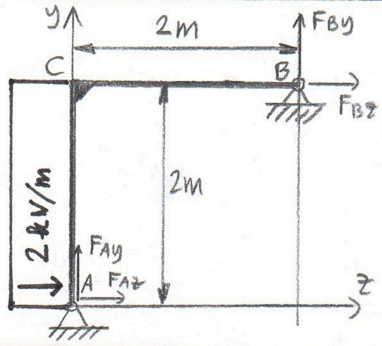
$$F_z = 0 = F_{Az}$$

$$\vec{F_A} = (-3\vec{e_y})\,kN\ ;\ \vec{F_B} = (22\vec{e_y})\,kN\ ;\ \vec{F_C} = (13\vec{e_y})\,kN$$

• eredeti szerkezet igénybevételi ábrái:



STATIKAILAG HATÁROZATLAN TÖRTVONALÚ TARTÓ



• ismeretlen támasztóerők: F_{Ay}, F_{Az}, F_{By}, F_{Bz} $\Rightarrow$ 4 db

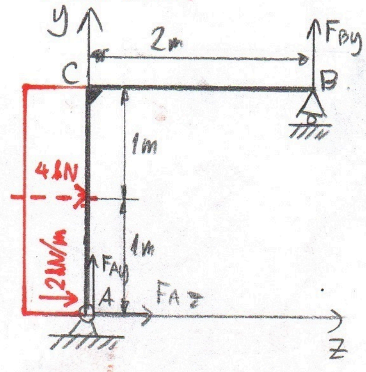
• statikai egyenletek: $F_y=0$, $F_z=0$, $M_a=0$ $\Rightarrow$ 3 db

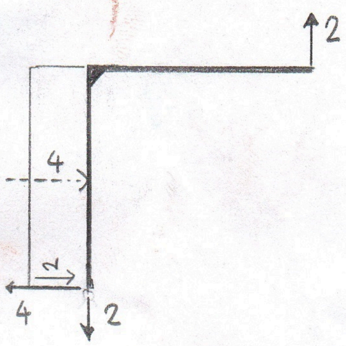
A tartó statikailag egyszeresen határozatlan

$$d=100\,mm \Rightarrow A=\frac{d^2\pi}{4}=\frac{0{,}1^2\pi}{4}=7{,}85398\cdot10^{-3}\ m^2$$

$$I_x=\frac{d^4\pi}{64}=\frac{0{,}1^4\pi}{64}=4{,}90873\cdot10^{-6}\ m^4$$

• eredeti terhelés a határozottá tett tartón





- támasztóerők:

$M_a=0=1\cdot4-2F_{By}$

$\Downarrow$

$F_{By}=\frac{4}{2}=2\,kN\,(\uparrow)$

$F_y=0=F_{Ay}+F_{By}$

$\Downarrow$

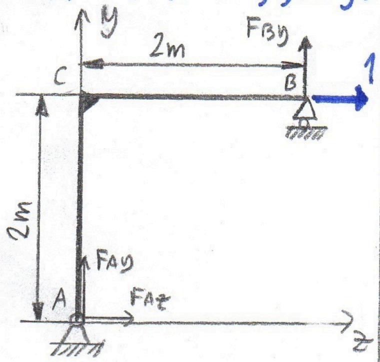
$F_{Ay}=-F_{By}=-2\,kN\,(\downarrow)$

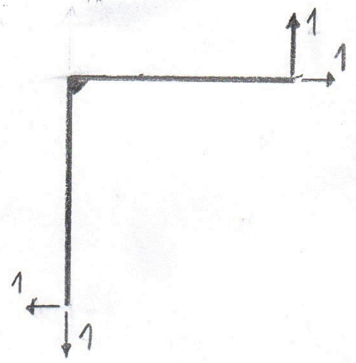
$F_z=0=F_{Az}+4$

$\Downarrow$

$F_{Az}=-4\,kN\,(\leftarrow)$

• az elhagyott támasztóerőnek megfelelő egységnyi nagyságú terhelés





- támasztóerők

$M_a=0=2\cdot1-2F_{By}$

$\Downarrow$

$F_{By}=\frac{2\cdot1}{2}=1\,(\uparrow)$

$F_y=0=F_{Ay}+F_{By}$

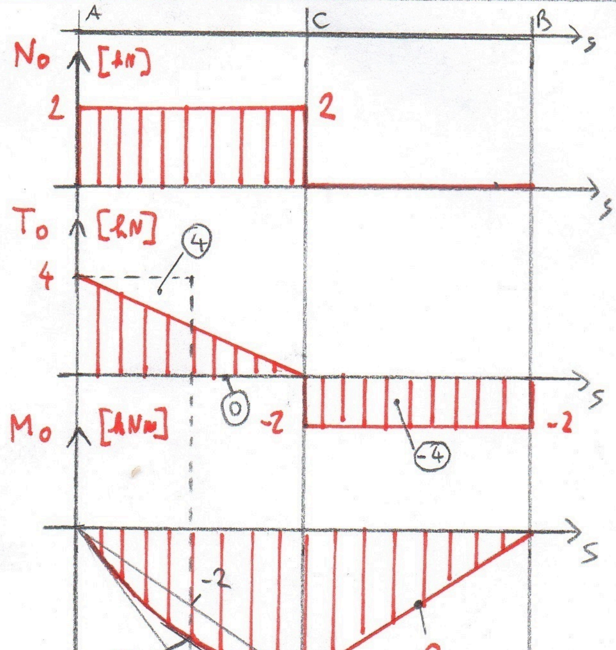
$\Downarrow$

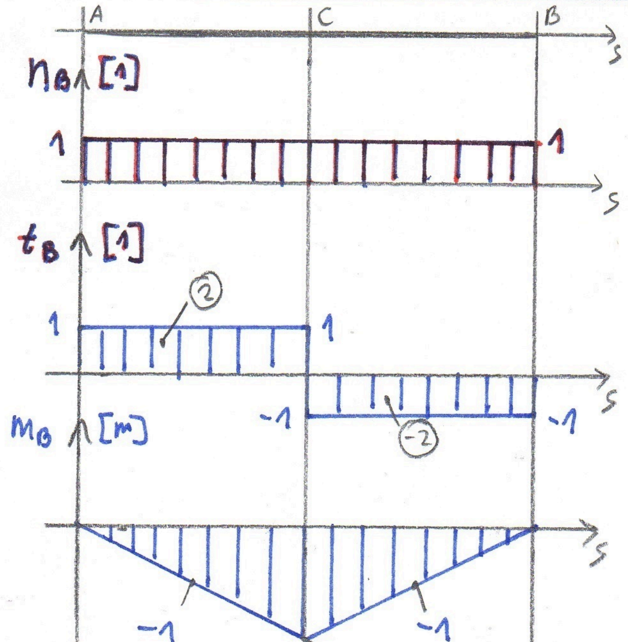
$F_{Ay}=-F_{By}=-1\,(\downarrow)$

$F_z=0=F_{Az}+1$

$\Downarrow$

$F_{Az}=-1\,(\leftarrow)$





• az alakváltozási energia:

$$U \approx U_N + U_H = \frac{1}{2}\int_A^B \frac{N^2}{AE}ds + \frac{1}{2}\int_A^B \frac{M_{hx}^2}{I_x E}ds = \frac{1}{2}\int_A^B \frac{(N_0 + F_{Bz} n_B)^2}{AE}ds + \frac{1}{2}\int_A^B \frac{(M_0 + F_{Bz} m_B)^2}{I_x E}ds$$

• Castigliano-tétel:

$$w_B = 0 = \frac{\partial U}{\partial F_{Bz}} = \int_A^B \frac{(N_0 + F_{Bz} n_B)\, n_B}{AE}ds + \int_A^B \frac{(M_0 + F_{Bz} m_B)\, m_B}{I_x E}ds \qquad / \cdot E$$

$$0 = \frac{1}{A}\int_A^B N_0 n_B\, ds + F_{Bz}\frac{1}{A}\int_A^B n_B^2\, ds + \frac{1}{I_x}\int_A^B M_0 m_B\, ds + F_{Bz}\frac{1}{I_x}\int_A^B m_B^2\, ds$$

$$F_{Bz} = \frac{-\frac{1}{A}\int_A^B N_0 n_B\, ds - \frac{1}{I_x}\int_A^B M_0 m_B\, ds}{\frac{1}{A}\int_A^B n_B^2\, ds + \frac{1}{I_x}\int_A^B m_B^2\, ds}$$

• integrálok kiszámítása:

$$\int_A^B N_0 n_B\, ds = \int_A^C N_0 n_B\, ds = \frac{2}{6}\Big[\overbrace{2\cdot 1 + 4\cdot 2\cdot 1 + 2\cdot 1}^{12}\Big] = 4$$

$$\int_A^B M_0 m_B\, ds = \int_A^C M_0 m_B\, ds + \int_C^B M_0 m_B\, ds = \frac{2}{6}\Big[0\cdot 0 + 4\cdot(-3)(-1) + (-4)(-2)\Big] + \frac{2}{6}\Big[-4(-2) + 4\cdot(-2)(-1) + 0\cdot 0\Big] =$$

$$= \frac{2}{6}\Big[20 + 16\Big] = 12$$

$$\int_A^B n_B^2\, ds = \frac{4}{6}\Big[1^2 + 4\cdot 1^2 + 1^2\Big] = 4$$

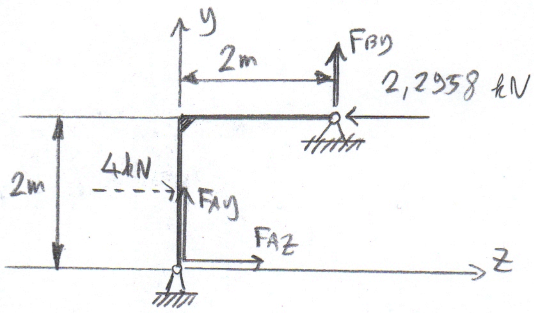
$$\int_A^B m_B^2\, ds = \int_A^C m_B^2\, ds + \int_C^B m_B^2\, ds = \frac{2}{6}\Big[0^2 + 4\cdot(-1)^2 + (-2)^2\Big] + \frac{2}{6}\Big[(-2)^2 + 4\cdot(-1)^2 + 0^2\Big] =$$

$$= \frac{2}{6}\Big[8 + 8\Big] = \frac{16}{3}$$

• behelyettesítve F_{Bz} képletébe:

$$F_{Bz} = \frac{-\frac{1}{A}\cdot\cancel{4} - \frac{1}{I_x}\cdot\cancel{12}^{\,3}}{\frac{1}{A}\cdot\cancel{4} + \frac{1}{I_x}\cdot\frac{\cancel{16}\,4}{3}} = \frac{-\frac{1}{7{,}85398\cdot 10^{-3}} - \frac{3}{4{,}90873\cdot 10^{-6}}}{\frac{1}{7{,}85398\cdot 10^{-3}} + \frac{4}{3\cdot 4{,}90873\cdot 10^{-6}}} = -2{,}2958\ kN \quad (\leftarrow)$$

többi támasztóerő $\Rightarrow$ statikai egyenletekből



$$F_z = 0 = F_{Az} + 4 - 2{,}2958 \Rightarrow F_{Az} = -1{,}7042\ kN\ (\leftarrow)$$

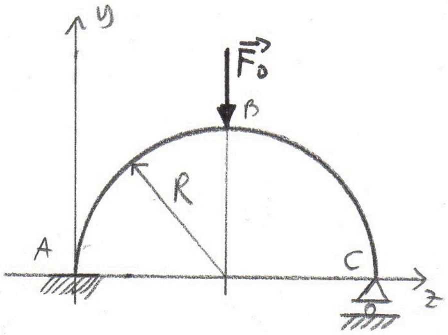
$$M_a = 0 = 1 \cdot 4 - 2F_{By} - 2 \cdot 2{,}2958 \Rightarrow F_{By} = \frac{2 \cdot 2{,}2958 - 1 \cdot 4}{-2} = -0{,}2158\ kN\ (\downarrow)$$

$$F_y = 0 = F_{Ay} - 0{,}2958 \Rightarrow F_{Ay} = 0{,}2958\ kN\ (\uparrow)$$

$$\vec{F_A} = \left(0{,}2958\,\vec{e_y} - 1{,}7042\,\vec{e_z}\right) kN$$

$$\vec{F_B} = \left(-0{,}2958\,\vec{e_y} - 2{,}2958\,\vec{e_z}\right) kN$$

STATIKAILAG HATÁROZATLAN SÍKGÖRBE RÚD



adott:

$R = 60\,mm$

$d = 24\,mm$

$F_0 = 7\,kN$

$E = 2\cdot10^5\,MPa$

feladat:

támasztóerők

• ismeretlen támasztóerők:

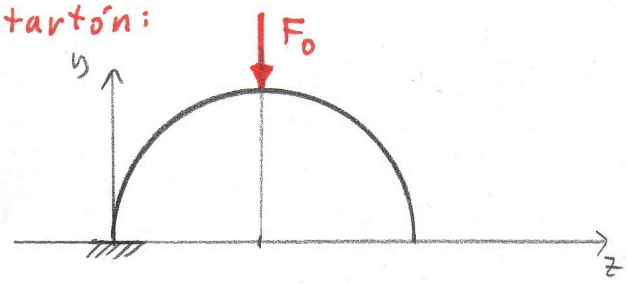
$F_{Ay}, F_{Az}, M_{Ax}, F_{Cy} \Rightarrow 4$ db

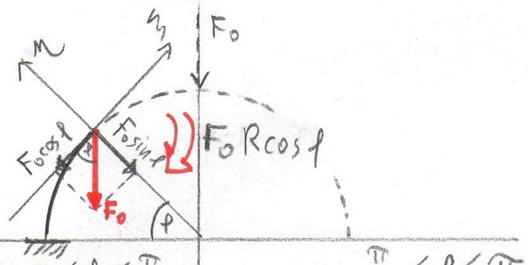
• statikai egyenletek:

$F_y = 0,\ F_z = 0,\ M_a = 0 \Rightarrow 3$ db

• A tartó statikailag egyszeresen határozatlan

• eredeti terhelés a határozottá tett tartón:





$0 < \varphi < \frac{\pi}{2}$

$N_0 = -F_0 \cos\varphi$

$T_{\eta 0} = F_0 \sin\varphi$

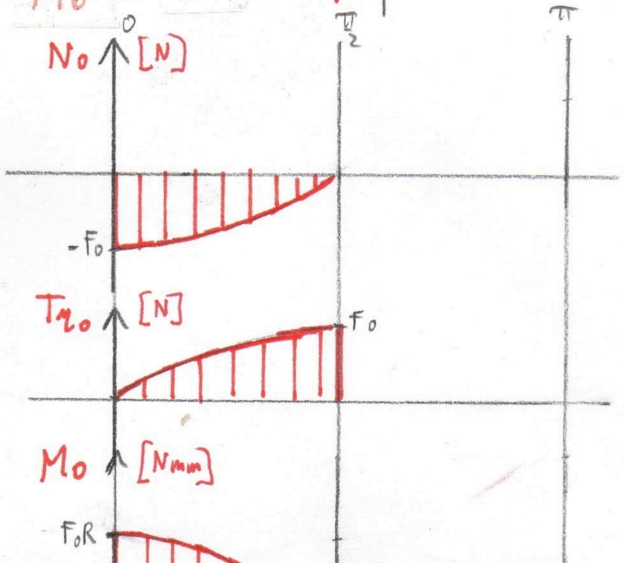
$M_0 = F_0 R \cos\varphi$

$\frac{\pi}{2} < \varphi < \pi$

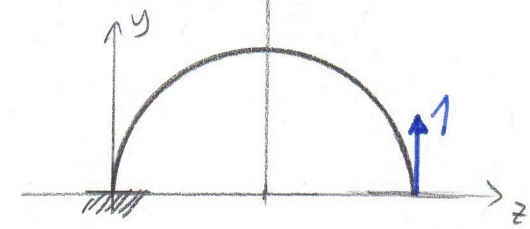
$N_0 = 0$

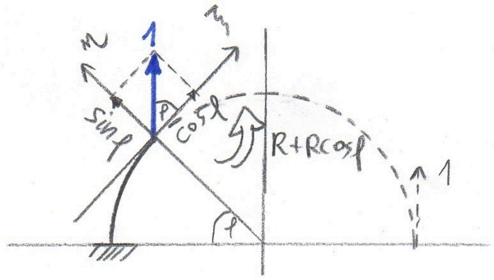
$T_{\eta 0} = 0$

$M_0 = 0$



• az elhagyott támasztóerőnek megfelelő egységnyi nagyságú terhelés:

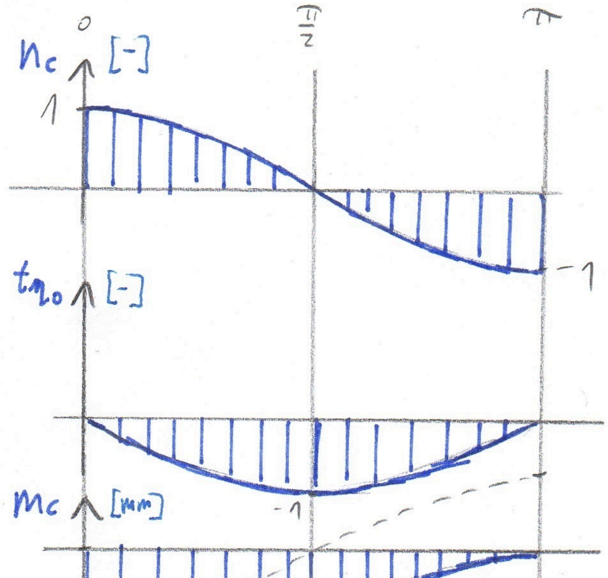




$n_c = \cos\varphi$

$t_{\eta c} = -\sin\varphi$

$m_c = -(R + R\cos\varphi) = -R\cos\varphi - R$



• az alakváltozási energia:

$$U \approx U_N + U_M = \frac{1}{2}\int_{(\ell)} \frac{N^2}{AE}\,ds + \frac{1}{2}\int_{(\ell)} \frac{M_{hx}^2}{I_x E}\,ds = \frac{1}{2}\int_{\varphi=0}^{\pi} \frac{(N_0 + F_{cy} n_c)^2}{AE} R\,d\varphi + \frac{1}{2}\int_{\varphi=0}^{\pi} \frac{(M_0 + F_{cy} m_c)^2}{I_x E} R\,d\varphi$$

• Castigliano-tétel:

$$v_c = 0 = \frac{\partial U}{\partial F_{cy}} = \int_{\varphi=0}^{\pi} \frac{(N_0 + F_{cy} n_c) n_c}{AE} R\,d\varphi + \int_{\varphi=0}^{\pi} \frac{(M_0 + F_{cy} m_c) m_c}{I_x E} R\,d\varphi \quad / \cdot E \,/ : R$$

$$0 = \frac{1}{A}\int_{\varphi=0}^{\pi} N_0 n_c\,d\varphi + F_{cy}\frac{1}{A}\int_{\varphi=0}^{\pi} n_c^2\,d\varphi + \frac{1}{I_x}\int_{\varphi=0}^{\pi} M_0 m_c\,d\varphi + F_{cy}\frac{1}{I_x}\int_{\varphi=0}^{\pi} m_c^2\,d\varphi$$

$$F_{cy} = \frac{-\frac{1}{A}\int_{\varphi=0}^{\pi} N_0 n_c\,d\varphi - \frac{1}{I_x}\int_{\varphi=0}^{\pi} M_0 m_c\,d\varphi}{\frac{1}{A}\int_{\varphi=0}^{\pi} n_c^2\,d\varphi + \frac{1}{I_x}\int_{\varphi=0}^{\pi} m_c^2\,d\varphi}$$

• Integrálok kiszámítása:

$$\int_{\varphi=0}^{\pi} N_0 n_c\,d\varphi = \int_{\varphi=0}^{\frac{\pi}{2}} -F_0\cos\varphi\cos\varphi\,d\varphi = -F_0\int_{\varphi=0}^{\frac{\pi}{2}} \cos^2\varphi\,d\varphi = -\frac{F_0}{2}\int_{\varphi=0}^{\frac{\pi}{2}} 1+\cos 2\varphi\,d\varphi =$$

$$= -\frac{F_0}{2}\left[\varphi + \frac{\sin 2\varphi}{2}\right]_{\varphi=0}^{\varphi=\frac{\pi}{2}} = -\frac{F_0}{2}\frac{\pi}{2} = -\frac{F_0\pi}{4}$$

$$\int_{\varphi=0}^{\pi} M_0 m_c\,d\varphi = \int_{\varphi=0}^{\frac{\pi}{2}} (F_0 R\cos\varphi)(-R\cos\varphi - R)\,d\varphi = \int_{\varphi=0}^{\frac{\pi}{2}} -F_0R^2\cos^2\varphi - F_0R^2\cos\varphi\,d\varphi =$$

$$= -\frac{F_0R^2}{2}\int_{\varphi=0}^{\frac{\pi}{2}} 1+\cos 2\varphi\,d\varphi - F_0R^2\int_{\varphi=0}^{\frac{\pi}{2}} \cos\varphi\,d\varphi = -\frac{F_0R^2}{2}\left[\varphi + \frac{\sin 2\varphi}{2}\right]_{\varphi=0}^{\varphi=\frac{\pi}{2}} - F_0R^2\left[\sin\varphi\right]_{\varphi=0}^{\varphi=\frac{\pi}{2}} =$$

$$= -\frac{F_0R^2}{2}\frac{\pi}{2} - F_0R^2 = -\frac{F_0R^2\pi}{4} - F_0R^2$$

$$\int_{\varphi=0}^{\pi} n_c^2\,d\varphi = \int_{\varphi=0}^{\pi} \cos^2\varphi\,d\varphi = \frac{1}{2}\int_{\varphi=0}^{\pi} 1+\cos 2\varphi\,d\varphi = \frac{1}{2}\left[\varphi + \frac{\sin 2\varphi}{2}\right]_{\varphi=0}^{\varphi=\pi} = \frac{\pi}{2}$$

$$\int_{\varphi=0}^{\pi} M_c^2\, d\varphi = \int_{\varphi=0}^{\pi} (-R\cos\varphi - R)^2\, d\varphi = \int_{\varphi=0}^{\pi} R^2\cos^2\varphi + 2R^2\cos\varphi + R^2\, d\varphi =$$

$$= \frac{R^2}{2}\int_{\varphi=0}^{\pi} 1+\cos 2\varphi\, d\varphi + 2R^2\int_{\varphi=0}^{\pi}\cos\varphi\, d\varphi + \int_{\varphi=0}^{\pi} R^2\, d\varphi =$$

$$= \frac{R^2}{2}\left[\varphi + \frac{\sin 2\varphi}{2}\right]_{\varphi=0}^{\varphi=\pi} + 2R^2\left[\sin\varphi\right]_{\varphi=0}^{\varphi=\pi} + \left[R^2\varphi\right]_{\varphi=0}^{\varphi=\pi} =$$

$$= \frac{R^2\pi}{2} + 0 + R^2\pi = \frac{3R^2\pi}{2}$$

- Keresztmetszeti jellemzők

$$A = \frac{d^2\pi}{4} = \frac{24^2\pi}{4} = 452,389\,mm^2$$

$$\frac{R}{d/2} = \frac{60}{24/2} = \frac{60}{12} = 5 \qquad 3<5 \Rightarrow I_r \approx I_x$$

$$I_x = \frac{d^4\pi}{64} = \frac{24^4\pi}{64} = 16286,016\,mm^4$$

- behelyettesítve F_{cy} képletébe:

$$F_{cy} = \frac{-\frac{1}{A}\left(-\frac{F_0\pi}{4}\right) - \frac{1}{I_x}\left(-\frac{F_0R^2\pi}{4} - F_0R^2\right)}{\frac{1}{A}\left(\frac{\pi}{2}\right) + \frac{1}{I_x}\left(\frac{3R^2\pi}{2}\right)} =$$

$$= \frac{\frac{F_0\pi}{4A} + \frac{F_0R^2(\pi+4)}{4I_x}}{\frac{\pi}{2A} + \frac{3R^2\pi}{2I_x}} =$$

$$= \frac{\frac{7000\pi}{4\cdot 452,389} + \frac{7000\cdot 60^2(\pi+4)}{4\cdot 16286,016}}{\frac{\pi}{2\cdot 452,389} + \frac{3\cdot 60^2\pi}{2\cdot 16286,016}} = 2654,929\,N\,(\uparrow)$$

- többi támasztóerő koordináta statikai egyenletekből:

$$F_y = 0 = F_{Ay} - F_0 + F_{cy} \Rightarrow F_{Ay} = F_0 - F_{cy} = 7000 - 2654,929 = 4345,071\,N\,(\uparrow)$$

$$F_z = 0 = F_{Az}$$

$$M_a = 0 = M_{Ax} + RF_0 - 2RF_{cy} \Rightarrow M_{Ax} = 2RF_{cy} - RF_0 =$$

$$= 2\cdot 60\cdot 2654,929 - 60\cdot 7000 = -101408,52\,Nmm\,(\circlearrowright)$$

$$\vec{F_A} = (4345,071\,\vec{e_y})\,N \quad \vec{F_c} = (2654,929\,\vec{e_y})\,N \quad \vec{M_A} = (-101408,52\,\vec{e_x})\,Nmm$$