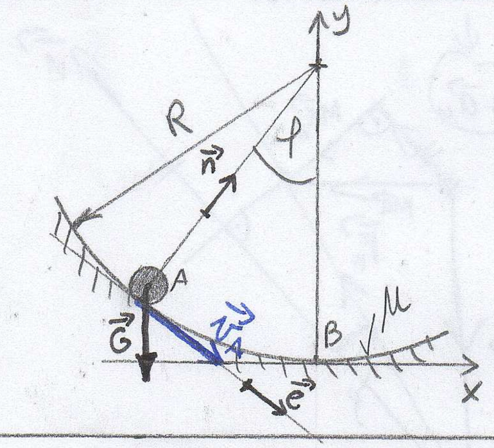
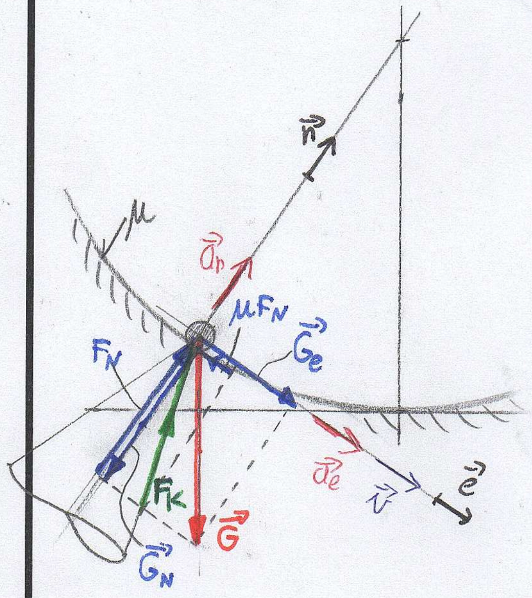
tömegpont körívpályán mozgása



- **adott**
  - $\vec{G} = (-60\vec{j})\,N$
  - $g = 10\,\frac{m}{s^2}$
  - $v_A = v_A\vec{e}$ $\quad v_A = 2\,\frac{m}{s}$
  - $\mu = tg\,\rho = 0{,}2$
  - $\vec{e} = 0{,}8\vec{i} - 0{,}6\vec{j}$
  - $\vec{n} = 0{,}6\vec{i} + 0{,}8\vec{j}$
  - $R = 2m$

- **Feladat**
  - $\vec{a} = ?$
  - $\vec{F_K} = ?$
  - $v_B = ?$ ha $\mu = 0$



- a tömegpontra ható külső erők: $\vec{F} = \vec{G} + \vec{F_K}$
  - $\vec{G} = G_e\vec{e} + G_n\vec{n}$
    ahol $G_e = \vec{G}\cdot\vec{e} = (-60\vec{j})\cdot(0{,}8\vec{i} - 0{,}6\vec{j}) = 36N$
    $G_n = \vec{G}\cdot\vec{n} = (-60\vec{j})\cdot(0{,}6\vec{i} + 0{,}8\vec{j}) = -48N$
    $\vec{G} = (36\vec{e} - 48\vec{n})\,N$
  - $\vec{F_K} = \vec{F_S} + \vec{F_N} = -\mu F_N\vec{e} + F_N\vec{n}$
- a tömeg: $G = mg \Rightarrow m = \frac{G}{g} = \frac{60}{10} = 6\,kg$
- a gyorsulás: $\vec{a} = \vec{a_e} + \vec{a_v} = a_e\vec{e} + a_n\vec{n}$

$$a_n = \frac{v^2}{R} = \frac{2^2}{2} = 2\,\frac{m}{s^2}$$

Impulzus tétel diff. alakja $\Rightarrow a_e, F_K$

$$m\vec{a} = \vec{F}$$
$$m\vec{a} = \vec{G} + \vec{F_K}$$
$$m(a_e\vec{e} + a_n\vec{n}) = (G_e\vec{e} + G_n\vec{n}) + (-\mu F_N\vec{e} + F_N\vec{n}) \quad /\cdot\vec{e}\ /\cdot\vec{n}$$

$/\cdot\vec{e} \Rightarrow$ 1) $m a_e = G_e - \mu F_N$

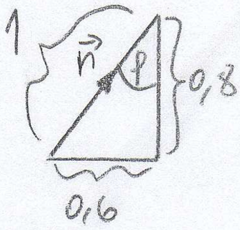
$/\cdot\vec{n} \Rightarrow$ 2) $m a_n = G_n + F_N \Longrightarrow F_N = m a_n - G_n = 6\cdot 2 - (-48) = 60N$

$$1) \Rightarrow a_e = \frac{G_e - \mu F_N}{m} = \frac{36 - 0{,}2\cdot 60}{6} = 4\,\frac{m}{s^2}$$

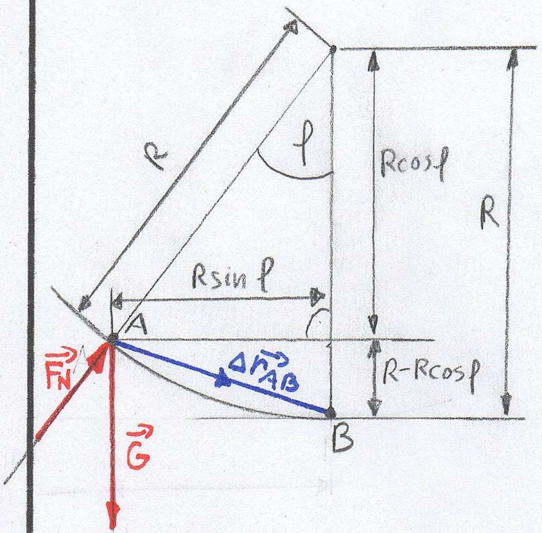
$$\underline{\underline{\vec{a}}} = a_e\vec{e} + a_n\vec{n} = \underline{\underline{(4\vec{e} + 2\vec{n})\,\frac{m}{s^2}}}$$

$$\underline{\underline{\vec{F_K}}} = -\mu F_N\vec{e} + F_N\vec{n} = -0{,}2\cdot 60\vec{e} + 60\vec{n} = \underline{\underline{(-12\vec{e} + 60\vec{n})\,N}}$$

Munkatétel $\Rightarrow v_B$



$$\operatorname{tg}\varphi = \frac{0{,}6}{0{,}8}$$
$$\sin\varphi = 0{,}6$$
$$\cos\varphi = 0{,}8$$



$$E_B - E_A = W_{AB}$$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \int_{\vec{r}_A}^{\vec{r}_B} \vec{F}\,d\vec{r}$$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \underbrace{\int_{\vec{r}_A}^{\vec{r}_B} \vec{F}_N\,d\vec{r}}_{=0\ (\vec{F}_N \perp d\vec{r})} + \vec{G}\underbrace{\int_{\vec{r}_A}^{\vec{r}_B} d\vec{r}}_{\Delta\vec{r}_{AB}}$$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \underbrace{-mg\vec{j}}_{\vec{G}}\underbrace{\left(R\sin\varphi\,\vec{i} - R(1-\cos\varphi)\vec{j}\right)}_{\Delta\vec{r}_{AB}}$$

$$\frac{1}{2}\cancel{m}v_B^2 - \frac{1}{2}\cancel{m}v_A^2 = \cancel{m}gR(1-\cos\varphi)$$

$$v_B = \sqrt{2gR(1-\cos\varphi) + v_A^2} =$$

$$= \sqrt{2\cdot 10\cdot 2(1-0{,}8) + 2^2} = \underline{\underline{3{,}46\ \frac{m}{s}}}$$

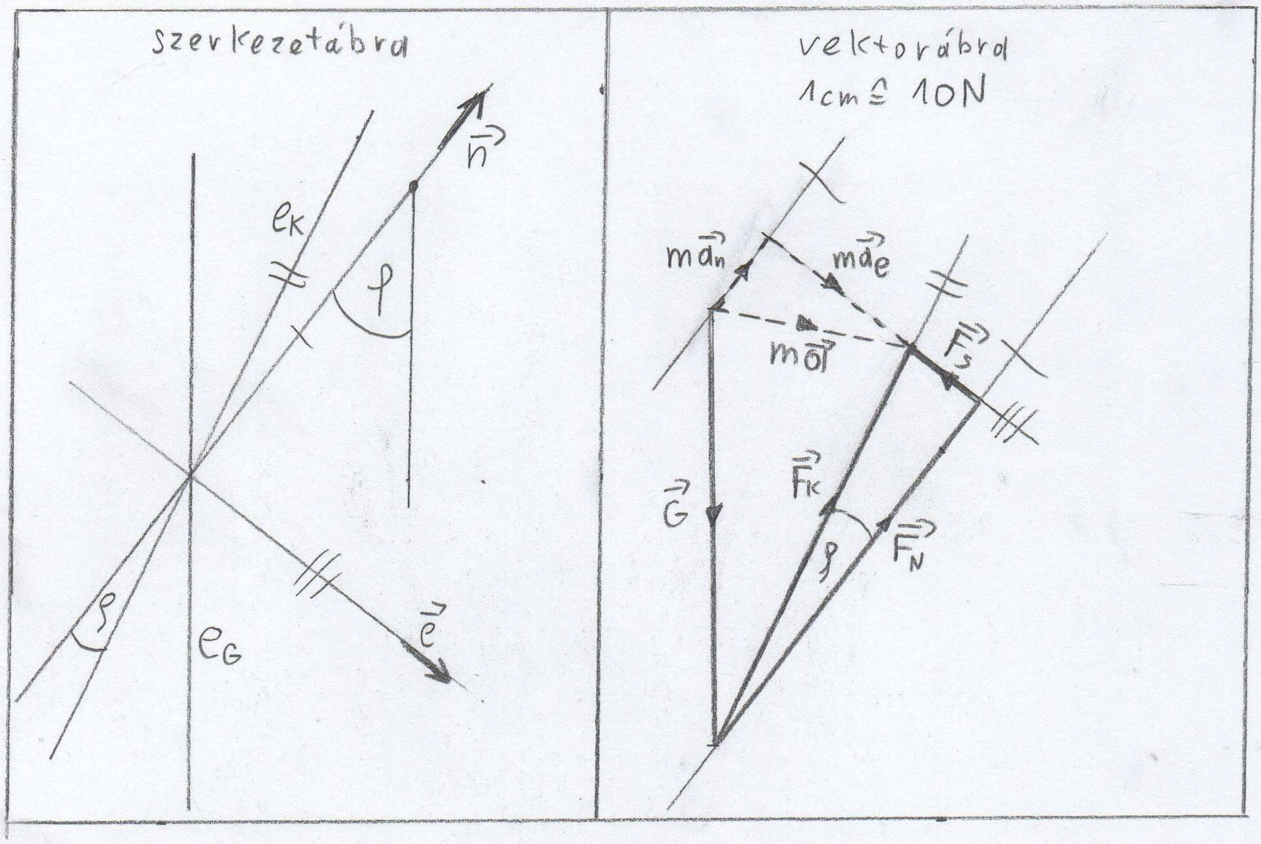
szerkesztés

$$\vec{F} = \dot{\vec{I}}$$

$$\vec{G} + \underbrace{\left(-\mu F_N \vec{e} + F_N \vec{n}\right)}_{\vec{F_K}} = \underbrace{m a_e \vec{e} + m a_n \vec{n}}_{m\vec{a}}$$

- A szerkesztéshez ki kell számítani $m\vec{a_n}$ értékét:

$$a_n = \frac{v^2}{R} = \frac{2^2}{2} = \frac{4}{2} = 2\,\frac{m}{s^2} \; ; \; m\vec{a_n} = m a_n \vec{n} = 6 \cdot 2 \cdot \vec{n} = (12\vec{n})\,N$$



Szerkesztés lépései:

I. szerkezetábra:

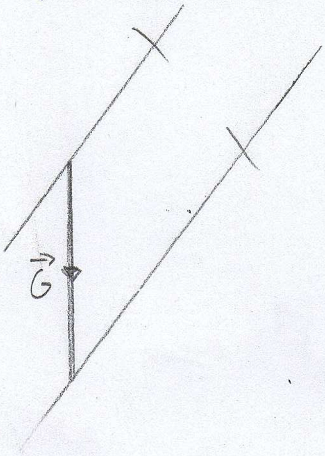
e, n, e_G, e_K hatásvonalak felvétele

$\uparrow$

ϱ szöget zár be $\vec{n}$-nel ($tg\,\varrho = 0{,}2$)

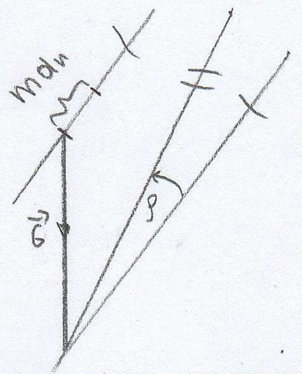
II. vektorábra

(1.) vegyük fel $\vec{G}$-t, majd 2 végpontján keresztül húzunk párhuzamost $\vec{n}$-nel

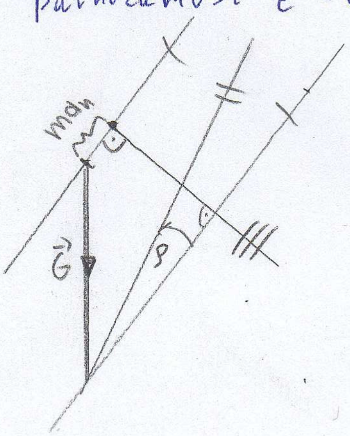


(2) • A fenti párhuzamoson mérjük fel a kiszámított ma_n-t

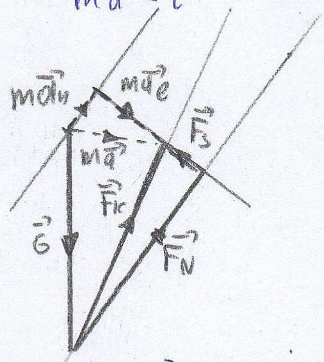
• A lenti párhuzamosra mérjük fel ϱ-t (vagy húzzunk párhuzamost a lenti ponton keresztül e_K-val)



(3.) Az ma_n végpontjából húzunk párhuzamost $\vec{e}$-vel



(4) húzzuk be a nyilakat és $m\vec{a}$-t

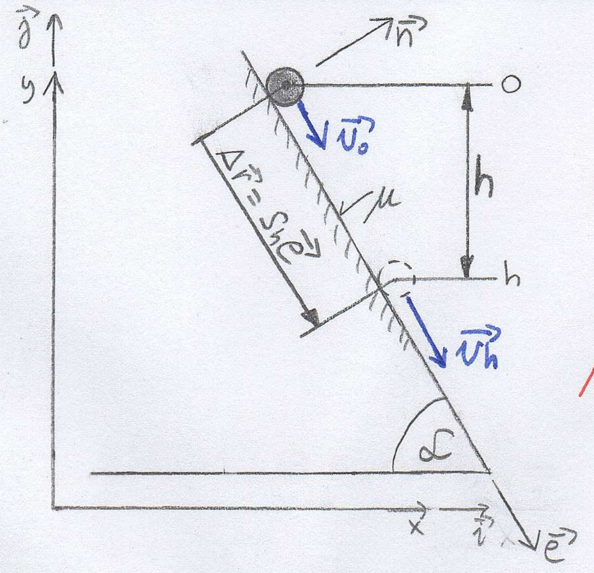


$\vec{F}_K = \vec{F}_N + \vec{F}_S$ $\longrightarrow$

$m\vec{a} = \vec{G} + \vec{F}_K$ $\longrightarrow$ A nyílFolyamok ütközők

$m\vec{a} = m\vec{a}_e + m\vec{a}_n$ $\nearrow$

tömegpont lejtőn mozgása



- **adott**

$m = 1\,kg$

$|\vec{v}_0| = 20\,\frac{m}{s}$

$\alpha = 60°$

$h = 2m$

$\mu = 0{,}2$

- **Feladat**

$\vec{F}_k = ?$ (kényszererő)

$\vec{a} = ?$

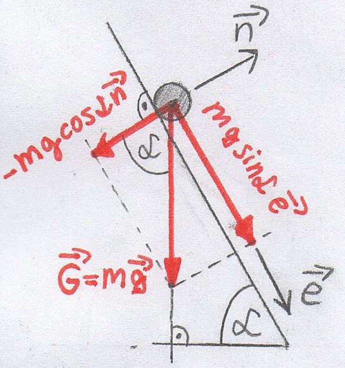
$\vec{v}_h = ?$

$t_h = ?$

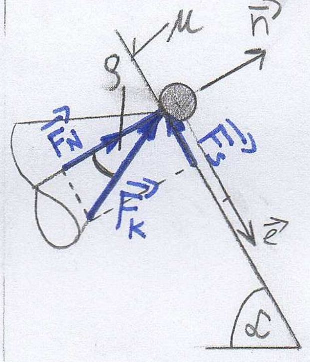
Impulzus tétel diff. alakja ⇒ $\vec{F}_k$ és $\vec{a}$

tömegpontra ható összes külsőerő = impulzus idő szerinti deriváltja

súlyerő + kényszererő



$\vec{G} = m\vec{g} = mg\sin\alpha\,\vec{e} - mg\cos\alpha\,\vec{n}$

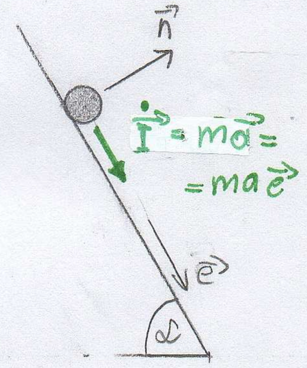


$tg\,\rho = \mu = \frac{F_S}{F_N} \Rightarrow F_S = \mu F_N$

F_S iránya mindig ellentétes a sebesség irányával

$\Rightarrow \vec{F}_S = -\mu F_N \vec{e}$

$\vec{F}_K = \vec{F}_S + \vec{F}_N = -\mu F_N \vec{e} + F_N \vec{n}$



$\dot{\vec{I}} = m\dot{\vec{v}} = m\vec{a} = ma\vec{e}$

$$\vec F = \dot{\vec I}$$

$$\vec F = m\vec a$$

$$\vec G + \vec F_k = m\vec a$$

$$(mg\sin\alpha\,\vec e - mg\cos\alpha\,\vec n) + (-\mu F_N\vec e + F_N\vec n) = ma\vec e \quad /\cdot\vec e\ /\cdot\vec n$$

$$/\cdot\vec e \Rightarrow 1)\ mg\sin\alpha - \mu F_N = ma$$

$$/\cdot\vec n \Rightarrow 2)\ -mg\cos\alpha + F_N = 0 \Rightarrow F_N = mg\cos\alpha$$

$$2)\to 1):\ \cancel{m}g\sin\alpha - \mu\cancel{m}g\cos\alpha = \cancel{m}a$$

$$a = g(\sin\alpha - \mu\cos\alpha) = 10\cdot(\sin 60^\circ - 0{,}2\cdot\cos 60^\circ) = 7{,}66\,\frac{m}{s^2}$$

$$a = 7{,}66\,\frac{m}{s^2}\,(\searrow);\quad \underline{\underline{\vec a}} = a\vec e = \underline{\underline{(7{,}66\,\vec e)\,\frac{m}{s^2}}}$$

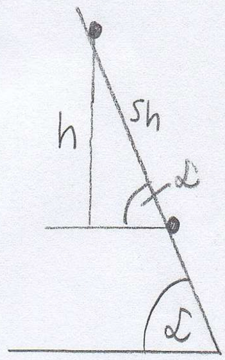
$$F_N = mg\cos\alpha = 1\cdot 10\cdot\cos 60^\circ = 10\cdot\frac{1}{2} = 5N$$

$$\underline{\underline{\vec F_k}} = \vec F_s + \vec F_N = -\mu F_N\vec e + F_N\vec n = -0{,}2\cdot 5\vec e + 5\vec n = \underline{\underline{(-1\vec e + 5\vec n)N}}$$

Munkatétel $\Rightarrow \vec v_h$

$$E_h - E_0 = W_{0h} = \int_{\vec r_0}^{\vec r_h}\vec F\,d\vec r_{0h} = \vec F\int_{\vec r_0}^{\vec r_h} d\vec r_{0h} = \vec F\,s_h\vec e$$

$$\sin\alpha = \frac{h}{s_h} \Rightarrow s_h = \frac{h}{\sin\alpha}$$



$$\frac{1}{2}mv_h^2 - \frac{1}{2}mv_0^2 = (\vec G + \vec F_k)\cdot(s_h\vec e)$$

$$\frac{1}{2}mv_h^2 - \frac{1}{2}mv_0^2 = \left[(mg\sin\alpha\,\vec e - mg\cos\alpha\,\vec n) + (-\mu F_N\vec e + F_N\vec n)\right]\cdot s_h\vec e$$

$$\frac{1}{2}mv_h^2 - \frac{1}{2}mv_0^2 = mg\sin\alpha\, s_h - 0 - \mu F_N s_h + 0$$

$$\frac{1}{2}\cancel{m}v_h^2 - \frac{1}{2}\cancel{m}v_0^2 = \cancel{m}g\,\cancel{\sin\alpha}\,\frac{h}{\cancel{\sin\alpha}} - \mu\, \cancel{m}g\cos\alpha\cdot\frac{h}{\sin\alpha}$$

$$v_h^2 - v_0^2 = 2gh(1 - \mu\,ctg\,\alpha)$$

$$v_h = \sqrt{2gh(1-\mu\,ctg\,\alpha) + v_0^2} = \sqrt{2\cdot 10\cdot 2(1 - 0{,}2\,ctg\,60^\circ) + 20^2} = 20{,}866\,\frac{m}{s}$$

$$\underline{\underline{\vec v_h = (20{,}866\,\vec e)\,\frac{m}{s}}}$$

Impulzus tétel integrál alakja $\Rightarrow t_h$

$$\frac{d\vec{I}}{dt} = \vec{F}$$

$$d\vec{I} = \vec{F}\,dt$$

$$\int_{\vec{I}_0}^{\vec{I}_h} d\vec{I} = \int_{t_0}^{t_h} \vec{F}\,dt$$

$$\vec{I}_h - \vec{I}_0 = \int_{t_0}^{t_h} \left(\vec{G} + \vec{F}_{fc}\right) dt$$

$$m v_h \vec{e} - m v_0 \vec{e} = \left(\vec{G} + \vec{F}_{fc}\right) \cdot \int_{t_0}^{t_h} dt$$

$$m v_h \vec{e} - m v_0 \vec{e} = \left(\vec{G} + \vec{F}_{fc}\right) \cdot \left(t_h - \overset{0}{t_0}\right)$$

$$m v_h \vec{e} - m v_0 \vec{e} = \left[\left(mg\sin\alpha\,\vec{e} - mg\cos\alpha\,\vec{n}\right) + \left(-\mu F_N \vec{e} + F_N \vec{n}\right)\right] \cdot t_h \qquad /\cdot\vec{e} \; /\cdot\vec{n}$$

$$/\cdot\vec{e} \Rightarrow 1)\quad m v_h - m v_0 = \left(mg\sin\alpha - \mu F_N\right) t_h$$

$$/\cdot\vec{n} \Rightarrow 2)\quad 0 = \left(-mg\cos\alpha + F_N\right) t_h \qquad \Rightarrow F_N = mg\cos\alpha$$

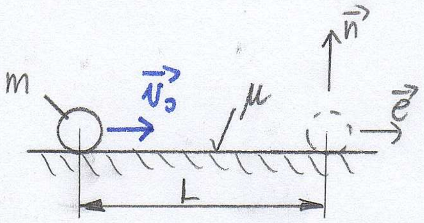
$$2) \to 1):\quad m v_h - m v_0 = \left(mg\sin\alpha - \mu\, mg\cos\alpha\right) t_h$$

$$\cancel{m}\left(v_h - v_0\right) = \cancel{m} g\left(\sin\alpha - \mu\cos\alpha\right) t_h$$

$$t_h = \frac{v_h - v_0}{g\left(\sin\alpha - \mu\cos\alpha\right)}$$

$$\underline{\underline{t_h}} = \frac{20{,}866 - 20}{10\left(\sin 60^\circ - 0{,}2 \cdot \cos 60^\circ\right)} = \underline{\underline{0{,}113\,s}}$$

tömegpont vízszintes pályán mozgása



• adott

$v_0 = 2\ \frac{m}{s}$

$\mu = 0,2$

$m = 10\ kg$

$g = 10\ m/s^2$

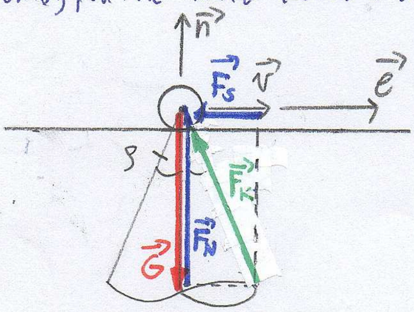
• Feladat

$\vec{a} = ?$

$t_1 = ?$ megállásig szükséges idő

$L = ?$ megállásig megtett út

A tömegpontra ható külső erők



$$\vec{F} = \vec{G} + \vec{F}_K$$

• $\vec{G} = -mg\vec{n}$

• $\vec{F}_K = F_S\vec{e} + F_N\vec{n} = -\mu F_N\vec{e} + F_N\vec{n}$

• Impulzus tétel diff alakja ⇒ $\vec{a}$

$$\vec{F} = \dot{\vec{I}}$$

$$\vec{F} = m\vec{a}$$

$$(-mg\vec{n}) + (-\mu F_N\vec{e} + F_N\vec{n}) = ma\vec{e} \qquad /\cdot\vec{e}\ /\cdot\vec{n}$$

$$\left.\begin{array}{l} /\cdot\vec{e} \Rightarrow 1)\ -\mu F_N = ma \\ /\cdot\vec{n} \Rightarrow 2)\ -mg + F_N = 0 \end{array}\right\} \Rightarrow F_N = mg$$

$$2) \to 1):\quad -\mu \cancel{m} g = \cancel{m} a$$

$$a = -\mu g = -0,2\cdot 10 = -2\ \frac{m}{s^2} \Rightarrow \underline{\underline{\vec{a}}} = a\vec{e} = \underline{\underline{(-2\vec{e})\frac{m}{s^2}}}$$

• Impulzus tétel integrál alakja ⇒ t_1

$$\vec{I}_1 - \vec{I}_0 = \int_{t_0=0}^{t_1} \vec{F}\,dt = \int_{t_0=0}^{t_1} (\vec{G} + \vec{F}_K)\,dt = (\vec{G} + \vec{F}_K)\int_{t_0=0}^{t_1} dt = (\vec{G} + \vec{F}_K)\,t_1$$

$$m\underbrace{v_1}_{0\ (\text{megáll})}\vec{e} - mv_0\vec{e} = \left[(-mg\vec{n}) + (-\mu F_N\vec{e} + F_N\vec{n})\right] t_1 \qquad /\cdot\vec{e}$$

$$-mv_0 = -\mu F_N t_1$$

$$\cancel{m} v_0 = \mu \cancel{m} g t_1$$

$$\underline{\underline{t_1}} = \frac{v_0}{\mu g} = \frac{2}{0,2\cdot 10} = \underline{\underline{1s}}$$

Munkatétel $\Rightarrow L$

$$E_1 - E_0 = W_{01}$$

- baloldal: $E_1 - E_0 = \frac{1}{2} m \underbrace{v_1^2}_{0} - \frac{1}{2} m v_0^2 = -\frac{1}{2} m v_0^2$

- jobboldal: $W_{01} = \int\limits_{\vec{r}_0=\vec{0}}^{\vec{r}_1} \vec{F} d\vec{r} \int\limits_{r_0=0}^{r_1} \left[(-mg\vec{n}) + (-\mu F_N \vec{e} + F_N \vec{n})\right] dr\, \vec{e} =$

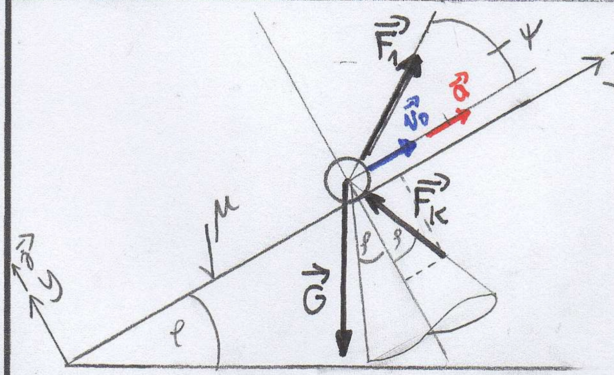
$$= \left[(-mg\vec{n}) + (-\mu F_N \vec{e} + F_N \vec{n})\right] \underbrace{\int\limits_{r_0=0}^{r_1} dr}_{L}\, \vec{e} = -\mu F_N L = -\mu m g L$$

$$E_1 - E_0 = W_{01}$$

$$-\frac{1}{2} \cancel{m} v_0^2 = -\mu \cancel{m} g L$$

$$\underline{\underline{L}} = \frac{v_0^2}{2\mu g} = \frac{2^2}{2 \cdot 0{,}2 \cdot 10} = \underline{\underline{1 m}}$$

tömegpont lejtőn mozgása – pályaelhagyás



• **adott**

$\rho = \psi = 30^\circ$

$m = 10\ kg$

$\mu = tg\,\rho = 0{,}2$

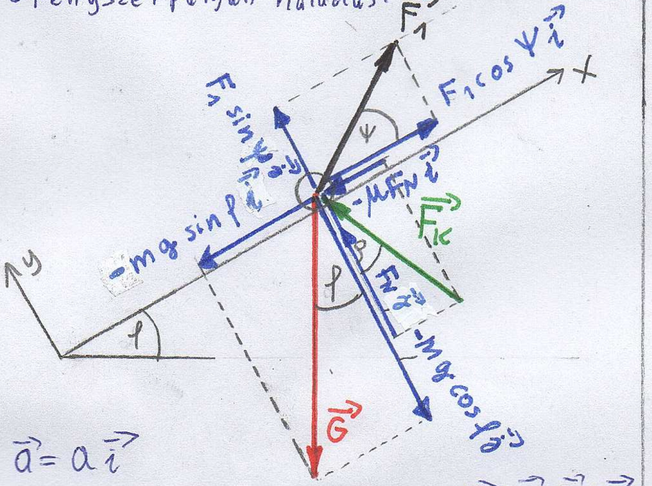
$\vec{v_0} = (8\vec{i})\ \frac{m}{s}$

• **Feladat**

a) Mekkora $\vec{F_1}$ erő esetén marad a tömegpont kényszerpályán

b) $\vec{a} = ?$ pályaelhagyásának pillanatában.

• Kényszerpályán haladás:



$\vec{a} = a\vec{i}$

– A tömegpontra ható erők: $\vec{F} = \vec{G} + \vec{F_K} + \vec{F_1}$

$\vec{G} = -mg\sin\rho\,\vec{i} - mg\cos\rho\,\vec{j}$

$\vec{F_K} = -\mu F_N \vec{i} + F_N \vec{j}$

$\vec{F_1} = F_1\cos\psi\,\vec{i} + F_1\sin\psi\,\vec{j}$

• **Impulzus tétel diff alakja**

$m\vec{a} = \vec{F}$

$ma\vec{i} = -mg\sin\rho\,\vec{i} - mg\cos\rho\,\vec{j} - \mu F_N\vec{i} + F_N\vec{j} + F_1\cos\psi\,\vec{i} + F_1\sin\psi\,\vec{j} \quad /\cdot\vec{i}\ /\cdot\vec{j}$

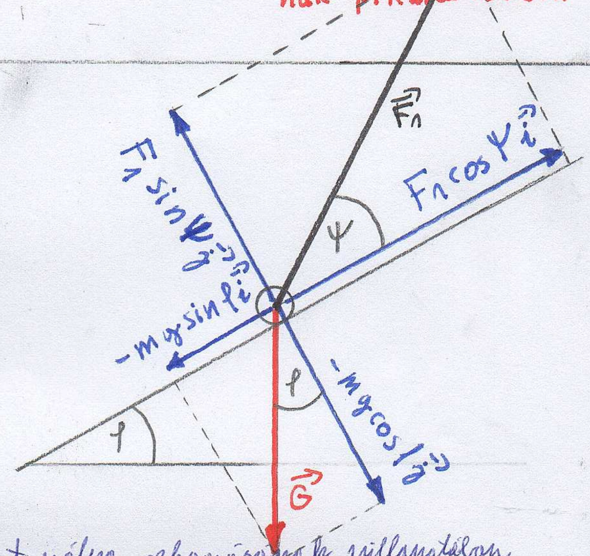
$\cdot\vec{i} \Rightarrow 1)\ ma = -mg\sin\rho - \mu F_N + F_1\cos\psi$

$\cdot\vec{j} \Rightarrow 2)\ 0 = -mg\cos\rho + F_N + F_1\sin\psi$

$2) \Rightarrow F_N = mg\cos\rho - F_1\sin\rho$

A kényszerpályán maradás feltétele:

$F_N \geq 0$



• A pálya elhagyásának pillanatában

$F_N = 0$

$mg\cos\rho - F_1\sin\rho = 0$

$F_1 = \frac{mg\cos\rho}{\sin\psi} =$

$= \frac{10\cdot 10\cdot\cos 30^\circ}{\sin 30^\circ} = 173{,}2\,N$

$\vec{F_1} = F_1\cos\psi\,\vec{i} + F_1\sin\psi\,\vec{j}$

$\vec{F_1} = 173{,}2\cdot\cos 30^\circ\,\vec{i} + 173{,}2\sin 30^\circ\,\vec{j} =$

$= \underline{\underline{(150\vec{i} + 86{,}6\vec{j})\,N}}$

$1) \Rightarrow ma = -mg\sin\rho - \mu F_N + F_1\cos\psi$

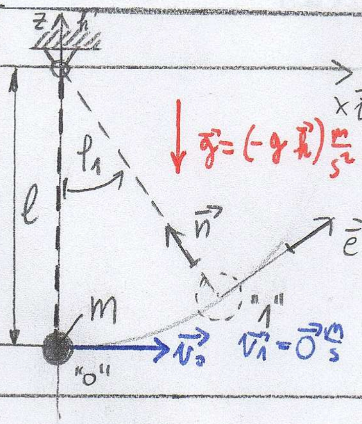
$a = \frac{-mg\sin\rho - \mu F_N + F_1\cos\psi}{m} =$

a pálya elhagyásának pillanatában: $F_N = 0$, $F_1 = 173{,}2$

$= \frac{-10\cdot 10\sin 30^\circ + 0{,}2\cdot 0 + 173{,}2\cos 30^\circ}{10} =$

$= 10\,\frac{m}{s^2} \Rightarrow \underline{\vec{a} = (10\vec{i})\,\frac{m}{s^2}}$

tömegpont körívpályán mozgása



• adott

$m = 1\,kg$

$v_0 = 4\,\frac{m}{s}$

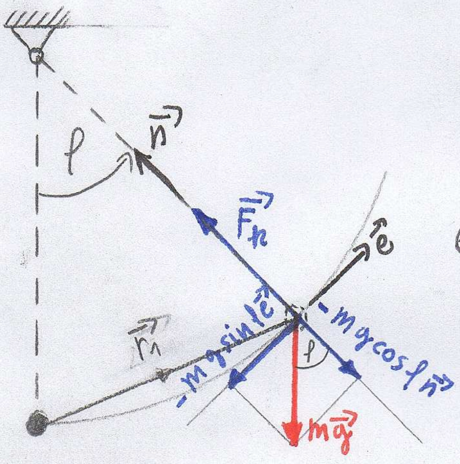
$\ell = 2m$

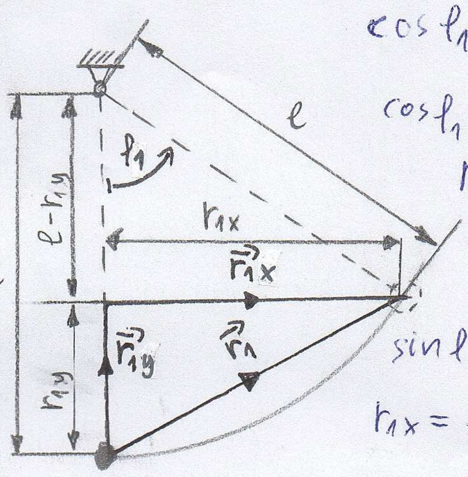
• Feladat

• $\varphi_{max} = ?$ (amikor $\vec{v}_1 = \vec{0}\,\frac{m}{s}$)

• $\vec{F}_{kötél} = \vec{F}_k(\varphi) = ?$

$\vec{F}_k(\varphi_0) = ?\,;\ \vec{F}_k(\varphi_{max}) = ?$





$$\cos\varphi_1 = \frac{\ell - r_{1y}}{\ell}$$

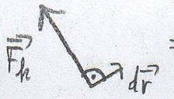
$$\cos\varphi_1 \cdot \ell = \ell - r_{1y}$$

$$r_{1y} = \ell(1-\cos\varphi_1)$$

$$\sin\varphi_1 = \frac{r_{1x}}{\ell}$$

$$r_{1x} = \ell\cdot\sin\varphi_1$$

• Munkatétel $\Rightarrow \varphi_{max}$



$$\Rightarrow \vec{F}_k \cdot d\vec{r} = F_k\vec{n}\cdot dr\vec{e} = F_k dr\underbrace{(\vec{n}\cdot\vec{e})}_{0}$$

$$E_1 - E_0 = W_{01} = \int_{\vec{r}_0}^{\vec{r}_1} \vec{F}\,d\vec{r}$$

• jobboldal:

$$\int_{\vec{r}_0}^{\vec{r}_1} \vec{F}\,d\vec{r} = \int_{\vec{r}_0}^{\vec{r}_1} (\vec{F}_k + \vec{G})\,d\vec{r} = \underbrace{\int_{\vec{r}_0}^{\vec{r}_1} \vec{F}_k\,d\vec{r}}_{} + \int_{\vec{r}_0}^{\vec{r}_1} \vec{G}\,d\vec{r} =$$

$$= \int_{\vec{r}_0}^{\vec{r}_1} m\vec{g}\,d\vec{r} = m\vec{g}\int_{\vec{r}_0}^{\vec{r}_1} d\vec{r} = m\vec{g}\,[\vec{r}]_{\vec{r}_0}^{\vec{r}_1} = m\vec{g}(\vec{r}_1 - \underset{0}{\vec{r}_0}) =$$

$$= m\vec{g}\,\vec{r}_1 = -mg\vec{k}\cdot\left[\ell\sin\varphi_1\,\vec{i} + \ell\cdot(1-\cos\varphi_1)\,\vec{k}\right] =$$

$$= -mg\ell(1-\cos\varphi_1)$$

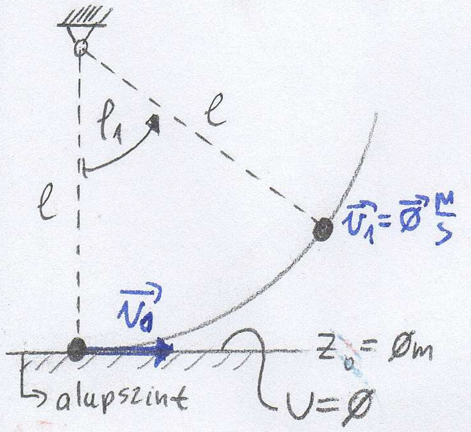
$$\overbrace{\frac{1}{2}mv_1^2}^{=0} - \frac{1}{2}\cancel{m}v_0^2 = -\cancel{m}g\ell(1-\cos\varphi_1) \quad / \cdot(-2)$$

$$v_0^2 = 2g\ell(1-\cos\varphi_1)$$

$$\frac{v_0^2}{2g\ell} = 1 - \cos\varphi_1$$

$$\cos\varphi_1 = 1 - \frac{v_0^2}{2g\ell} = 1 - \frac{4^2}{2\cdot 10\cdot 2} = 0{,}6 \Rightarrow \overset{\varphi_{max}}{\varphi_1} = \arccos 0{,}6 = \underline{\underline{53{,}13^\circ}}$$

Energiamegmaradás → φ_{max}



$$E_0 + U_0 = E_1 + U_1$$

$$\frac{1}{2} m v_0^2 + m g \underset{0}{\underset{\shortparallel}{z_0}} = \frac{1}{2} m \underset{0}{\underset{\shortparallel}{v_1}}^2 + m g z_1$$

$$\frac{1}{2} m v_0^2 = m g z_1$$

$$\frac{1}{2} \cancel{m} v_0^2 = \cancel{m} g \ell (1 - \cos \varphi_1)$$

$$v_0^2 = 2 g \ell - 2 g \ell \cos \varphi_1$$

$$2 g \ell \cos \varphi_1 = 2 g \ell - v_0^2$$

$$\cos \varphi_1 = 1 - \frac{v_0^2}{2 g \ell} = 1 - \frac{4^2}{2 \cdot 10 \cdot 2} = 0{,}6$$

$$\varphi_1 = \varphi_{max} = \arccos 0{,}6 = \underline{\underline{53{,}13^\circ}}$$

Impulzus tétel + Munkatétel → $\vec{F}_k(\varphi)$

- Impulzus tétel:

$$\vec{F} = \dot{\vec{I}}$$

$$\vec{F}_k + \vec{G} = m \vec{a}$$

$$F_k \vec{n} + (-mg \sin \varphi\, \vec{e} - mg \cos \varphi\, \vec{n}) = m a_e \vec{e} + m a_n \vec{n} \quad / \cdot \vec{e}\ / \cdot \vec{n}$$

(1) $-mg \sin \varphi = m a_e \rightarrow a_e = -g \sin \varphi \quad \vec{a}_e = -g \sin \varphi\, \vec{e}$

(2) $F_k - mg \cos \varphi = m a_n \rightarrow F_k = mg \cos \varphi + m a_n$

$$F_k = m (g \cos \varphi + a_n)$$

(1) $\boxed{F_k = m \left(g \cos \varphi + \frac{v^2}{\ell} \right)}$

- Munkatételből (előző pont)

$$\frac{1}{2} \cancel{m} v^2 - \frac{1}{2} \cancel{m} v_0^2 = -\cancel{m} g \ell (1 - \cos \varphi)$$

(2) $\boxed{v^2 = v_0^2 - 2 g \ell (1 - \cos \varphi)}$

(2)→(1)

- tehát: $\boxed{\vec{F}_k(\varphi) = F_k \vec{n} = m \left(g \cos \varphi + \frac{v_0^2 - 2 g \ell (1 - \cos \varphi)}{\ell} \right) \vec{n}}$

φ_0 esetén $v = v_0 = 4 \frac{m}{s}$

$$\rightarrow F_k(\varphi_0) = m \left(g \cos \varphi_0 + \frac{v_0^2}{\ell} \right) = 1 \cdot \left(10 \cdot 1 + \frac{4^2}{2} \right) = \underline{\underline{18N}}$$

$$\vec{F}_k(\varphi_0) = (18 \vec{n}) N$$

$\varphi_1 = \varphi_{max}$ esetén $v = v_1 = 0 \frac{m}{s}$

$$\rightarrow F_k(\varphi_1) = m \left(g \cos \varphi + \frac{0^2}{\ell} \right) = mg \cos \varphi = 1 \cdot 10 \cdot \overbrace{\cos 53{,}13}^{0{,}6} = \underline{\underline{6N}}$$

$$\vec{F}_k(\varphi_1) = (6 \vec{n}) N$$