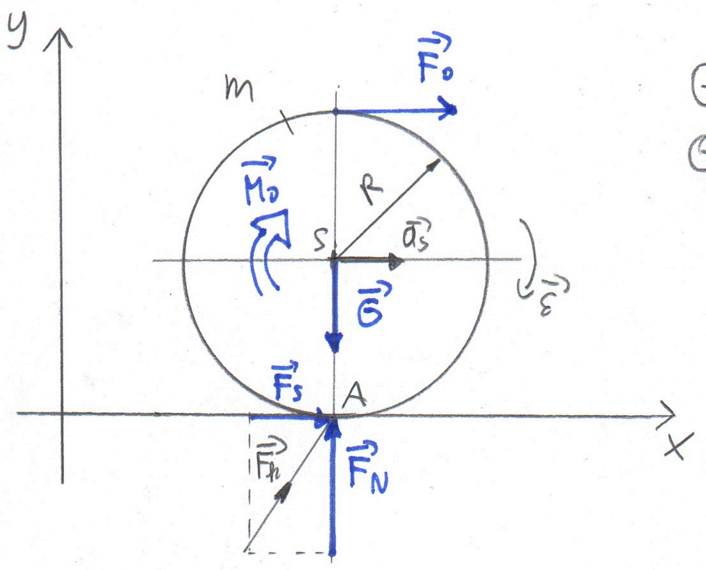
TISZTA GÖRDÜLÉS



$\oplus\ \vec{a}_s \to \ominus\ \vec{\varepsilon}$

$\ominus\ \vec{a}_s \to \oplus\ \vec{\varepsilon}$

F_s és F_N függetlenek

$\Downarrow$

számításnál célszerű mindkettőt $\oplus$-nak feltételezni

$$\vec{F}_h = F_s \vec{e}_x + F_N \vec{e}_y$$

ismert: $R, \vec{M}_0, \vec{F}_0, m$

Feladat: $\vec{a}_s, \vec{\varepsilon}, \vec{F}_h, \mu_0^{min}$

Forgó mozgás

$\Downarrow$

Perdület tétel

$$\dot{\vec{\Pi}}_A = \vec{M}_A$$

$$\underline{\underline{J}}_A \vec{\varepsilon} + \underbrace{\vec{\omega} \times \vec{\Pi}_A}_{=\vec{0}\ (\vec{\omega} \parallel \vec{\Pi}_A)} = \vec{M}_A \quad / \cdot \vec{e}_z$$

$$J_a \varepsilon = M_a$$

Haladó mozgás

$\Downarrow$

Impulzus tétel

$$\dot{\vec{I}} = \vec{F}$$

$$m\vec{a}_s = \vec{F} \quad / \cdot \vec{e}_x \ / \cdot \vec{e}_y$$

$$\left.\begin{array}{l} 1)\ m a_s = F_x \to F_s \\ 2)\ 0 = F_y \to F_N \end{array}\right\} \Rightarrow \vec{F}_h$$

$$a_s = -\varepsilon R$$

$$\mu_0^{min} = \frac{|F_s|}{|F_N|}$$

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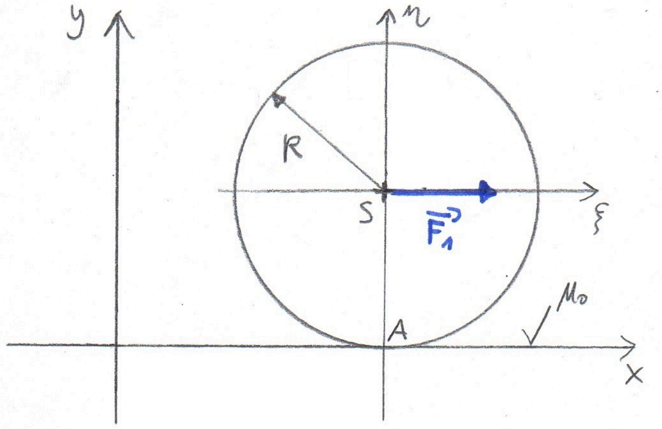


adott:

$R = 0{,}4\,m$

$m = 2\,kg$

$\vec{F_1} = (12\vec{e_x})\,N$

$\vec{\omega_0} = (-4\vec{e_z})\,\frac{rad}{s}$

Feladat:

$\vec{\varepsilon} = ?\ ,\ \vec{a_S} = ?\ ,\ \vec{F_k} = ?\ ,\ \mu_0^{min} = ?$

① Perdület tétel $\Rightarrow$ $\vec{\varepsilon}$ szöggyorsulás

$$\dot{\vec{\Pi_A}} = \vec{M_A}$$

$$\underline{\underline{J}}_A\vec{\varepsilon} + \underbrace{\vec{\omega}\times\vec{\Pi_A}}_{=\vec{0}\ (\vec{\omega}\parallel\vec{\Pi_A})} = \vec{M_A} \quad /\cdot\vec{e_z}$$

$$J_a\,\varepsilon = M_a$$

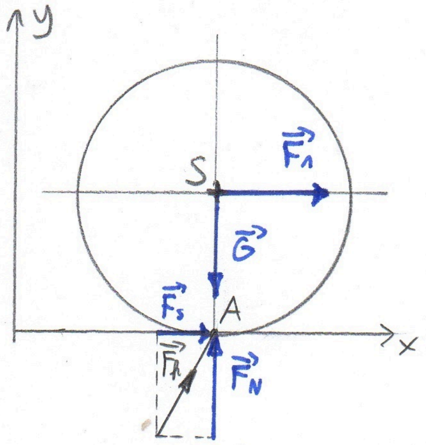
$$\frac{3}{2}mR^2\varepsilon = -RF_1$$

$$\varepsilon = \frac{-2F_1}{3mR} = \frac{-2\cdot12}{3\cdot2\cdot0{,}4} = -10\,\frac{rad}{s^2} \qquad \vec{\varepsilon} = \varepsilon\vec{e_z} = (-10\vec{e_z})\,\frac{rad}{s^2}$$

② súlypont gyorsulása:

$$a_S = -\varepsilon R = -(-10)\cdot0{,}4 = 4\,\frac{m}{s^2} \qquad \vec{a_S} = a_S\vec{e_x} = (4\vec{e_x})\,\frac{m}{s^2}$$

③ Impulzus tétel $\Rightarrow$ $\vec{F_k}$ kényszererő



$$\dot{\vec{I}} = \vec{F}$$

$$m\vec{a_s} = \vec{F_1} + \vec{G} + \vec{F_k}$$

$$ma_s\vec{e_x} = F_1\vec{e_x} - mg\,\vec{e_y} + F_s\vec{e_x} + F_N\vec{e_y} \quad /\cdot\vec{e_x}\ /\cdot\vec{e_y}$$

$/\cdot\vec{e_x} \Rightarrow ma_s = F_1 + F_s$

$F_s = ma_s - F_1 =$

$= 2\cdot4 - 12 = -4\,N$

$/\cdot\vec{e_y} \Rightarrow 0 = -mg + F_N$

$F_N = mg = 2\cdot10 = 20\,N$

$$\vec{F_k} = F_s\vec{e_x} + F_N\vec{e_y} = (-4\vec{e_x} + 20\vec{e_y})\,N$$

④ a megcsúszásmentes tiszta gördüléshez szükséges minimális tapadási súrlódási tényező:

$$\mu_0^{min} = \frac{|F_s|}{|F_N|} = \frac{|-4|}{|20|} = 0{,}2$$

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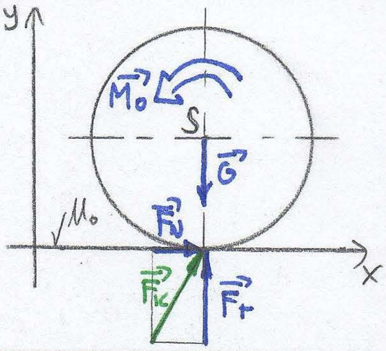


• adott

$\vec{M_0} = (36\vec{k})\,Nm$

$R = 0{,}3\,m$

$J_S = 1{,}2\,kgm^2$

$\mu_0 = 0{,}4$

$\mu = 0{,}3$

$m = 20\,kg$

• Feladat

a) $\vec{\varepsilon} = ?$ $\vec{a_S} = ?$

b) $\vec{F_K} = ?$

c) $\vec{a}_{S max}, \vec{\varepsilon}_{max}, \vec{M}_{0 max} = ?$

henger még ne csússzon meg

a) Perdület tétel A pontra

$$\dot{\Pi}_A = \vec{M_A}$$

$$J_A \vec{\varepsilon} + \underbrace{\vec{\omega} \times \vec{\Pi_A}}_{=\vec{0}} = \vec{M_A}$$

$$J_a \vec{\varepsilon} = \vec{M_A} \quad / \cdot \vec{k}$$

$$J_a \varepsilon = M_a$$

$$(J_S + mR^2) \cdot \varepsilon = M_0$$

$$\varepsilon = \frac{M_0}{J_S + mR^2} = \frac{36}{1{,}2 + 20 \cdot 0{,}3^2} = 12 \frac{rad}{s^2}$$

$$\vec{\varepsilon} = \varepsilon \vec{k} = (12\vec{k}) \frac{rad}{s^2}$$

$$a_S = -\varepsilon R = -12 \cdot 0{,}3 = -3{,}6 \frac{m}{s^2}$$

$$\vec{a_S} = a_S \vec{i} = (-3{,}6\vec{i}) \frac{m}{s^2}$$

b) Impulzus tétel

$$\dot{\vec{I}} = \vec{F}$$

$$m\vec{a_S} = \vec{G} + \vec{F_K}$$

$$-ma_S \vec{i} = -mg\vec{j} + F_T \vec{i} + F_N \vec{j} \quad / \cdot \vec{i} / \cdot \vec{j}$$

$$\left.\begin{aligned} (1)\quad -ma_S &= F_T \\ (2)\quad 0 &= -mg + F_N \end{aligned}\right\}$$

$$(1) \to F_T = -ma_S = -20 \cdot 3{,}6 = -72\,N$$

$$(2) \to F_N = mg = 20 \cdot 10 = 200\,N$$

$$\vec{F_K} = F_T \vec{i} + F_N \vec{j} = (-72\vec{i} + 200\vec{j})\,N$$

c) A megcsúszás határán

$$F_{T max} = \mu_0 F_N = 0{,}4 \cdot 200 = 80\,N$$

impulzus tétel:

$$F_{T max} = -m a_{S max}$$

$$\mu_0 \underbrace{\cancel{m}g}_{F_N} = -\cancel{m} a_{S max}$$

$$a_{S max} = -\mu_0 g = -0{,}4 \cdot 10 = -4 \frac{m}{s^2}$$

$$\vec{a}_{S max} = (-4\vec{i}) \frac{m}{s^2}$$

$$\varepsilon_{max} = -\frac{a_{S max}}{R} = -\frac{-4}{0{,}3} = 13{,}\dot{3} \frac{rad}{s^2}$$

$$\vec{\varepsilon}_{max} = (13{,}\dot{3}\,\vec{k}) \frac{rad}{s^2}$$

perdület tétel:

$$J_a \varepsilon_{max} = M_{0 max}$$

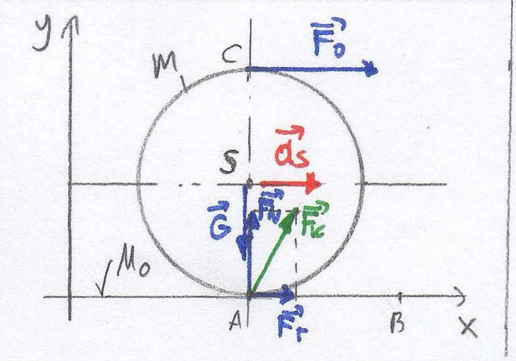
$$M_{0 max} = (J_S + mR^2) \cdot \varepsilon_{max} =$$

$$= (1{,}2 + 20 \cdot 0{,}3^2) \cdot 13{,}\dot{3} =$$

$$= 40\,Nm$$

$$\vec{M}_{0 max} = M_{0 max} \vec{k} = (40\vec{k})\,Nm$$

TISZTA GÖRDÜLÉS



• adott

$\vec{a}_S = (8\vec{i})\frac{m}{s^2}$

$l_{AB} = 2m$

$R = 0{,}1m$

$m = 30kg$

$g = 10\frac{m}{s^2}$

• Feladat

a) $\vec{F_0} = ?$

b) $\vec{F_K} = ?$

c) $\mu_{0min} = ?$

d) $W_{AB} = ?$

a) $\varepsilon = -\frac{a_S}{R} = -\frac{8}{0{,}1} = -80\frac{rad}{s^2}$

$\vec{\varepsilon} = \varepsilon\vec{k} = (-80\vec{k})\frac{rad}{s^2}$

Perdület tétel A pontra

$\dot{\vec{\Pi}}_A = \vec{M}_A$

$\underline{\underline{J_A}}\vec{\varepsilon} + \underbrace{\vec{\omega}\times\vec{\Pi}_A}_{=\vec{0}} = \vec{M}_A$

$J_a\vec{\varepsilon} = \vec{M}_A \quad /\cdot\vec{k}$

$J_a\varepsilon = M_a$

$\frac{3}{2}mR^2\varepsilon = -2RF_0$

$F_0 = \frac{-\frac{3}{2}mR^2\varepsilon}{2R} = \frac{-\frac{3}{2}\cdot30\cdot0{,}1\cdot(-80)}{2} = 180N$

$\underline{\underline{\vec{F_0} = (180\vec{i})N}}$

b) Impulzus tétel

$\dot{\vec{I}} = \vec{F}$

$m\vec{a_S} = \vec{F_0} + \vec{G} + \vec{F_K}$

$ma_S\vec{i} = F_0\vec{i} - mg\vec{j} + F_T\vec{i} + F_N\vec{j} \quad /\cdot\vec{i}/\vec{j}$

(1) $ma_S = F_0 + F_T$

(2) $0 = -mg + F_N$

(1)→ $F_T = ma_S - F_0 = 30\cdot8 - 180 = 60N$

(2)→ $F_N = mg = 30\cdot10 = 300N$

$\vec{F_K} = F_T\vec{i} + F_N\vec{j} = \underline{\underline{(60\vec{i} + 300\vec{j})N}}$

ellenőrzés => perdület tétel S pontra

$J_S\vec{\varepsilon} = \vec{M}_S \quad /\cdot\vec{k}$

$-\frac{1}{2}mR^2\varepsilon = -RF_0 + RF_T$

$F_T = -\frac{1}{2}mR\varepsilon + F_0 = -\frac{1}{2}30\cdot0{,}1\cdot80 + 180 = 60N$ ✓

c) A csúszásmentes gördüléshez szükséges minimális nyugvásbeli súrlódási tényező:

$\mu_{0min} = \frac{|F_T|}{|F_N|} = \frac{60}{300} = \underline{\underline{0{,}2}}$

d) l_{AB} szakaszon végzett munka:

$W_{AB} = \int_{t_A}^{t_B} P\,dt = \int_{t_A}^{t_B}\sum_{i=1}^{3}\vec{F_i}\cdot\vec{v_i}\,dt =$

$= \int_{t_A}^{t_B}\left(\vec{F_0}\cdot\vec{v_C} + \underbrace{\vec{G}\cdot\vec{v_S}}_{=0\ \perp} + \underbrace{\vec{F_K}\cdot\vec{v_A}}_{=0\ \ =0}\right)dt =$

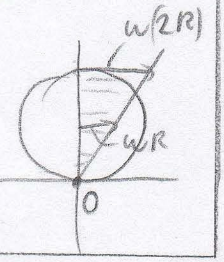
$= \int_{t_A}^{t_B}\vec{F_0}\cdot\vec{v_C}\,dt = \int_{t_A}^{t_B}\vec{F_0}\cdot2\cdot\vec{v_S}\,dt =$

$= 2\vec{F_0}\int_{t_A}^{t_B}\vec{v_S}\,dt = 2\vec{F_0}\int_{t_A}^{t_B}\frac{d\vec{r}}{dt}dt =$

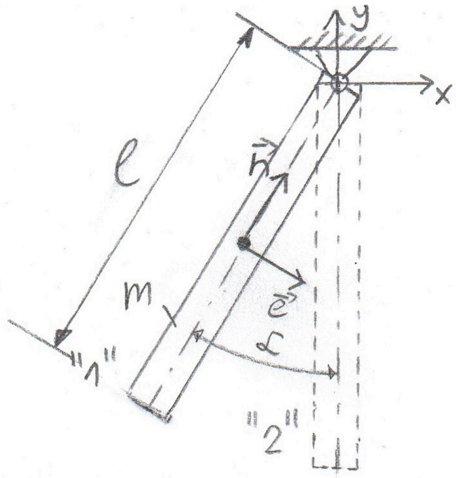
$= 2\cdot\vec{F_0}\int_{\vec{r}_A}^{\vec{r}_B}d\vec{r} = 2\cdot\vec{F_0}\cdot\Delta\vec{r}_{AB} =$

$= 2F_0\vec{i}\cdot l_{AB}\vec{i} = 2F_0 l_{AB} =$

$= 2\cdot180\cdot2 = \underline{\underline{720J}}$



FIZIKAI INGA



• adott

$m = 2\,kg$

$\ell = 2\,m$

$g = 10\,\frac{m}{s^2}$

$\alpha = 30^\circ$

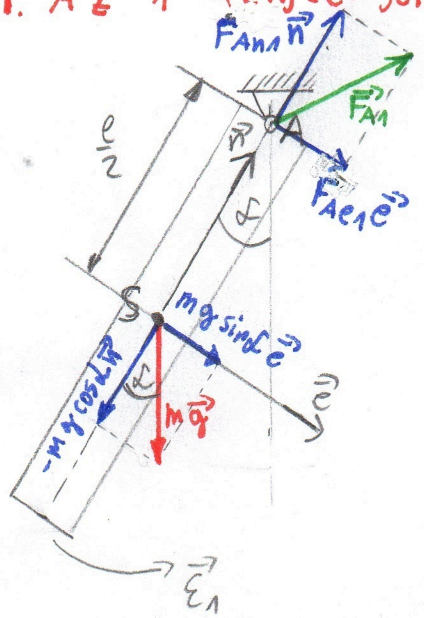
$v_{S1} = 0\,\frac{m}{s}$

• Feladat

a) $\vec{a}_{S1} = ?\quad \vec{F}_{A1} = ?$

b) $\vec{a}_{S2} = ?\quad \vec{F}_{A2} = ?$

1. Az "1" helyzet jellemzői:



• Az ingára ható erők:

$$\vec{F}_{A1} = F_{A1}\vec{e} + F_{A1}\vec{n}$$

$$m\vec{g} = mg\sin\alpha\,\vec{e} - mg\cos\alpha\,\vec{n}$$

• A súlypont gyorsulása:

$$\vec{a}_{S1} = a_{Se1}\vec{e} + a_{Sn1}\vec{n}$$

szöggyorsulás

$$\vec{\varepsilon}_1 = \varepsilon_1\vec{k}$$

$$\vec{\omega}_1 = \omega_1\vec{k} = \vec{0}$$

↑ szögsebesség

$$\boxed{a_{Se1} = \frac{\ell}{2}\varepsilon_1}$$

[Mivel a ⊕ érintő irányú gyorsuláshoz ⊕ szöggyorsulás tartozik. Ellenkező esetben $a_{Se1} = -\frac{\ell}{2}\varepsilon_1$ lenne]

$$\boxed{a_{Sn1} = \frac{v_{S1}^2}{\ell/2} = \frac{2v_{S1}^2}{\ell}} = \frac{2\cdot 0^2}{2} = 0\,\frac{m}{s^2}$$

[Ha ω_1 lenne adott akkor: $a_{Sn1} = \frac{2v_{S1}^2}{\ell} = \frac{2\left(\frac{\ell}{2}\right)^2\omega_1^2}{\ell} = \frac{2\frac{\ell^2}{4}\omega_1^2}{\ell} = \boxed{\frac{\ell}{2}\omega_1^2}$]

1.1: $\vec{\varepsilon}_1$ meghatározása → perdület tétel A pontra

$$\dot{\vec{\Pi}}_{A1} = \vec{M}_{A1}$$

$$J_A\vec{\varepsilon}_1 + \underbrace{\vec{\omega}_1 \times \vec{\Pi}_{A1}}_{=\vec{0}} = \vec{M}_{A1} \;/\cdot\vec{e}_z$$

$$J_A\varepsilon_1 = M_{A1}$$

$$\frac{m\ell^2}{3}\varepsilon_1 = \frac{\ell}{2}mg\sin\alpha$$

$$\varepsilon_1 = \frac{3g\sin\alpha}{2\ell} = \frac{3\cdot 10\cdot\sin 30^\circ}{2\cdot 2} = 3{,}75\,\frac{rad}{s^2}$$

1.2: $\vec{a}_{S1}$ meghatározása

$$a_{Se1} = \frac{l}{2}\varepsilon_1 = \frac{2}{2}\cdot 3{,}75 = 3{,}75\ \frac{m}{s^2}$$

$$\vec{a}_S = a_{Se1}\vec{e} + a_{Sn1}\vec{n} = \underline{\underline{\left(3{,}75\vec{e} + 0\vec{n}\right)\frac{m}{s^2}}}$$

1.3: $\vec{F}_{A1}$ meghatározása → impulzus tétel

$$\dot{\vec{I}} = \vec{F}$$

$$m\vec{a}_{S1} = \vec{F}_{A1} + m\vec{g}$$

$$m a_{Se1}\vec{e} = F_{Ae1}\vec{e} + F_{An1}\vec{n} + mg\sin\alpha\,\vec{e} - mg\cos\alpha\,\vec{n} \quad /\cdot\vec{e}\,/\cdot\vec{n}$$

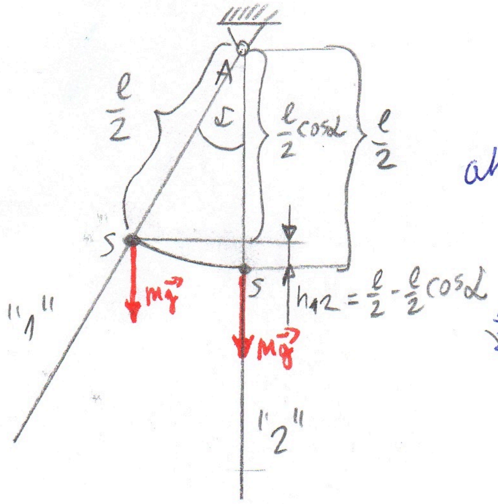
$$\left.\begin{aligned} (1)\quad m a_{Se1} &= F_{Ae1} + mg\sin\alpha \\ (2)\quad 0 &= F_{An1} - mg\cos\alpha \end{aligned}\right\}$$

$$(1) \rightarrow F_{Ae1} = m a_{Se1} - mg\sin\alpha = m(a_{Se1} - g\sin\alpha) =$$
$$= 2\cdot(3{,}75 - 10\cdot\sin 30^\circ) = -2{,}5\,N$$

$$(2) \rightarrow F_{An1} = mg\cos\alpha = 2\cdot 10\cdot\cos 30^\circ = 17{,}32\,N$$

$$\vec{F}_{A1} = F_{Ae1}\vec{e} + F_{An1}\vec{n} = \underline{\underline{\left(-2{,}5\vec{e} + 17{,}32\vec{n}\right)N}}$$

2. "1" helyzet → "2" helyzet ($\vec{\omega}_2$ meghatározása)



Munkatétel

$$E_2 - E_1 = W_{12}$$

ahol: $W_{12} = \vec{F}\cdot\Delta\vec{r} = mg(-\vec{e}_y)\cdot h_{12}(-\vec{e}_y) = mgh_{12}$

$$E_2 - E_1 = mgh_{12}$$

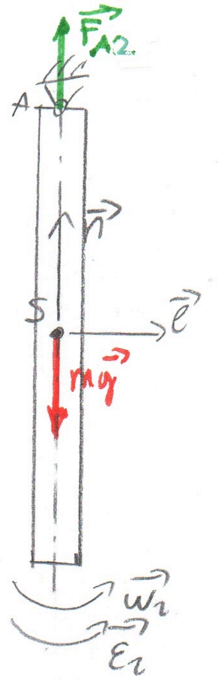
$$\frac{1}{2}J_a\omega_2^2 - \underbrace{\frac{1}{2}J_a\omega_1^2}_{=0} = mg\frac{l}{2}(1-\cos\alpha)$$

$$\omega_2 = \sqrt{\frac{mgl(1-\cos\alpha)}{J_a}} =$$

$$= \sqrt{\frac{2\cdot 10\cdot 2(1-\cos 30^\circ)}{2{,}\dot{6}}} = 1{,}418\ \frac{rad}{s}$$

$$J_a = \frac{ml^2}{3} = \frac{2\cdot 2^2}{3} = 2{,}\dot{6}$$

3. A "2" helyzet jellemzői:



3.1: $\vec{\varepsilon}_2$ meghatározása → perdülettétel A pontra

$$\dot{\vec{\Pi}}_{A2} = \vec{M}_{A2}$$

$$J_A \vec{\varepsilon}_2 + \underbrace{\vec{\omega}_2 \times \vec{\Pi}_{A2}}_{=\vec{0}} = \vec{M}_{A2} \quad / \cdot \vec{e}_z$$

$$J_A \varepsilon_2 = M_A$$

$$J_A \varepsilon_2 = 0$$

$$\varepsilon_2 = 0 \ \frac{rad}{s^2}$$

3.2: $\vec{a}_{S2}$ meghatározása

$$a_{Se2} = \frac{l}{2}\varepsilon_2 = \frac{2}{2}\cdot 0 = 0 \ \frac{m}{s^2}$$

$$a_{Sn2} = \frac{2 v_{S2}^2}{l} = \frac{l}{2}\omega_2^2 = \frac{2}{2}\cdot 1{,}418^2 = 2{,}01 \ \frac{m}{s^2}$$

$$\vec{a}_{S2} = a_{Se2}\vec{e} + a_{Sn2}\vec{n} = \underline{\underline{(0\vec{e} + 2{,}01\vec{n})\ \frac{m}{s^2}}}$$

3.3: $\vec{F}_{A2}$ meghatározása → impulzustétel

$$\dot{\vec{I}} = \vec{F}$$

$$m\vec{a}_{S2} = \vec{F}_{A2} + m\vec{g}$$

$$m a_{Sn2}\vec{n} = F_{Ae2}\vec{e} + F_{An2}\vec{n} - mg\vec{n} \quad / \cdot \vec{e} / \cdot \vec{n}$$

(1) $0 = F_{Ae2}$

(2) $m a_{Sn2} = F_{An2} - mg$

(1) → $F_{Ae2} = 0\ N$

(2) → $F_{An2} = m a_{Sn2} + mg = m(a_{Sn2} + g) = 2\cdot(2{,}01 + 10) = 24{,}02 N$

$$\vec{F}_{A2} = F_{Ae2}\vec{e} + F_{An2}\vec{n} = \underline{\underline{(0\vec{e} + 24{,}02\vec{n})\ N}}$$