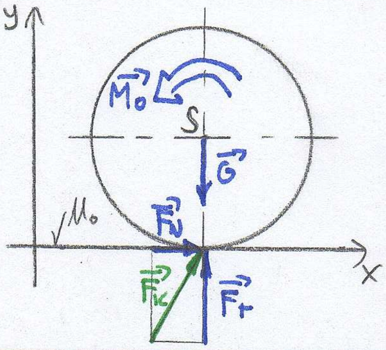


• adott

$\vec{M_0} = (36\vec{k})\,Nm$

$R = 0{,}3\,m$

$J_S = 1{,}2\,kgm^2$

$\mu_0 = 0{,}4$

$\mu = 0{,}3$

$m = 20\,kg$

• Feladat

a) $\vec{\varepsilon} = ?$ $\vec{a_S} = ?$

b) $\vec{F_K} = ?$

c) $\vec{a}_{Smax}, \vec{\varepsilon}_{max}, \vec{M}_{0max} = ?$

$\Downarrow$

henger még ne csússzon meg

a) Perdület tétel A pontra

$$\dot{\vec{\Pi}}_A = \vec{M_A}$$

$$J_A \cdot \vec{\varepsilon} + \underbrace{\vec{\omega} \times \vec{\Pi_A}}_{=\vec{0}} = \vec{M_A}$$

$$J_a \vec{\varepsilon} = \vec{M_A} \quad / \cdot \vec{k}$$

$$J_a \varepsilon = M_a$$

$$(J_S + mR^2) \cdot \varepsilon = M_0$$

$$\varepsilon = \frac{M_0}{J_S + mR^2} = \frac{36}{1{,}2 + 20 \cdot 0{,}3^2} = 12 \frac{rad}{s^2}$$

$$\vec{\varepsilon} = \varepsilon \vec{k} = (12\vec{k}) \frac{rad}{s^2}$$

$$a_S = -\varepsilon R = -12 \cdot 0{,}3 = -3{,}6 \frac{m}{s^2}$$

$$\vec{a}_S = a_S \vec{i} = (-3{,}6\vec{i}) \frac{m}{s^2}$$

b) Impulzus tétel

$$\dot{\vec{I}} = \vec{F}$$

$$m\vec{a}_S = \vec{G} + \vec{F_K}$$

$$-ma_S\vec{i} = -mg\vec{j} + F_T\vec{i} + F_N\vec{j} \quad / \cdot \vec{i} / \cdot \vec{j}$$

$$\left.\begin{aligned} &(1)\; -ma_S = F_T \\ &(2)\quad 0 = -mg + F_N \end{aligned}\right\}$$

$$(1) \rightarrow F_T = -ma_S = -20 \cdot 3{,}6 = -72\,N$$

$$(2) \rightarrow F_N = mg = 20 \cdot 10 = 200\,N$$

$$\vec{F_K} = F_T\vec{i} + F_N\vec{j} = (-72\vec{i} + 200\vec{j})\,N$$

c) A megcsúszási határnál

$$F_{Tmax} = \mu_0 F_N = 0{,}4 \cdot 200 = 80\,N$$

impulzus tétel:

$$F_{Tmax} = -m a_{Smax}$$

$$\mu_0 \underbrace{\cancel{m}g}_{F_N} = -\cancel{m}\, a_{Smax}$$

$$a_{Smax} = -\mu_0 g = -0{,}4 \cdot 10 = -4 \frac{m}{s^2}$$

$$\vec{a}_{Smax} = (-4\vec{i}) \frac{m}{s^2}$$

$$\varepsilon_{max} = -\frac{a_{Smax}}{R} = -\frac{-4}{0{,}3} = 13{,}\dot{3} \frac{rad}{s^2}$$

$$\vec{\varepsilon}_{max} = (13{,}\dot{3}\vec{k}) \frac{rad}{s^2}$$

perdület tétel:

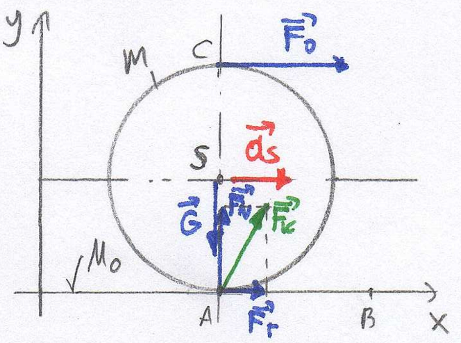
$$J_a \varepsilon_{max} = M_{0max}$$

$$M_{0max} = (J_S + mR^2) \cdot \varepsilon_{max} =$$

$$= (1{,}2 + 20 \cdot 0{,}3^2) \cdot 13{,}\dot{3} =$$

$$= 40\,Nm$$

$$\vec{M}_{0max} = M_{0max}\vec{k} = (40\vec{k})\,Nm$$



• **adott**

$\vec{a}_s = (8\vec{i})\ \frac{m}{s^2}$

$l_{AB} = 2m$

$R = 0{,}1m$

$m = 30kg$

$g = 10\ \frac{m}{s^2}$

• **Feladat**

a) $\vec{F}_0 = ?$

b) $\vec{F}_K = ?$

c) $\mu_{0min} = ?$

d) $W_{AB} = ?$

a) $\varepsilon = -\frac{a_s}{R} = -\frac{8}{0{,}1} = -80\ \frac{rad}{s^2}$

$\vec{\varepsilon} = \varepsilon\vec{k} = (-80\vec{k})\ \frac{rad}{s^2}$

Perdület tétel A pontra

$\dot{\vec{\Pi}}_A = \vec{M}_A$

$J_A\vec{\varepsilon} + \underbrace{\vec{\omega}\times\vec{\Pi}_A}_{=\vec{0}} = \vec{M}_A$

$J_a\vec{\varepsilon} = \vec{M}_A \quad /\cdot\vec{k}$

$J_a\varepsilon = M_a$

$\frac{3}{2}mR^2\varepsilon = -2RF_0$

$F_0 = \frac{-\frac{3}{2}mR^2\varepsilon}{2R} = \frac{-\frac{3}{2}\cdot30\cdot0{,}1\cdot(-80)}{2} = 180N$

$\underline{\vec{F}_0 = (180\vec{i})N}$

b) Impulzus tétel

$\dot{\vec{I}} = \vec{F}$

$m\vec{a}_s = \vec{F}_0 + \vec{G} + \vec{F}_K$

$ma_s\vec{i} = F_0\vec{i} - mg\vec{j} + F_T\vec{i} + F_N\vec{j} \quad /\cdot\vec{i}/\vec{j}$

$\left.\begin{aligned} &(1)\ ma_s = F_0 + F_T \\ &(2)\ 0 = -mg + F_N \end{aligned}\right\}$

$(1) \Rightarrow F_T = ma_s - F_0 = 30\cdot8 - 180 = 60N$

$(2) \Rightarrow F_N = mg = 30\cdot10 = 300N$

$\vec{F}_K = F_T\vec{i} + F_N\vec{j} = \underline{(60\vec{i} + 300\vec{j})N}$

ellenőrzés $\Rightarrow$ perdület tétel S pontra

$J_S\vec{\varepsilon} = \vec{M}_S \quad /\cdot\vec{k}$

$-\frac{1}{2}mR^2\varepsilon = -RF_0 + RF_T$

$F_T = -\frac{1}{2}mR\varepsilon + F_0 = -\frac{1}{2}30\cdot0{,}1\cdot80 + 180 = 60N$ ✓

c) A csúszásmentes gördüléshez szükséges minimális nyugvásbeli súrlódási tényező:

$\mu_{0min} = \frac{|F_T|}{|F_N|} = \frac{60}{300} = \underline{\underline{0{,}2}}$

d) l_{AB} szakaszon végzett munka:

$W_{AB} = \int_{t_A}^{t_B} P\,dt = \int_{t_A}^{t_B}\sum_{i=1}^{3}\vec{F}_i\cdot\vec{v}_i\,dt =$

$= \int_{t_A}^{t_B}\left(\vec{F}_0\cdot\vec{v}_C + \underbrace{\vec{G}\cdot\vec{v}_S}_{=0\ \perp} + \underbrace{\vec{F}_K\cdot\vec{v}_A}_{=0\ \ =0}\right)dt =$

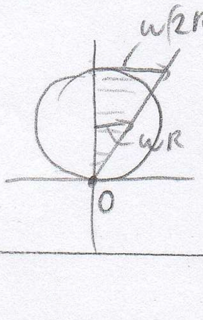
$= \int_{t_A}^{t_B}\vec{F}_0\cdot\vec{v}_C\,dt = \int_{t_A}^{t_B}\vec{F}_0\cdot2\cdot\vec{v}_S\,dt =$

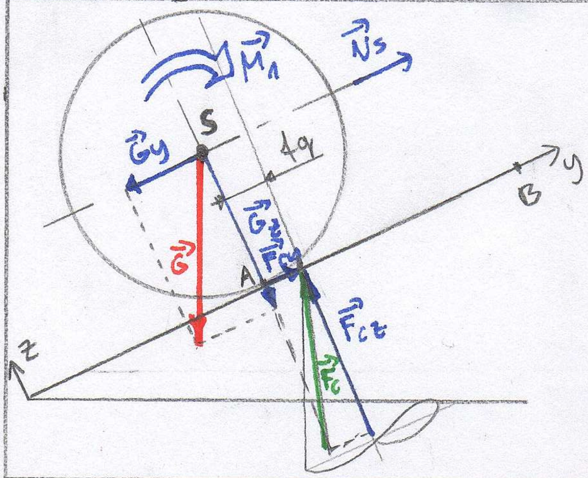
$= 2\vec{F}_0\int_{t_A}^{t_B}\vec{v}_S\,dt = 2\vec{F}_0\int_{t_A}^{t_B}\frac{d\vec{r}}{dt}\,dt =$

$= 2\cdot\vec{F}_0\int_{\vec{r}_A}^{\vec{r}_B}d\vec{r} = 2\cdot\vec{F}_0\cdot\Delta\vec{r}_{AB} =$

$= 2F_0\vec{i}\cdot l_{AB}\vec{i} = 2F_0 l_{AB} =$

$= 2\cdot180\cdot2 = \underline{\underline{720\,J}}$





• adott

$\vec{G} = (-100\vec{j} - 240\vec{k})\,N$

$f_y = 5\,mm$

$\vec{M}_1 = (-59\vec{i})\,Nm$

$R = 0{,}5\,m$

$\mu = 0{,}4$

$\mu_0 = 0{,}5$

$s_{AB} = 3{,}9\,m$

$\vec{\omega}_B = (-4{,}9\vec{i})\,\frac{rad}{s}$

• Feladat

a) $\vec{\varepsilon} = ?$

$\vec{F}_c = ?$

b) $\vec{\omega}_A = ?$ hogy B pontban $\vec{\omega}_B$ legyen a szögsebesség

$$m = \frac{G}{g} = \frac{\sqrt{100^2 + 240^2}}{10} = 26\,kg$$

a)

• Perdület tétel A pontra

$$\dot{\vec{\Pi}}_A = \vec{M}_A$$

$$J_A\vec{\varepsilon} + \underbrace{\vec{\omega} \times \vec{\Pi}_A}_{=\vec{0}} = \vec{M}_A$$

$$J_A\vec{\varepsilon} = \vec{M}_A \quad / \cdot \vec{i}$$

$$J_A \varepsilon = M_A$$

① $\boxed{\frac{3}{2}mR^2\varepsilon = -M_1 + RG_y + f_y F_{cz}}$

• impulzus tétel

$$\dot{\vec{I}} = \vec{F}$$

$$m\cdot\vec{a}_S = \vec{G} + \vec{F}_c$$

$$m a_S \vec{j} = -G_y\vec{j} - G_z\vec{k} + F_{cy}\vec{j} + F_{cz}\vec{k} \quad /\cdot\vec{j}, \cdot\vec{k}$$

② $\boxed{m a_S = -G_y + F_{cy}}$

③ $\boxed{0 = -G_z + F_{cz}}$

---

④ $\boxed{a_S = -\varepsilon R}$

③ → $F_{cz} = G_z = 240\,N$

① → $\varepsilon = \dfrac{-M_1 + RG_y + f_y F_{cz}}{\frac{3}{2}mR^2} =$

$$= \frac{-59 + 0{,}5\cdot 100 + 0{,}005\cdot 240}{\frac{3}{2}\cdot 26\cdot 0{,}5^2} =$$

$$= -0{,}8\,\frac{rad}{s^2} \Rightarrow \vec{\varepsilon} = (-0{,}8\vec{i})\,\frac{rad}{s^2}$$

④ → $a_S = -\varepsilon R = -(-0{,}8)\cdot 0{,}5 = 0{,}4\,\frac{m}{s^2}$

② → $F_{cy} = m a_S + G_y = 26\cdot 0{,}4 + 100 = 110{,}4\,N$

$$\vec{F}_c = (110{,}4\vec{j} + 240\vec{k})\,N$$

Megcsúszás ellenőrzése:

A maximálisan átvihető tapadás:

$$F_{T\max} = \mu_0 \underset{F_{cz}}{F_N} = 0{,}5\cdot 240 = 120\,N$$

$$F_T = F_{cy} = 110{,}4\,N$$

$F_T < F_{T\max}$ ⇒ nincs megcsúszás

b) S_{AB} út során végzett munka

$$W_{AB} = \int_{t_A}^{t_B} P \, dt$$

• ahol: $P = \vec{G} \cdot \vec{v_s} + \vec{M_1} \vec{\omega} + \vec{M_A} \cdot \vec{\omega} =$

$$= (G_y \vec{j} - G_z \vec{k}) \cdot (v_s \vec{j}) + (-M_1 \vec{i}) \cdot (-\omega \vec{i}) + (F_{Cz} f_g \vec{i}) \cdot (-\omega \vec{i}) =$$

$$= -G_y v_s + M_1 \omega - F_{Cz} f_g \omega = -G_y v_s + (M_1 - F_{Cz} f_g) \omega$$

• tehát

$$W_{AB} = \int_{t_A}^{t_B} -G_y v_s + (M_1 - F_{Cz} f_g) \omega \, dt = -G_y \int_{t_A}^{t_B} v_s dt + (M_1 - F_{Cz} f_g) \int_{t_A}^{t_B} \omega \, dt =$$

$$= -G_y S_{AB} + (M_1 - F_{Cz} f_g) \varphi_{AB} = -G_y S_{AB} + (M_1 - F_{Cz} f_g) \frac{S_{AB}}{R} =$$

$$= -100 \cdot 3{,}9 + (59 - 240 \cdot 0{,}005) \cdot \frac{3{,}9}{0{,}5} = 60{,}847$$

• Munkatétel

$$E_B - E_A = W_{AB}$$

$$\frac{1}{2} J_a \omega_B^2 - \frac{1}{2} J_a \omega_A^2 = W_{AB}$$

$$J_a \omega_B^2 - J_a \omega_A^2 = 2 W_{AB}$$

$$J_a \omega_A^2 = J_a \omega_B^2 - 2 W_{AB}$$

$$\omega_A = \sqrt{\frac{J_a \omega_B^2 - 2 W_{AB}}{J_a}} = \sqrt{\omega_B^2 - \frac{2 W_{AB}}{J_a}} =$$

$$J_a \leftarrow \frac{3}{2} m R^2$$

$$= \sqrt{\omega_B^2 - \frac{4 W_{AB}}{3 m R^2}} = \sqrt{(-4{,}9)^2 - \frac{4 \cdot 60{,}84}{3 \cdot 26 \cdot 0{,}5^2}} =$$

$$= 3{,}395 \ \frac{\text{rad}}{\text{s}}$$

$$\vec{\omega_A} = -\omega_A \vec{i} = \underline{\underline{(-3{,}395 \vec{i}) \ \frac{\text{rad}}{\text{s}}}}$$