

1.0

# EGYSZÁBADSÁGFOKÚ REZGÖRENPSZER MOZGÁSEGYENLETE

• általános koordinátek

$$q_r = y_B \quad - B \text{ pont elmozdulása}$$

$$\dot{q}_r = \dot{y}_B (v_B) \quad - B \text{ pont sebessége}$$

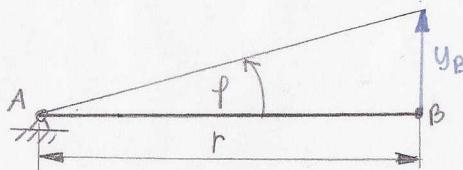
$$\ddot{q}_r = \ddot{y}_B (a_B) \quad - B \text{ pont gyorsulása}$$

$$q_r = \varphi \quad \text{nögelfordulás}$$

$$\dot{q}_r = \dot{\varphi} (\omega) \quad \text{nögselenség}$$

$$\ddot{q}_r = \ddot{\varphi} (\epsilon) \quad \text{nögyorsulás}$$

• önmítőges általános koordináták körül



$$y_B = r \cdot \dot{\varphi}$$

$$\dot{y}_B = r \cdot \ddot{\varphi}$$

$$\ddot{y}_B = r \cdot \ddot{\varphi}$$

$$\text{Lagrange egyenlet: } \frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}_r} \right] - \frac{\partial E}{\partial q_r} = Q_c + Q_k + Q_g$$

## 1. $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$ meghatározása

1. lépés: rendszer össenergiaja

a) tömegpont: $E = \frac{1}{2} m v^2$	$\dot{q} = \dot{y}_A$	$\dot{q} = \dot{\varphi}$
	$\frac{1}{2} m \dot{y}_A^2$	$\frac{1}{2} m (r \cdot \dot{\varphi})^2$

b) merev test: $E = \frac{1}{2} J_a \omega^2$	$\frac{1}{2} J_a \left( \frac{y_B}{r} \right)^2$	$\frac{1}{2} J_a \dot{\varphi}^2$
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az energia általános alakja:  $E = \frac{1}{2} \cdot M_{red} \cdot \dot{q}^2$

2. lépés: összehasonlítás:  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = M_{red} \cdot \ddot{q}$

2.  $\frac{\partial E}{\partial q}$  meghatározása  $\frac{\partial E}{\partial q} = 0$

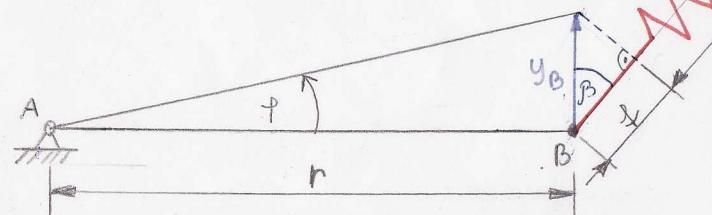
## 3. $Q_c$ meghatározása

1. lépés: nyívenergia:  $U = \frac{1}{2} \frac{f^2}{c}$

f = nögy horizontálisra

c = nyívfállamoló

f meghatározása:



$$f = y_B \cdot \cos \beta$$

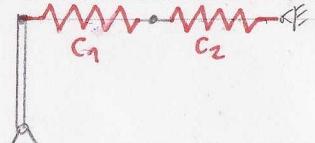
$$y_B = r \cdot \dot{\varphi}$$

$$f = r \cdot \dot{\varphi} \cdot \cos \beta$$

2. lépés: nyívenergia

$$Q_c = - \frac{\partial U}{\partial q_r} = - \frac{1}{c_{red}} \cdot q_r$$

sorba kapcsolt nyírok:



$$U = \frac{1}{2} \cdot \frac{f^2}{c_1 + c_2}$$

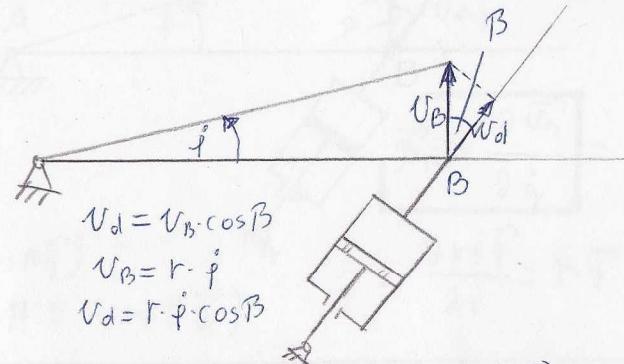
## 4. $Q_k$ meghatározása

1. lépés: villapítási energia

$$D = \frac{1}{2} \frac{1}{2} V_d^2$$

2. lépés: villapító elv

$$Q_k = - \frac{\partial D}{\partial \dot{q}} = - k_{red} \cdot \dot{q}$$



$$\vec{P}_B = \frac{\partial \vec{V}_B}{\partial \dot{q}}$$

## 5. $Q_g$ meghatározása

$$Q_g = \vec{F}_g(t) \cdot \vec{P}_B$$

vaaz

$$Q_g = \vec{M}_g(t) \cdot \vec{b}$$

$$\vec{b} = \frac{\partial \vec{w}}{\partial \dot{q}}$$

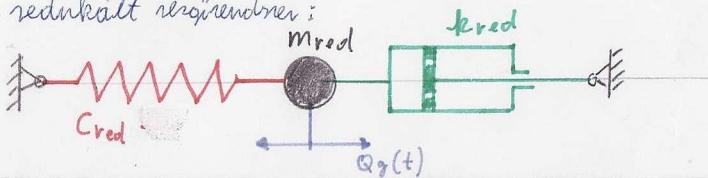
• lehelyettesítés Lagrange egyenlete

$$m_{red} \cdot \ddot{q} - 0 = - \frac{1}{c_{red}} \cdot q_r - k_{red} \cdot \dot{q} + Q_g(t)$$

• rendszer az egyenleteit:

$$m_{red} \ddot{q} + k_{red} \dot{q} + \frac{1}{c_{red}} q_r = Q_g(t)$$

• a redukált rezgésszabvány:



## • általános koordinátaik

$$\dot{q}_r = y_B \quad - B \text{ pont elmozdulása}$$

$$\ddot{q}_r = \ddot{y}_B (U_B) \quad - B \text{ pont sebessége}$$

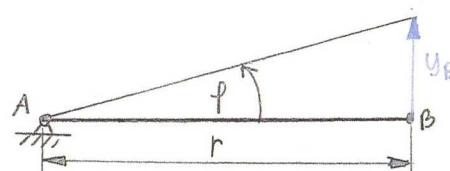
$$\dddot{q}_r = \dddot{y}_B (U_B) \quad - B \text{ pont gyorsulása}$$

$$\dot{q}_r = f \quad \text{szögelmozgás}$$

$$\ddot{q}_r = \ddot{f} (\omega) \quad \text{szögsebesség}$$

$$\dddot{q}_r = \dddot{f} (\varepsilon) \quad \text{szöggyorsulás}$$

## • összetett általános koordináták körül



$$y_B = r \cdot f$$

$$\dot{y}_B = r \cdot \dot{f}$$

$$\ddot{y}_B = r \cdot \ddot{f}$$

$$\bullet \text{Lagrange egyenlet: } \frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}_r} \right] - \frac{\partial E}{\partial q_r} = Q_c + Q_k + Q_g$$

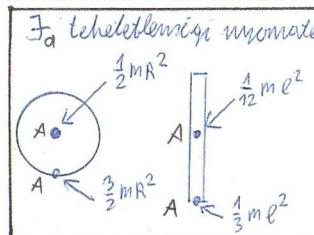
(1)  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}_r} \right]$  meghatározása

## 1. lépés: rendszer össnenegyéja

	$\dot{q} = y_A$	$\dot{q} = \dot{f}$
a) tömegpont: $E = \frac{1}{2} m v^2$	$\frac{1}{2} m \dot{y}_B^2$	$\frac{1}{2} m (r \cdot \dot{f})^2$
b) merev test: $E = \frac{1}{2} J_A \omega^2$	$\frac{1}{2} J_A (\dot{y}_B / r)^2$	$\frac{1}{2} J_A \dot{f}^2$

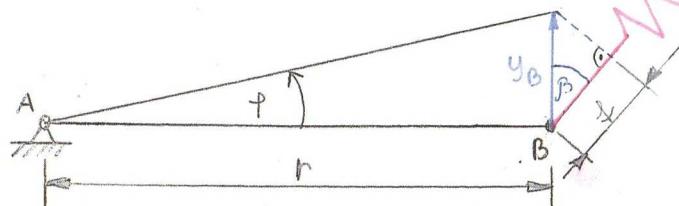
az energia általános alakja:  $E = \frac{1}{2} \cdot M_{red} \cdot \dot{q}^2$

2. lépés: összefüggés:  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}_r} \right] = M_{red} \cdot \ddot{q}_r$

(2)  $\frac{\partial E}{\partial q_r}$  meghatározása  $\frac{\partial E}{\partial q_r} = 0$ (3)  $Q_c$  meghatározása

1. lépés: mojenegyia:  $U = \frac{1}{2} \frac{f^2}{c}$

f meghatározása:



$$f = \text{magó hosszmódra}$$

$$c = \text{magóállandó}$$

$$f = y_B \cdot \cos \beta$$

$$y_B = r \cdot f$$

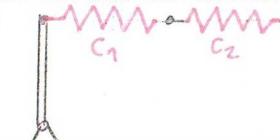
$$f = r \cdot \dot{f} \cdot \cos \beta$$

## 2. lépés: nyírő

$$Q_c = - \frac{\partial U}{\partial q_r} = - \frac{1}{c_{red}} \cdot q_r$$

szabályozott nyírő:

$$U = \frac{1}{2} \cdot \frac{f^2}{c_1 + c_2}$$

(4)  $Q_k$  meghatározása

$$Q_k = \vec{F}_k \cdot \vec{\beta}_B$$

$$\vec{F}_k = -k \cdot \vec{U}_{dry}$$

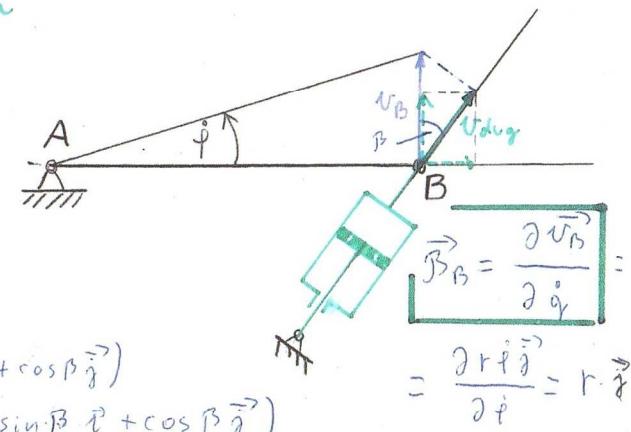
$$U_{dry} = U_B \cdot \cos \beta$$

$$U_B = r \cdot \dot{f}$$

$$U_{dry} = r \cdot \dot{f} \cdot \cos \beta$$

$$\vec{U}_{dry} = U_{dry} (\sin \beta \hat{i} + \cos \beta \hat{j})$$

$$\vec{U}_{dry} = r \cdot \dot{f} \cdot \cos \beta \cdot (\sin \beta \hat{i} + \cos \beta \hat{j})$$

(5)  $Q_g$  meghatározása

$$Q_g = \vec{F}_g(t) \cdot \vec{\beta}_B$$

$$\text{már } Q_g = \vec{M}_g(t) \cdot \vec{b}$$

$$\vec{\beta}_B = \frac{\partial \vec{U}_B}{\partial \dot{q}_r}$$

$$\vec{b} = \frac{\partial \vec{\omega}}{\partial \dot{q}_r}$$

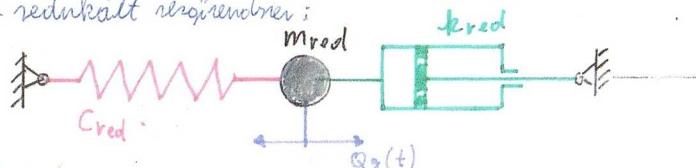
## • lehelyetterítés Lagrange egyenlete

$$m_{red} \ddot{q}_r = - \frac{1}{c_{red}} \cdot q_r - k_{red} \cdot \dot{q}_r + Q_g(t)$$

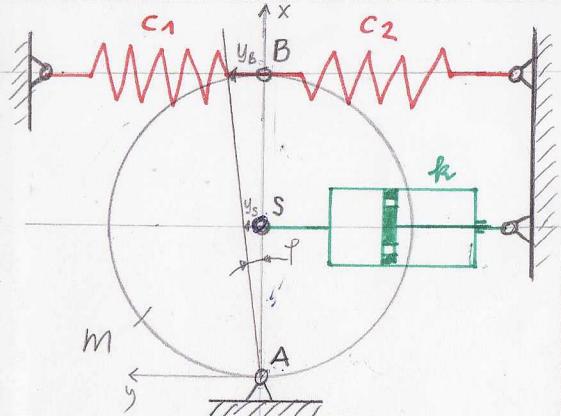
## • rendszer az egyenleteit:

$$m_{red} \ddot{q}_r + k_{red} \dot{q}_r + \frac{1}{c_{red}} q_r = Q_g(t)$$

## • a rendszer által megadott:



# 1.1 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



a) 1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} \int_a \dot{\varphi}^2 = \frac{1}{2} \left( \frac{3}{2} MR^2 \right) \dot{\varphi}^2$$

$$\frac{\partial E}{\partial \dot{\varphi}} = \frac{3}{2} MR^2 \dot{\varphi}$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \frac{3}{2} MR^2 \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$

3.  $Q_c$  meghatározása

$$U = U_1 + U_2 = \frac{1}{2} \cdot \frac{y_B^2}{c_1} + \frac{1}{2} \cdot \frac{y_B^2}{c_2} =$$

$$= \frac{1}{2} \cdot \frac{(2R\dot{\varphi})^2}{c_1} + \frac{1}{2} \cdot \frac{(2R\dot{\varphi})^2}{c_2} =$$

$$= \frac{1}{2} \cdot \left( \frac{4R^2\dot{\varphi}^2}{c_1} + \frac{4R^2\dot{\varphi}^2}{c_2} \right) = \frac{2R^2\dot{\varphi}^2}{c_1} + \frac{2R^2\dot{\varphi}^2}{c_2}$$

$$Q_c = - \frac{\partial U}{\partial \dot{\varphi}} = - \frac{\partial}{\partial \dot{\varphi}} \left( \frac{2R^2\dot{\varphi}^2}{c_1} + \frac{2R^2\dot{\varphi}^2}{c_2} \right) =$$

adott:  $c_1, c_2, k, m, R$

• az általános hozzáírása:

a) $q = \varphi$	b) $q = y_B$
$\dot{q} = \dot{\varphi}$	$\dot{q} = \dot{y}_B$ ( $v_B$ )
$\ddot{q} = \ddot{\varphi}$ ( $\epsilon$ )	$\ddot{q} = \ddot{y}_B$ ( $a_B$ )

• Lagrange egyenlet:

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_c + Q_k$$

$$= - \left( \frac{4R^2 l}{c_1} + \frac{4R^2 l}{c_2} \right) = - \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi}$$

$$Q_c = - \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi}$$

4.  $Q_k$  meghatározása

$$D = \frac{1}{2} k V_d^2 = \frac{1}{2} k (R \cdot \dot{\varphi})^2 = \frac{1}{2} k \cdot R^2 \dot{\varphi}^2$$

$$Q_k = - \frac{\partial D}{\partial \dot{\varphi}} = - k R^2 \dot{\varphi}$$

$$Q_k = - k R^2 \dot{\varphi}$$

• lehelyettesítés Lagrange egyenlete

$$\frac{3}{2} M R^2 \ddot{\varphi} - 0 = - \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi} - k R^2 \dot{\varphi}$$

• rendezze az egyenletet:

$$\frac{3}{2} M R^2 \cdot \ddot{\varphi} + k \cdot R^2 \cdot \dot{\varphi} + \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi} = 0$$

$M_{red}$   $k_{red}$   $\frac{1}{c_{red}}$

• a redukált rezgésszövrendel:



$$M_{red} \cdot \ddot{q} + k_{red} \cdot \dot{q} + \frac{1}{c_{red}} \cdot q = 0$$

b) 1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} \int_a \dot{\varphi}^2 = \frac{1}{2} \left( \frac{3}{2} M R^2 \right) \omega^2 = \frac{1}{2} \left( \frac{3}{2} M R^2 \right) \cdot \left( \frac{\dot{y}_B}{2R} \right)^2 = \frac{1}{2} \cdot \left( \frac{3}{8} M \right) \dot{y}_B^2$$

$$\frac{\partial E}{\partial \dot{y}_B} = \frac{3}{8} M \dot{y}_B$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{y}_B} \right] = \frac{3}{8} M \ddot{y}_B$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \frac{3}{8} M \ddot{y}_B$$

4.  $Q_k$  meghatározása

$$D = \frac{1}{2} k V_d^2 = \frac{1}{2} k \left( \frac{\dot{y}_B}{2} \right)^2 = \frac{1}{2} k \cdot \frac{1}{4} \cdot \dot{y}_B^2$$

$$Q_k = - \frac{\partial D}{\partial \dot{y}_B} = - k \cdot \frac{1}{4} \cdot \dot{y}_B$$

$$Q_k = - k \cdot \frac{1}{4} \cdot \dot{y}_B$$

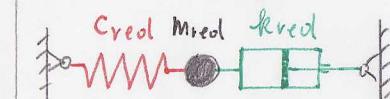
• lehelyettesítés Lagrange egyenlete

$$\frac{3}{8} M \ddot{y}_B - 0 = - \left( \frac{1}{c_1} + \frac{1}{c_2} \right) y_B - \frac{k}{4} \cdot y_B$$

• rendezze az egyenletet

$$\underbrace{\frac{3}{8} M \ddot{y}_B}_{M_{red}} + \underbrace{\frac{k}{4} \cdot y_B}_{k_{red}} + \underbrace{\left( \frac{1}{c_1} + \frac{1}{c_2} \right) y_B}_{\frac{1}{c_{red}}} = 0$$

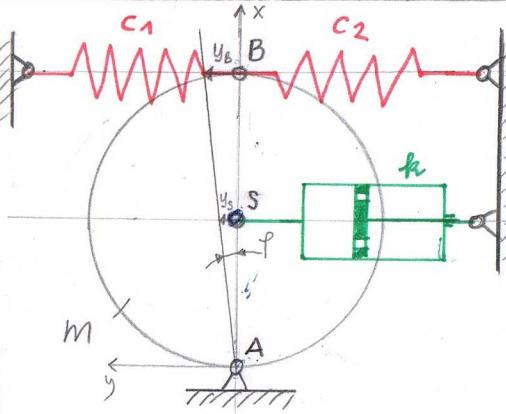
• a redukált rezgésszövrendel:



$$M_{red} \ddot{q} + k_{red} \cdot \dot{q} + \frac{1}{c_{red}} \cdot q = 0$$

1.1

# EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



a) 1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} J_a \cdot \dot{\varphi}^2 = \frac{1}{2} \left( \frac{3}{2} m R^2 \right) \dot{\varphi}^2$$

$$\frac{\partial E}{\partial \dot{\varphi}} = \frac{3}{2} m R^2 \dot{\varphi}$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\varphi}} \right] = \frac{3}{2} m R^2 \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$

3.  $Q_c$  meghatározása

$$U = U_1 + U_2 = \frac{1}{2} \cdot \frac{y_B^2}{c_1} + \frac{1}{2} \cdot \frac{y_B^2}{c_2} =$$

$$= \frac{1}{2} \cdot \frac{(2R\dot{\varphi})^2}{c_1} + \frac{1}{2} \cdot \frac{(2R\dot{\varphi})^2}{c_2} =$$

$$= \frac{1}{2} \cdot \left( \frac{4R^2\dot{\varphi}^2}{c_1} + \frac{4R^2\dot{\varphi}^2}{c_2} \right) = \frac{2R^2\dot{\varphi}^2}{c_1} + \frac{2R^2\dot{\varphi}^2}{c_2}$$

$$Q_c = - \frac{\partial U}{\partial \dot{\varphi}} = - \frac{\partial}{\partial \dot{\varphi}} \left( \frac{2R^2\dot{\varphi}^2}{c_1} + \frac{2R^2\dot{\varphi}^2}{c_2} \right) =$$

adott:  $c_1, c_2, k, m, R$

• az általános koordináták:

a) $q = \varphi$	b) $q = y_B$
$\dot{q} = \dot{\varphi}(\omega)$	$\dot{q} = \dot{y}_B(v_B)$
$\ddot{q} = \ddot{\varphi}(\epsilon)$	$\ddot{q} = \ddot{y}_B(a_B)$

• Lagrange egyenlet:

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_c + Q_k$$

$$= - \left( \frac{4R^2\dot{\varphi}}{c_1} + \frac{4R^2\dot{\varphi}}{c_2} \right) = - \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi}$$

$$Q_c = - \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi}$$

4.  $Q_k$  meghatározása

$$Q_k = \vec{F}_k \cdot \vec{B}_S$$

$$\vec{F}_k = -k \cdot \vec{v}_{\text{dmg}} = -k \cdot \vec{v}_S = -k \cdot (R \cdot \dot{\varphi}) \vec{j}$$

$$\vec{B}_S = \frac{\partial \vec{v}_S}{\partial \dot{\varphi}} = \frac{\partial (R \cdot \dot{\varphi} \vec{j})}{\partial \dot{\varphi}} = R \cdot \vec{j}$$

$$Q_k = -k \cdot R \cdot \dot{\varphi} \cdot \vec{j} \cdot R \cdot \vec{j} = -k R^2 \dot{\varphi}$$

$$Q_k = -k R^2 \dot{\varphi}$$

• lehetségteljes Lagrange egyenlete

$$\frac{3}{2} m R^2 \ddot{\varphi} - 0 = - \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi} - k R^2 \dot{\varphi}$$

• rendezi az egyenletet:

$$\frac{3}{2} m R^2 \cdot \ddot{\varphi} + k \cdot R^2 \cdot \dot{\varphi} + \left( \frac{4R^2}{c_1} + \frac{4R^2}{c_2} \right) \cdot \dot{\varphi} = 0$$

$$m_{\text{red}} \quad k_{\text{red}} \quad \frac{1}{c_{\text{red}}}$$

• a rezonkált rezgőrendszer:



$$m_{\text{red}} \cdot \ddot{q} + k_{\text{red}} \cdot \dot{q} + \frac{1}{c_{\text{red}}} \cdot q = 0$$

b) 1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} J_a \omega^2 = \frac{1}{2} \left( \frac{3}{2} m R^2 \right) \cdot \omega^2 = \frac{1}{2} \left( \frac{3}{2} m R^2 \right) \cdot \left( \frac{\dot{y}_B}{2R} \right)^2 = \frac{1}{2} \cdot \left( \frac{3}{8} m \right) \cdot \dot{y}_B^2$$

$$\frac{\partial E}{\partial \dot{y}_B} = \frac{3}{8} m \dot{y}_B$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{y}_B} \right] = \frac{3}{8} m \ddot{y}_B$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{y}_B} \right] = \frac{3}{8} m \ddot{y}_B$$

4.  $Q_k$  meghatározása

$$Q_k = \vec{F}_k \cdot \vec{B}_S$$

$$\vec{F}_k = -k \cdot \vec{v}_{\text{dmg}} = -k \cdot \vec{v}_S =$$

$$= -k \left( \frac{\dot{y}_B}{2} \right) \vec{j} = \frac{\partial \vec{v}_S}{\partial \dot{y}_B} = \frac{\partial \left( \frac{\dot{y}_B}{2} \vec{j} \right)}{\partial \dot{y}_B} = \frac{1}{2} \vec{j}$$

$$Q_k = -k \cdot \frac{\dot{y}_B}{2} \vec{j} \cdot \frac{1}{2} \vec{j} = -k \cdot \frac{1}{4} \cdot \dot{y}_B$$

$$Q_k = -k \cdot \frac{1}{4} \cdot \dot{y}_B$$

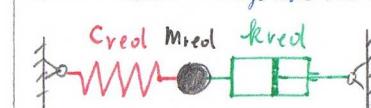
• lehetségteljes Lagrange egyenlete

$$\frac{3}{8} m \ddot{y}_B - 0 = - \left( \frac{1}{c_1} + \frac{1}{c_2} \right) y_B - \frac{k}{4} \cdot \dot{y}_B$$

• rendezi az egyenletet

$$\underbrace{\frac{3}{8} m \ddot{y}_B}_{m_{\text{red}}} + \underbrace{\frac{k}{4} \cdot \dot{y}_B}_{k_{\text{red}}} + \underbrace{\left( \frac{1}{c_1} + \frac{1}{c_2} \right) y_B}_{\frac{1}{c_{\text{red}}}} = 0$$

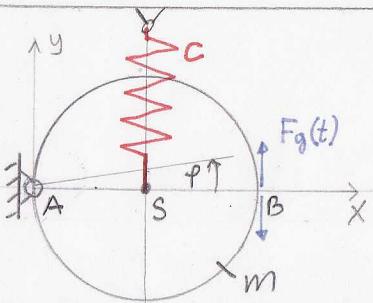
• a rezonkált rezgőrendszer:



$$m_{\text{red}} \ddot{q} + k_{\text{red}} \cdot \dot{q} + \frac{1}{c_{\text{red}}} \cdot q = 0$$

## EGYSZABADSÁGFOKÚ REZGÖRENDSZER

## MOZGÁSEGYENLETE



adott:  $C, m, R, F_g(t) = F_{go} \cdot \sin(\omega t + \epsilon)$

- az általános koordináta:  
 $q = \varphi \quad \dot{q} = \dot{\varphi} \quad \ddot{q} = \ddot{\varphi}$

Lagrange egyenlet

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_c + Q_g$$

1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} \mathcal{J}_a \omega^2 = \frac{1}{2} \left( \frac{3}{2} m R^2 \right) \dot{\varphi}^2$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \frac{3}{2} m R^2 \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$

3.  $Q_c$  meghatározása

$$U = \frac{1}{2} \cdot \frac{(R\dot{\varphi})^2}{C} = \frac{1}{2} \frac{R^2}{C} \cdot \dot{\varphi}^2$$

$$Q_c = - \frac{\partial U}{\partial \dot{\varphi}} = - \frac{R^2}{C} \cdot \dot{\varphi}$$

$$Q_c = - \frac{R^2}{C} \cdot \dot{\varphi}$$

4.  $Q_g$  meghatározása

$$Q_g = \vec{F}_g \cdot \vec{B}_B$$

$$\vec{F}_g = F_{go} \cdot \sin(\omega t + \epsilon) \hat{j}$$

$$\vec{B}_B = \frac{\partial \vec{B}}{\partial \dot{\varphi}} = \frac{\partial (2R\dot{\varphi} \cdot \hat{j})}{\partial \dot{\varphi}} = 2R\hat{j}$$

$$Q_g = \vec{F}_g \cdot \vec{B}_B = 2R F_g(t)$$

$$Q_g = 2R F_g(t)$$

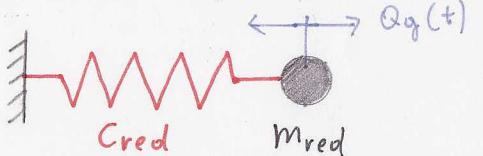
• lehelyettesítve Lagrange egyenlettel:

$$\frac{3}{2} m R^2 \ddot{\varphi} - 0 = - \frac{R^2}{C} \cdot \dot{\varphi} + 2R F_g(t)$$

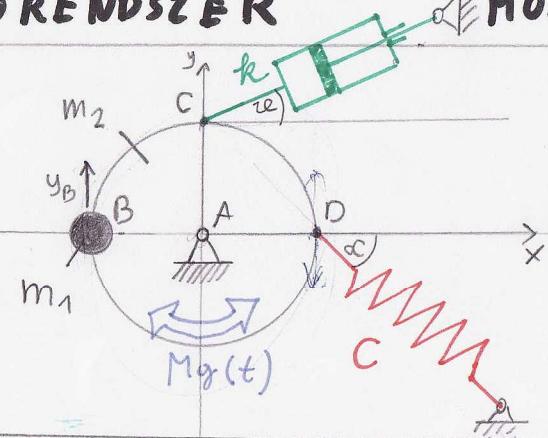
• rendszír az egyenletet:

$$\underbrace{\frac{3}{2} m R^2 \ddot{\varphi}}_{M_{red}} + \underbrace{\frac{R^2}{C} \cdot \dot{\varphi}}_{Q_g(+)} = \underbrace{2R F_g(t)}_{Q_g(+)}$$

• a redukált rezgőrendszer:



$$M_{red} \ddot{q}_r + \frac{1}{C_{red}} \cdot q_r = Q_g(t)$$



1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

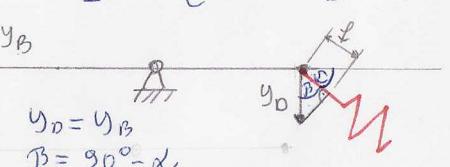
$$\begin{aligned} E &= E_1 + E_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \mathcal{J}_a \omega^2 = \\ &= \frac{1}{2} m_1 \dot{y}_B^2 + \frac{1}{2} \left( \frac{1}{2} m_2 R^2 \right) \cdot \left( \frac{\dot{y}_B}{R} \right)^2 = \\ &= \frac{1}{2} \left( m_1 + \frac{1}{2} m_2 R^2 \right) \cdot \dot{y}_B^2 \end{aligned}$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \left( m_1 + \frac{1}{2} m_2 \right) \ddot{y}_B$$

2.  $\frac{\partial E}{\partial q} = 0$

3.  $Q_c$  meghatározása

$$U = \frac{1}{2} \cdot \frac{(y_B \cdot \cos \beta)^2}{C} = \frac{1}{2} \cdot \frac{\cos^2 \beta}{C} \cdot y_B^2$$



$$Q_c = - \frac{\partial U}{\partial y_B} = - \frac{\cos^2 \beta}{C} \cdot y_B$$

$$Q_c = - \frac{\cos^2 \beta}{C} \cdot y_B$$

adott:  $m_1, m_2, C, k, \mathcal{J}, \omega, R, F_g(t) = F_{go} \cdot \sin(\omega t + \epsilon)$

az általános koordináta:  
 $q = y_B \quad \dot{q} = \dot{y}_B (v_B) \quad \ddot{q} = \ddot{y}_B (\alpha_B)$

Lagrange egyenlet

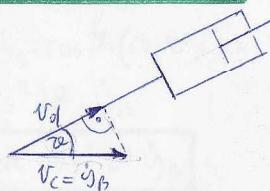
$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - 0 = Q_c + Q_k + Q_g$$

4.  $Q_k$  meghatározása

$$\begin{aligned} D &= \frac{1}{2} k v_d^2 = \frac{1}{2} k (\dot{y}_B \cos \beta)^2 = \\ &= \frac{1}{2} k \cdot \dot{y}_B^2 \cdot \cos^2 \beta \end{aligned}$$

$$Q_k = - \frac{\partial D}{\partial y_B} = - k \cdot \cos^2 \beta \cdot \dot{y}_B$$

$$Q_k = - k \cdot \cos^2 \beta \cdot \dot{y}_B$$



5.  $Q_g$  meghatározása

$$Q_g = \vec{M}_g(t) \cdot \vec{B}$$

$$\vec{M}_g(t) = M_g(t) \vec{k} = M_{go} \sin(\omega t + \epsilon) \vec{k}$$

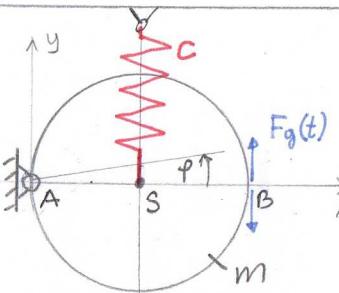
$$\vec{B} = \frac{\partial \vec{B}}{\partial \dot{y}_B} = \frac{\partial \left( \frac{y_B}{R} \vec{k} \right)}{\partial \dot{y}_B} = \frac{1}{R} \vec{k}$$

$$Q_g = \frac{1}{R} M_g(t)$$

• lehelyettesítve Lagrange egyenlettel

1.2

# E G Y S Z A B D S Á G F O K Ú R E Z G Ö R E N D S Z E R



dolott:  $c, m, R, F_g(t) = F_{g0} \cdot \sin(\omega t + \epsilon)$

- az általános koordináta:  
 $q_r = q \quad \dot{q}_r = \dot{q}(\omega) \quad \ddot{q}_r = \ddot{q}(\epsilon)$

- Lagrange egyenlet  
 $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_c + Q_g$

①.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} \exists_a \omega^2 = \frac{1}{2} \left( \frac{3}{2} m R^2 \right) \dot{q}^2$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \frac{3}{2} m R^2 \ddot{q}$$

②.  $\frac{\partial E}{\partial q} = 0$

③.  $Q_c$  meghatározása

$$U = \frac{1}{2} \cdot \frac{(R \dot{q})^2}{c} = \frac{1}{2} \frac{R^2}{c} \cdot \dot{q}^2$$

$$Q_c = - \frac{\partial U}{\partial \dot{q}} = - \frac{R^2}{c} \cdot \dot{q}$$

$$Q_c = - \frac{R^2}{c} \cdot \dot{q}$$

④.  $Q_g$  meghatározása

$$Q_g = \vec{F}_g \cdot \vec{\beta}_B$$

$$\vec{F}_g = F_{g0} \cdot \sin(\omega t + \epsilon) \hat{j}$$

$$\vec{\beta}_B = \frac{\partial \vec{v}_B}{\partial \dot{q}} = \frac{\partial (2R \dot{q} \hat{j})}{\partial \dot{q}} = 2R \hat{j}$$

$$Q_g = \vec{F}_g \cdot \vec{\beta}_B = 2R F_g(t)$$

$$Q_g = 2R F_g(t)$$

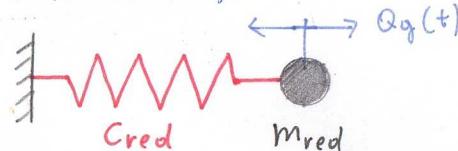
- lehejtésterűse Lagrange egyenlete:  
 $\frac{3}{2} m R^2 \ddot{q} - 0 = - \frac{R^2}{c} \cdot \dot{q} + 2R F_g(t)$

rendszerei egyenleteit:

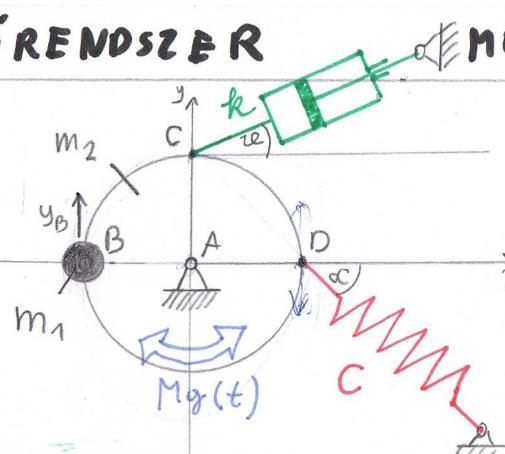
$$\frac{3}{2} m R^2 \ddot{q} + \frac{R^2}{c} \cdot \dot{q} = 2R F_g(t)$$

M<sub>red</sub>      C<sub>red</sub>

a rendszertartalmú rezgőrendszer:



$$M_{red} \cdot \ddot{q}_g + \frac{1}{C_{red}} \cdot q_g = Q_g(t)$$



dolott:  $m_1, m_2, c, k, \omega, \varphi, R, M_g(t) = M_{g0} \cdot \sin(\omega t + \epsilon)$

- az általános koordináta:  
 $q = \varphi \quad \dot{q} = \dot{\varphi}(\omega) \quad \ddot{q} = \ddot{\varphi}(\epsilon)$

- Lagrange egyenlet  
 $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$

①.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

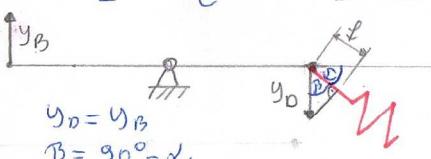
$$E = E_1 + E_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \exists_a \omega^2 = \frac{1}{2} m_1 \dot{y}_B^2 + \frac{1}{2} \left( \frac{1}{2} m_2 R^2 \right) \cdot \left( \frac{\dot{y}_B}{R} \right)^2 = \frac{1}{2} \left( m_1 + \frac{1}{2} m_2 R^2 \right) \cdot \dot{y}_B^2$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \left( m_1 + \frac{1}{2} m_2 R^2 \right) \ddot{y}_B$$

②.  $\frac{\partial E}{\partial q} = 0$

③.  $Q_c$  meghatározása

$$U = \frac{1}{2} \cdot \frac{(y_B \cos \beta)^2}{c} = \frac{1}{2} \cdot \frac{\cos^2 \beta}{c} \cdot y_B^2$$



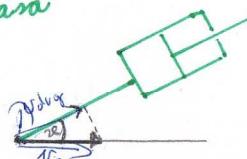
$$Q_c = - \frac{\partial U}{\partial y_B} = - \frac{\cos^2 \beta}{c} \cdot y_B$$

$$Q_c = - \frac{\cos^2 \beta}{c} \cdot y_B$$

④.  $Q_k$  meghatározása

$$Q_k = \vec{F}_k \cdot \vec{\beta}_C$$

$$\vec{F}_k = -k \vec{v}_{dy}$$



$$|v_{dy}| = |v_B| = |\dot{y}_B|$$

$$v_{dy} = v_B \cdot \cos 2\varphi = \dot{y}_B \cdot \cos 2\varphi$$

$$\vec{v}_{dy} = v_{dy} \cdot (\cos 2\varphi \hat{i} + \sin 2\varphi \hat{j})$$

$$\vec{\beta}_C = \frac{\partial \vec{v}_C}{\partial \dot{q}} = \frac{\partial (\dot{y}_B \hat{i})}{\partial \dot{q}} = \hat{i}$$

$$Q_k = -k \cdot \dot{y}_B \cdot \cos 2\varphi \cdot (\cos 2\varphi \hat{i} + \sin 2\varphi \hat{j}) \cdot \hat{i} = -k \cdot \cos^2 2\varphi \cdot \dot{y}_B$$

$$Q_k = -k \cos^2 2\varphi \dot{y}_B$$

⑤.  $Q_g$  meghatározása

$$Q_g = \vec{M}_g(t) \cdot \vec{b}$$

$$\vec{M}_g(t) = M_g(t) \hat{k} = M_{g0} \sin(\omega t + \epsilon) \hat{k}$$

$$\vec{b} = \frac{\partial \vec{v}}{\partial \dot{q}} = \frac{\partial (\frac{\dot{y}_B}{R} \hat{k})}{\partial \dot{q}} = \frac{1}{R} \hat{k}$$

$$Q_g = \frac{1}{R} M_g(t)$$

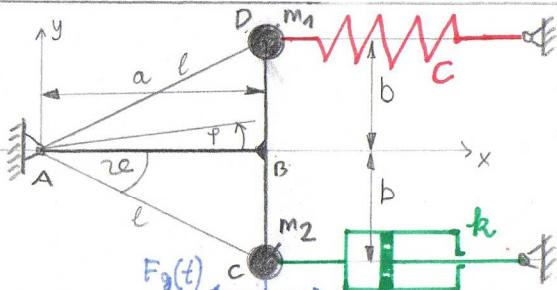
lehejtésterűse Lagrange egyenlete

...



1.3

# EGYSZABADSÁGFOKÚ REZGÖRENDSZER MORGÁSEGYENLETE



adott:  $m_1, m_2, a, b, c, k, \omega$ ,  
 $F_g(t) = F_{g0} \sin(\omega t + \varepsilon)$

az általános hozzáírás:  
 $\ddot{q} = \dot{\varphi}, \ddot{q} = \dot{\varphi}(\omega), \ddot{q} = \dot{\varphi}(\varepsilon)$

Lagrange egyenletei:

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_C + Q_K + Q_g$$

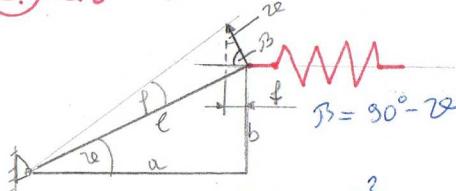
1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$\begin{aligned} E &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \\ &= \frac{1}{2} m_1 (\ell \dot{\varphi})^2 + \frac{1}{2} m_2 (\ell \dot{\varphi})^2 = \\ &= \frac{1}{2} m_1 ((a^2 + b^2) \dot{\varphi}^2) + \frac{1}{2} m_2 ((a^2 + b^2) \dot{\varphi}^2) = \\ &= \frac{1}{2} (m_1 + m_2) (a^2 + b^2) \dot{\varphi}^2 \end{aligned}$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = (m_1 + m_2) (a^2 + b^2) \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$

3.  $Q_C$  meghatározása



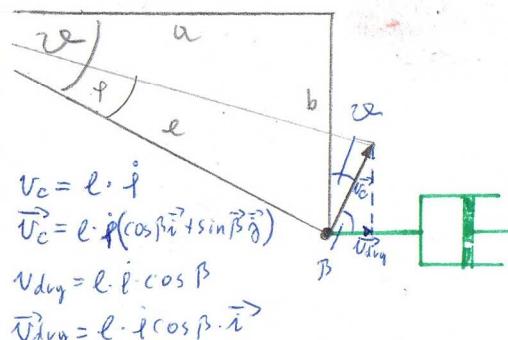
$$\begin{aligned} U &= \frac{1}{2} \frac{l^2}{c} = \frac{1}{2} \cdot \frac{(l \cdot \ell \cos \beta)^2}{c} = \\ &= \frac{1}{2} \cdot \frac{(a^2 + b^2) \cdot \ell^2 \cdot \cos^2 \beta}{c} = \\ &= \frac{1}{2} \cdot \frac{(a^2 + b^2) \cdot \cos^2 \beta}{c} \cdot \ell^2 \end{aligned}$$

$$Q_C = - \frac{\partial U}{\partial \varphi} = - \frac{(a^2 + b^2) \cos^2 \beta}{c} \cdot \ell$$

$$Q_C = - \frac{(a^2 + b^2) \cos^2 \beta}{c} \cdot \ell$$

4.  $Q_K$  meghatározása

$$Q_K = \vec{F}_k \cdot \vec{P}_c$$



$$\begin{aligned} \vec{F}_k &= -k \cdot \vec{U}_{dry} = -k \cdot \ell \cdot \dot{\varphi} \cdot \cos \beta \cdot \vec{i} \\ \vec{P}_c &= \frac{\partial \vec{v}_c}{\partial \dot{\varphi}} = \frac{\partial (\ell \cdot \dot{\varphi} \cdot (\cos \beta \vec{i} + \sin \beta \vec{j}))}{\partial \dot{\varphi}} = \\ &= \ell (\cos \beta \vec{i} + \sin \beta \vec{j}) \end{aligned}$$

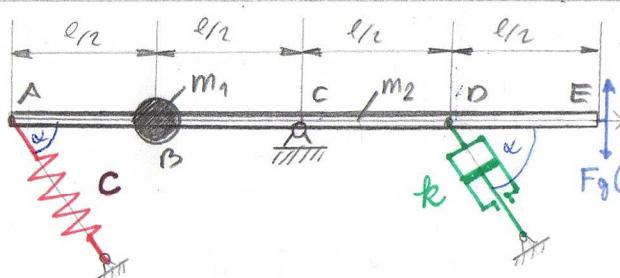
$$\begin{aligned} Q_K &= -k \ell \cos \beta \cdot \ell \cdot (\cos \beta \vec{i} + \sin \beta \vec{j}) = \\ &= -k \ell^2 \cos^2 \beta \dot{\varphi} \\ Q_K &= -k \ell^2 \cos^2 \beta \dot{\varphi} \end{aligned}$$

5.  $Q_g$  meghatározása

$$\begin{aligned} Q_g &= \vec{F}_g(t) \cdot \vec{P}_c \\ \vec{F}_g(t) &= F_{g0} \sin(\omega t + \varepsilon) \vec{i} \end{aligned}$$

$$\vec{P}_c = \frac{\partial \vec{v}_c}{\partial \dot{\varphi}} = \ell (\cos \beta \vec{i} + \sin \beta \vec{j})$$

$$Q_g = F_g(t) \cdot \ell \cdot \cos \beta$$



adott:  $m_1, m_2, l, c, k, \omega$ ,  
 $F_g(t) = F_{g0} \sin(\omega t + \varepsilon)$

az általános hozzáírás:  
 $\ddot{q} = \dot{\varphi}, \ddot{q} = \dot{\varphi}(\omega), \ddot{q} = \dot{\varphi}(\varepsilon)$

Lagrange egyenlet

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_C + Q_K + Q_g$$

1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$\begin{aligned} E &= E_1 + E_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} \vec{v}_c \cdot \vec{w}^2 = \\ &= \frac{1}{2} m_1 \left( \frac{\ell}{2} \cdot \dot{\varphi} \right)^2 + \frac{1}{2} \left( \frac{1}{12} m_2 (2\ell)^2 \right) \dot{\varphi}^2 = \\ &= \frac{1}{2} \cdot \left( \frac{m_1}{4} + \frac{m_2}{3} \right) \cdot \ell^2 \cdot \dot{\varphi}^2 \end{aligned}$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \left( \frac{m_1}{4} + \frac{m_2}{3} \right) \ell^2 \cdot \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$

3.  $Q_C$  meghatározása

$$\begin{aligned} U &= \frac{1}{2} \frac{\ell^2}{c} = \frac{1}{2} \cdot \frac{(\ell \cdot \ell \cos \beta)^2}{c} = \\ &= \frac{1}{2} \cdot \frac{\ell^2 \cos^2 \beta}{c} \cdot \ell^2 \end{aligned}$$



$$Q_C = - \frac{\partial U}{\partial \varphi} = - \frac{\ell^2 \cos^2 \beta}{c} \cdot \ell$$

$$Q_C = - \frac{\ell^2 \cos^2 \beta}{c} \cdot \dot{\varphi}$$

$$V_{dry} = \frac{\ell}{2} \cdot \dot{\varphi} \cdot \cos \beta$$

$$\begin{aligned} \vec{U}_{dry} &= V_{dry} \cdot (\sin \beta \vec{i} + \cos \beta \vec{j}) \\ \vec{F}_k &= -k \cdot \vec{U}_{dry} = -k \cdot \frac{\ell}{2} \cdot \dot{\varphi} \cdot (\sin \beta \vec{i} + \cos \beta \vec{j}) \end{aligned}$$

$$\begin{aligned} \vec{P}_D &= \frac{\partial \vec{v}_D}{\partial \dot{\varphi}} = \frac{\partial (\frac{\ell}{2} \cdot \dot{\varphi} \cdot \vec{i})}{\partial \dot{\varphi}} = \frac{\ell}{2} \cdot \vec{i} \\ Q_K &= -k \cdot \frac{\ell}{2} \cdot \dot{\varphi} \cdot (\sin \beta \vec{i} + \cos \beta \vec{j}) \cdot \frac{\ell}{2} \cdot \vec{i} = \\ &= -\frac{\ell^2 \cdot \cos^2 \beta \cdot \dot{\varphi}}{2} \end{aligned}$$

$$Q_K = -k \cdot \frac{\ell^2}{4} \cdot \cos^2 \beta \cdot \dot{\varphi}$$

5.  $Q_g$  meghatározása

$$Q_g = \vec{F}_g \cdot \vec{P}_E$$

$$\vec{F}_g = F_{g0} \sin(\omega t + \varepsilon) \vec{i}$$

$$\vec{P}_E = \frac{\partial \vec{v}_E}{\partial \dot{\varphi}} = \frac{\partial (\ell \cdot \dot{\varphi} \vec{i})}{\partial \dot{\varphi}} = \ell \cdot \vec{i}$$

$$Q_g = F_{g0} \sin(\omega t + \varepsilon) \cdot \ell \cdot \vec{i}$$

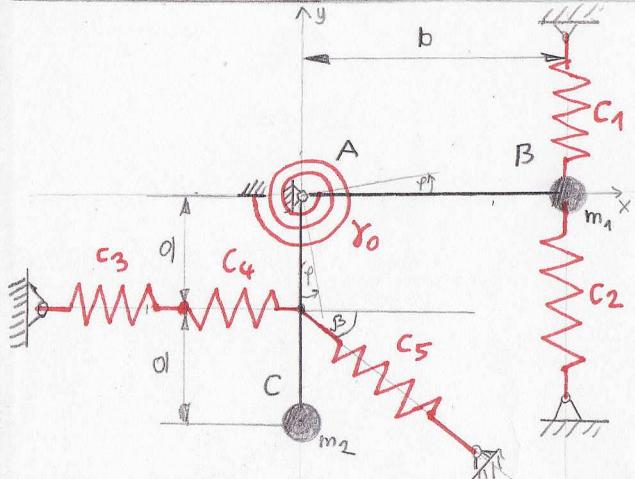
$$Q_g = F_g(t) \cdot \ell$$

4.  $Q_K$  meghatározása

$$\begin{aligned} Q_K &= \vec{F}_k \cdot \vec{P}_D \\ \vec{F}_k &= -k \cdot \vec{U}_{dry} \end{aligned}$$

1.4

# EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\varphi}} \right]$  meghatározása

$$\begin{aligned} E &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \\ &= \frac{1}{2} m_1 (b \dot{\varphi})^2 + \frac{1}{2} m_2 (2a \dot{\varphi})^2 = \\ &= \frac{1}{2} (m_1 b^2 + m_2 4a^2) \dot{\varphi}^2 \end{aligned}$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\varphi}} \right] = (m_1 b^2 + m_2 4a^2) \ddot{\varphi}$$

$$2. \frac{\partial E}{\partial \varphi} = 0$$

3.  $Q_c$  meghatározása

$$\begin{aligned} U &= U_1 + U_2 + U_3 + U_4 + U_5 + U_6 = \\ &= \frac{1}{2} \cdot \frac{(b \dot{\varphi})^2}{c_1} + \frac{1}{2} \cdot \frac{(b \dot{\varphi})^2}{c_2} + \frac{1}{2} \cdot \frac{(a \dot{\varphi})^2}{c_3+c_4} + \\ &+ \frac{1}{2} \cdot \frac{(a \dot{\varphi} \cos \beta)^2}{c_5} + \frac{1}{2} \cdot \frac{\dot{\varphi}^2}{\gamma_0} = \\ &= \frac{1}{2} \cdot \left[ \frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2 \cos^2 \beta}{c_5} + \frac{1}{\gamma_0} \right] \dot{\varphi}^2 \end{aligned}$$

$$Q_c = - \frac{\partial U}{\partial \dot{\varphi}}$$

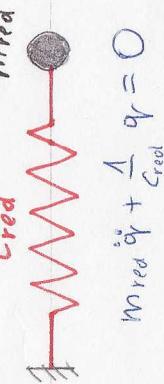
adott:  $m_1, m_2, \alpha_1, b, c_1, c_2, c_3, c_4, c_5, \gamma_0, \beta$

- az általános koordináta:  
 $\varphi = \varphi$  (szögeltudnás)  
 $\dot{\varphi} = \dot{\varphi}(\omega)$  (szögsebesség)  
 $\ddot{\varphi} = \ddot{\varphi}(\ddot{\varphi})$  (szöggyorsulás)
- Lagrange egyenlet:

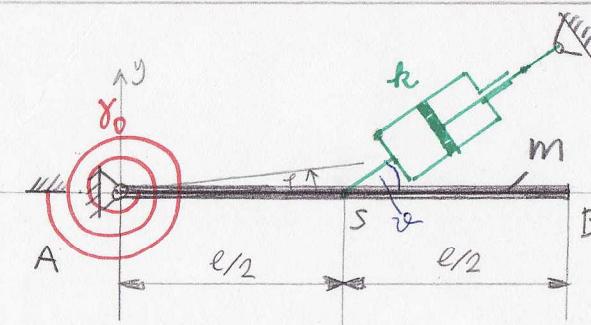
$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\varphi}} \right] - \frac{\partial E}{\partial \varphi} = Q_c$$

$$Q_c = - \left( \frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{\alpha^2}{c_3+c_4} + \frac{\alpha^2 \cos^2 \beta}{c_5} + \frac{1}{\gamma_0} \right) \dot{\varphi}$$

- elhelyettesítés a Lagrange egyenlethez  
 $(m_1 b^2 + m_2 4a^2) \ddot{\varphi} + \frac{\alpha^2 \cos^2 \beta}{c_5} \dot{\varphi} + \frac{\alpha^2}{c_3+c_4} \dot{\varphi} + \frac{b^2}{c_1} \dot{\varphi} + \frac{b^2}{c_2} \dot{\varphi} + \frac{1}{\gamma_0} \dot{\varphi} = 0$
- rendezve az egyenletet  
 $(m_1 b^2 + m_2 4a^2) \dot{\varphi} + \frac{b^2}{c_1} \dot{\varphi} + \frac{b^2}{c_2} \dot{\varphi} + \frac{\alpha^2 \cos^2 \beta}{c_5} \dot{\varphi} + \frac{\alpha^2}{c_3+c_4} \dot{\varphi} + \frac{1}{\gamma_0} \dot{\varphi} = 0$
- a redukált rezgésszámra:  
Cred



$$m_{red} \dot{\varphi} + \frac{1}{C_{red}} \varphi = 0$$



1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\varphi}} \right]$  meghatározása

$$E = \frac{1}{2} J_a \omega^2 = \frac{1}{2} \left( \frac{1}{3} m e^2 \right) \dot{\varphi}^2$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\varphi}} \right] = \frac{1}{3} m e^2 \ddot{\varphi}$$

$$2. \frac{\partial E}{\partial \varphi} = 0$$

3.  $Q_k$  meghatározása

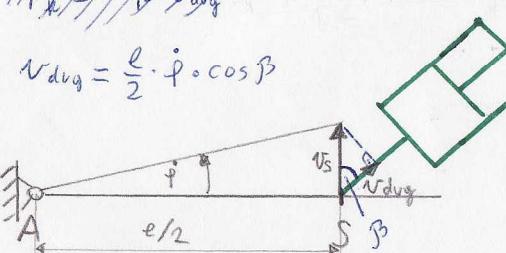
$$U = \frac{1}{2} \cdot \frac{\dot{\varphi}^2}{\gamma_0} \quad Q_k = - \frac{\partial U}{\partial \dot{\varphi}}$$

$$Q_k = - \frac{1}{\gamma_0} \cdot \dot{\varphi}$$

4.  $Q_n$  meghatározása

$$\begin{aligned} Q_n &= \vec{F}_n \cdot \vec{\beta} \\ \vec{F}_n &= -k \frac{\dot{\varphi}}{\sqrt{1+\dot{\varphi}^2}} \vec{v}_{dmg} \end{aligned}$$

$$v_{dmg} = \frac{e}{2} \cdot \dot{\varphi} \cdot \cos \beta$$



$$\vec{F}_n = v_{dmg} (\sin \beta \vec{i} + \cos \beta \vec{j})$$

adott:  $m, e, k, \gamma_0, \ddot{\varphi}$   
az általános koordináta  
 $\varphi = \varphi$   $\dot{\varphi} = \dot{\varphi}(\omega)$   $\ddot{\varphi} = \ddot{\varphi}(\ddot{\varphi})$

Lagrange egyenlet:  
 $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\varphi}} \right] - \frac{\partial E}{\partial \varphi} = Q_c + Q_k$

$$D = \frac{1}{2} h v_d^2 = \frac{1}{2} h (v_s \cdot \cos \beta)^2 =$$

$$= \frac{1}{2} h \cdot \left( \frac{e}{2} \dot{\varphi} \cos \beta \right)^2 =$$

$$= \frac{1}{2} h \cdot \frac{e^2}{4} \cdot \dot{\varphi}^2 \cos^2 \beta$$

$$Q_k = - \frac{\partial D}{\partial \dot{\varphi}} = - h \cdot \frac{e^2}{4} \cdot \cos^2 \beta \cdot \dot{\varphi}$$

$$Q_k = - \frac{e^2}{4} \cdot \cos^2 \beta \cdot \dot{\varphi}$$

elhelyettesítés a Lagrange egyenlethez  
 $\frac{1}{3} m e^2 \ddot{\varphi} - 0 = - \frac{1}{\gamma_0} \cdot \dot{\varphi} - k \frac{e^2}{4} \cdot \cos^2 \beta \cdot \dot{\varphi}$

rendezve az egyenlethez

$$\frac{1}{3} m e^2 \ddot{\varphi} + k \frac{e^2}{4} \cdot \cos^2 \beta \cdot \dot{\varphi} + \frac{1}{\gamma_0} \cdot \dot{\varphi} = 0$$

$$M_{red} \quad f_{red} \quad \frac{1}{C_{red}}$$

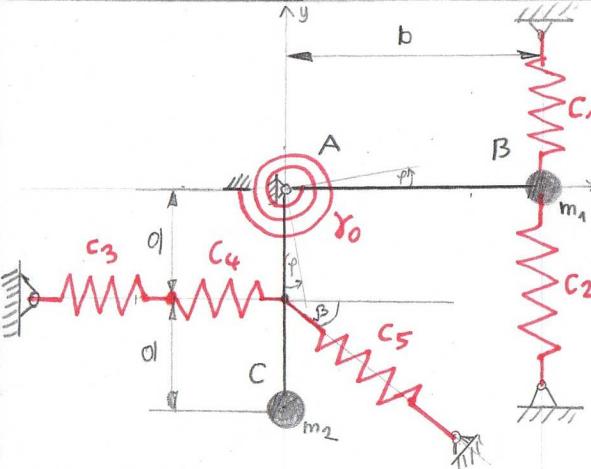
a redukált rezgésszám



$$M_{red} \ddot{\varphi} + f_{red} \dot{\varphi} + \frac{1}{C_{red}} \varphi = 0$$

1.4

# EGYSZABA DSÁGFOKÚ REZGÖRENDSZER MOZGÁSEGYENLETE



1.  $\frac{\partial}{\partial t} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$\begin{aligned} E &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \\ &= \frac{1}{2} m_1 (b \dot{\varphi})^2 + \frac{1}{2} m_2 (2a \dot{\varphi})^2 = \\ &= \frac{1}{2} (m_1 b^2 + m_2 4a^2) \dot{\varphi}^2 \end{aligned}$$

$$\frac{\partial}{\partial t} \left[ \frac{\partial E}{\partial \dot{q}} \right] = (m_1 b^2 + m_2 4a^2) \ddot{\varphi}$$

$$2. \frac{\partial E}{\partial q} = 0$$

3.  $Q_c$  meghatározása

$$\begin{aligned} U &= U_1 + U_2 + U_3 + U_4 + U_5 + U_6 = \\ &= \frac{1}{2} \cdot \frac{(b \dot{\varphi})^2}{c_1} + \frac{1}{2} \cdot \frac{(b \dot{\varphi})^2}{c_2} + \frac{1}{2} \cdot \frac{(a \dot{\varphi})^2}{c_3+c_4} + \\ &+ \frac{1}{2} \cdot \frac{(a \dot{\varphi} \cos \beta)^2}{c_5} + \frac{1}{2} \cdot \frac{\dot{\varphi}^2}{\gamma_0} = \\ &= \frac{1}{2} \cdot \left[ \frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2 \cos^2 \beta}{c_5} + \frac{1}{\gamma_0} \right] \dot{\varphi}^2 \end{aligned}$$

$$Q_c = - \frac{\partial U}{\partial \dot{\varphi}}$$

adott:  $m_1, m_2, \alpha, b, c_1, c_2, c_3, c_4, c_5, \gamma_0, \beta$

- az általános kordináta:  
 $q = \varphi$  (rögzítetlenül)
- $\dot{q} = \dot{\varphi} (\omega)$  (rögzítésig)
- $\ddot{q} = \ddot{\varphi} (\varepsilon)$  (rögzítéssel)

• Lagrange egyenlet:

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_c$$

$$Q_c = - \left( \frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2 \cos^2 \beta}{c_5} + \frac{1}{\gamma_0} \right) \dot{\varphi}$$

$$\left. \begin{aligned} &\frac{\partial}{\partial t} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \\ &\frac{\partial}{\partial \dot{q}} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \\ &\frac{\partial}{\partial q} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \end{aligned} \right\} \text{crel}$$

• behelyettesítés a Lagrange egyenletbe:  
 $(m_1 b^2 + m_2 4a^2) \ddot{\varphi} + \frac{1}{\gamma_0} \dot{\varphi} = 0$

• rögzítési erők meghatározása:  
 $(m_1 b^2 + m_2 4a^2) \dot{\varphi} + \frac{1}{\gamma_0} \dot{\varphi} = 0$

$$M_{red} \dot{\varphi} + \frac{1}{C_{red}} \dot{\varphi} = 0$$

1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} J_a \omega^2 = \frac{1}{2} \left( \frac{1}{3} m e^2 \right) \dot{\varphi}^2$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \frac{1}{3} m e^2 \ddot{\varphi}$$

$$2. \frac{\partial E}{\partial q} = 0$$

3.  $Q_c$  meghatározása

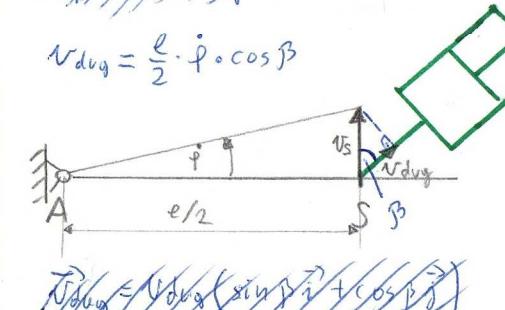
$$U = \frac{1}{2} \cdot \frac{\dot{\varphi}^2}{\gamma_0} \quad Q_c = - \frac{\partial U}{\partial \dot{\varphi}}$$

$$Q_c = - \frac{1}{\gamma_0} \cdot \dot{\varphi}$$

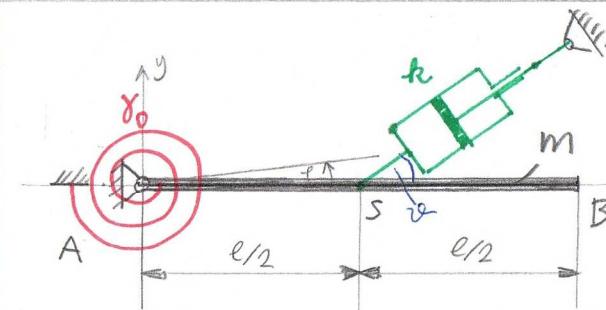
4.  $Q_k$  meghatározása

$$\begin{aligned} Q_k &= \vec{F}_k \cdot \vec{s}_k \\ \vec{F}_k &= -k \vec{v}_{deg} \end{aligned}$$

$$v_{deg} = \frac{e}{2} \cdot \dot{\varphi} \cdot \cos \beta$$



$$T_{red} \dot{\varphi} = v_{deg} (\sin \beta \vec{i} + \cos \beta \vec{j})$$



adott:  $m, e, k, \gamma_0, \dot{\varphi}$

• az általános kordináta  
 $q = \varphi \quad \dot{q} = \dot{\varphi} (\omega) \quad \ddot{q} = \ddot{\varphi} (\varepsilon)$

• Lagrange egyenlet:  
 $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_c + Q_k$

$$\vec{F}_k = -k \vec{v}_{deg} = -k \cdot \frac{e}{2} \cdot \dot{\varphi} \cos \beta \cdot (\sin \beta \vec{i} + \cos \beta \vec{j}) =$$

$$\vec{p}_s = \frac{\partial v_s}{\partial \dot{\varphi}} = \frac{\partial \left( \frac{e}{2} \cdot \dot{\varphi} \vec{j} \right)}{\partial \dot{\varphi}} = \frac{e}{2} \vec{j}$$

$$\begin{aligned} Q_k &= -k \cdot \frac{e}{2} \cdot \dot{\varphi} \cos \beta \sin \beta \vec{i} + \cos^2 \beta \vec{j} = \\ &= -k \cdot \frac{e}{2} \cdot \dot{\varphi} \cos^2 \beta \cdot \frac{e}{2} = -k \cdot \frac{e^2}{4} \cos^2 \beta \cdot \dot{\varphi} \end{aligned}$$

$$Q_k = -k \frac{e^2}{4} \cos^2 \beta \cdot \dot{\varphi}$$

• lehelyettesítés a Lagrange egyenletbe  
 $\frac{1}{3} m e^2 \ddot{\varphi} - 0 = -\frac{1}{\gamma_0} \dot{\varphi} - k \frac{e^2}{4} \cos^2 \beta \cdot \dot{\varphi}$

• rendszere az egyszerűbb

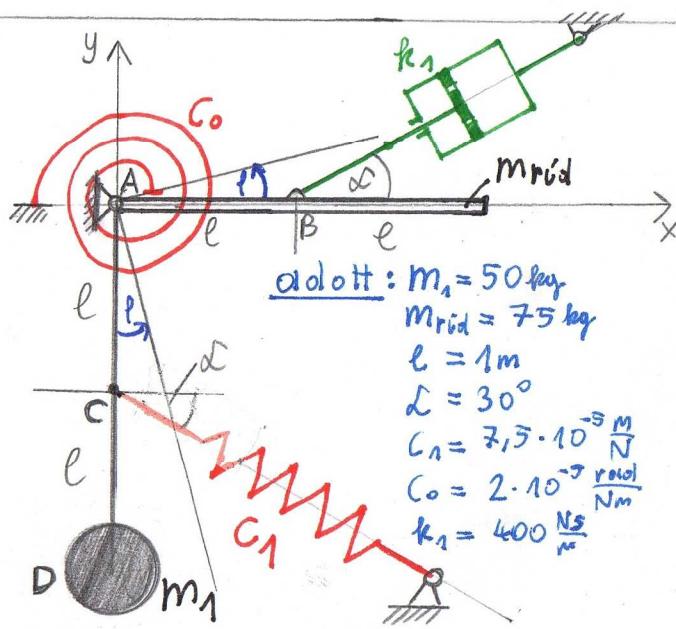
$$\frac{1}{3} m e^2 \ddot{\varphi} + k \frac{e^2}{4} \cos^2 \beta \dot{\varphi} + \frac{1}{\gamma_0} \dot{\varphi} = 0$$

$M_{red}$   $\ddot{\varphi}$   $\vec{s}_k$   $\frac{1}{C_{red}}$

• a rezgési rendszerekben

$$M_{red} \ddot{\varphi} + k_{red} \dot{\varphi} + \frac{1}{C_{red}} \dot{\varphi} = 0$$

$$M_{red} \ddot{\varphi} + k_{red} \dot{\varphi} + \frac{1}{C_{red}} \dot{\varphi} = 0$$



• általános koordináta:

$$q = \varphi \quad - \text{szögelfordulás}$$

$$\dot{q} = \dot{\varphi} = \omega \quad - \text{szögssebesség}$$

$$\ddot{q} = \ddot{\varphi} = \epsilon \quad - \text{szöggyorsulás}$$

• Lagrange egyenlet:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) - \frac{\partial E}{\partial q} = Q_c + Q_k$$

• Vissza helyettesítve a Lagrange egyenletheit:

$$m_{\text{red}} \ddot{\varphi} - 0 = -\frac{1}{C_{\text{red}}} \dot{\varphi} - k_{\text{red}} \dot{\varphi}$$

$$m_{\text{red}} \ddot{\varphi} + k_{\text{red}} \dot{\varphi} + \frac{1}{C_{\text{red}}} \dot{\varphi} = 0$$

$$300 \ddot{\varphi} + 100 \dot{\varphi} + 60000 \dot{\varphi} = 0$$

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) - \frac{\partial E}{\partial q} = Q_c + Q_k$$

$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right)$  meghatározása

$$E = \frac{1}{2} m_1 v_0^2 + \frac{1}{2} J_0 \omega^2$$

$$E = \frac{1}{2} m_1 \dot{x}_0^2 + \frac{1}{2} \left[ \frac{1}{3} m_{\text{red}} (2e)^2 \right] \dot{\varphi}^2$$

$$E = \frac{1}{2} m_1 (2e \dot{\varphi})^2 + \frac{1}{2} \left[ \frac{1}{3} m_{\text{red}} 4e^2 \right] \dot{\varphi}^2$$

$$E = \frac{1}{2} \left[ m_1 4e^2 + \frac{4}{3} m_{\text{red}} e^2 \right] \dot{\varphi}^2$$

$$\frac{\partial E}{\partial \dot{q}} = \frac{\partial}{\partial \dot{\varphi}} \left[ \frac{1}{2} \left[ m_1 4e^2 + \frac{4}{3} m_{\text{red}} e^2 \right] \dot{\varphi}^2 \right] = \left[ m_1 4e^2 + \frac{4}{3} m_{\text{red}} e^2 \right] \dot{\varphi}$$

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) = \underbrace{\left[ m_1 4e^2 + \frac{4}{3} m_{\text{red}} e^2 \right]}_{m_{\text{red}}} \ddot{\varphi}$$

$$m_{\text{red}} = M_1 4e^2 + \frac{4}{3} M_{\text{red}} e^2 = 50 \cdot 4 \cdot 1^2 + \frac{4}{3} 75 \cdot 1^2 = 300 \text{ kgm}^2$$

$Q_c$  meghatározása

$$U = U_0 + U_1$$

$$U = \frac{1}{2} \frac{p^2}{C_0} + \frac{1}{2} \frac{t_1^2}{C_1}$$

$$U = \frac{1}{2} \frac{p^2}{C_0} + \frac{1}{2} \frac{(e \dot{\varphi} \cos \alpha)^2}{C_1}$$

$$x_c = e \cdot \dot{\varphi}$$

$$\dot{x}_1 = x_c \cdot \cos \alpha$$

$$U = \frac{1}{2} \left[ \frac{1}{C_0} + \frac{e^2 \cos^2 \alpha}{C_1} \right] p^2$$

$$Q_c = -\frac{\partial U}{\partial p} = -\frac{\partial U}{\partial \dot{\varphi}} = -\left[ \frac{1}{C_0} + \frac{e^2 \cos^2 \alpha}{C_1} \right] p$$

$$Q_c = -\left[ \frac{1}{C_0} + \frac{e^2 \cos^2 \alpha}{C_1} \right] \dot{\varphi}$$

$$\frac{1}{C_{\text{red}}} = \frac{1}{C_0} + \frac{e^2 \cos^2 \alpha}{C_1} = \frac{1}{2 \cdot 10^{-5}} + \frac{1^2 \cos^2 30^\circ}{2 \cdot 5 \cdot 10^{-5}} = 6 \cdot 10^5 \text{ Nm}$$

$$C_{\text{red}} = \frac{1}{6 \cdot 10^5} = 1,6 \cdot 10^{-6} \frac{1}{\text{Nm}}$$

$Q_k$  meghatározása

$$V_B = e \dot{\varphi} \quad \vec{v}_B = e \dot{\varphi} \hat{j}$$

$$V_d = e \dot{\varphi} \cos 60^\circ \quad \vec{v}_d = e \dot{\varphi} \hat{i} + e \dot{\varphi} \cos 60^\circ \hat{j}$$

$$\vec{v}_{\text{ol}} = \frac{p}{q} e \dot{\varphi} \hat{i} + \frac{p}{q} e \dot{\varphi} \cos 60^\circ \hat{j}$$

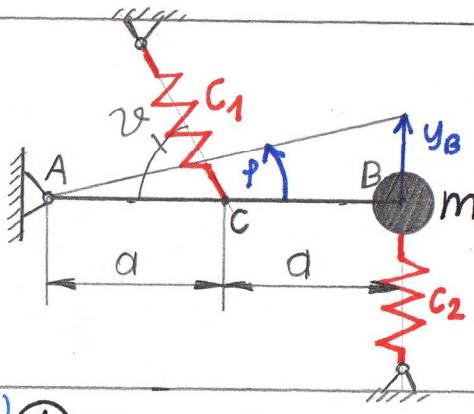
$$\vec{F}_k = -k \vec{v}_{\text{ol}} = -k \left( \frac{p}{q} e \dot{\varphi} \hat{i} + \frac{p}{q} e \dot{\varphi} \cos 60^\circ \hat{j} \right) \dot{\varphi}$$

$$\vec{P}_B = \frac{\partial \vec{v}_B}{\partial \dot{\varphi}} = \frac{\partial e \dot{\varphi} \hat{j}}{\partial \dot{\varphi}} = e \hat{j}$$

$$Q_k = \vec{F}_k \cdot \vec{P}_B = -k \left( \frac{p}{q} e \dot{\varphi} \hat{i} + \frac{p}{q} e \dot{\varphi} \cos 60^\circ \hat{j} \right) \dot{\varphi} \cdot e \hat{j} = -k \frac{1}{q} e^2 \dot{\varphi}$$

$$Q_k = -\frac{1}{q} k e^2 \dot{\varphi}$$

$$k_{\text{red}} = -\frac{1}{q} k e^2 = -\frac{1}{q} \cdot 400 \cdot 1^2 = 100 \text{ Nms}$$



a) ①  $m_{\text{red}}$  meghatározása

$$\begin{aligned} E &= \frac{1}{2} m v_B^2 = \frac{1}{2} m \dot{y}_B^2 = \\ &= \frac{1}{2} m (2a \dot{q})^2 = \frac{1}{2} m 4a^2 \dot{q}^2 \\ \frac{\partial E}{\partial q} &= \frac{\partial E}{\partial \dot{q}} = 2 \cdot \frac{1}{2} \cdot 4m a^2 \dot{q} = 4m a^2 \dot{q} \\ \frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) &= \frac{d}{dt} 4m a^2 \dot{q} = 4m a^2 \ddot{q} \end{aligned}$$

$$\boxed{\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) = 4m a^2 \ddot{q} = m_{\text{red}} \ddot{q}}$$

$$2) \frac{\partial E}{\partial q} = \frac{\partial E}{\partial \dot{q}} = 0$$

③  $C_{\text{red}}$  meghatározása

$$U = U_1 + U_2 = \frac{1}{2} \frac{f_1^2}{C_1} + \frac{1}{2} \frac{f_2^2}{C_2}$$

• adott:  $a, \varphi, m, C_1, C_2$

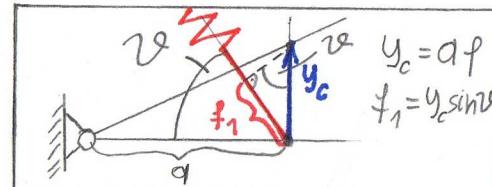
• általános koordináta:

a) $q = \dot{q}$	b) $q = y_B$
$\dot{q} = \ddot{q} = \omega$	$\dot{q} = \dot{y}_B = v_B$
$\ddot{q} = \ddot{q} = \varepsilon$	$\ddot{q} = \ddot{y}_B = \alpha_B$

• Lagrange egyenlete:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) - \frac{\partial E}{\partial q} = Q_C$$

$$= \frac{1}{2} \frac{(a f \sin \varphi)^2}{C_1} + \frac{1}{2} \frac{(2a \dot{q})^2}{C_2} =$$



$$= \frac{1}{2} \left[ \frac{a^2 \sin^2 \varphi}{C_1} + \frac{4a^2 \dot{q}^2}{C_2} \right] \dot{q}^2$$

$$\begin{aligned} Q_C &= - \frac{\partial U}{\partial q} = - \frac{\partial U}{\partial \dot{q}} \frac{1}{2} \left( \frac{a^2 \sin^2 \varphi}{C_1} + \frac{4a^2 \dot{q}^2}{C_2} \right) \dot{q}^2 = \\ &= - \left( \frac{a^2 \sin^2 \varphi}{C_1} + \frac{4a^2 \dot{q}^2}{C_2} \right) \dot{q} \end{aligned}$$

$$\boxed{Q_C = - \left( \frac{a^2 \sin^2 \varphi}{C_1} + \frac{4a^2 \dot{q}^2}{C_2} \right) \dot{q} = - \frac{1}{C_{\text{red}}} \dot{q}}$$

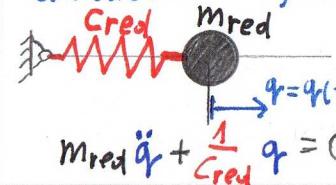
• behelyettesítve a Lagrange egyenletbe:

$$4m a^2 \ddot{q} - 0 = - \left( \frac{a^2 \sin^2 \varphi}{C_1} + \frac{4a^2 \dot{q}^2}{C_2} \right) \dot{q}$$

• átrendezve az egyenletet:

$$\begin{aligned} 4m a^2 \ddot{q} + \left( \frac{a^2 \sin^2 \varphi}{C_1} + \frac{4a^2 \dot{q}^2}{C_2} \right) \dot{q} &= 0 \\ m_{\text{red}} \ddot{q} + \frac{1}{C_{\text{red}}} \dot{q} &= 0 \end{aligned}$$

• el redukált rezgőrendszer:



b) ①  $m_{\text{red}}$  meghatározása

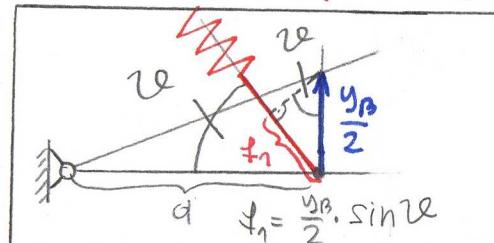
$$\begin{aligned} E &= \frac{1}{2} m v_B^2 = \frac{1}{2} m \dot{y}_B^2 \\ \frac{\partial E}{\partial q} &= \frac{\partial E}{\partial \dot{q}} = 2 \cdot \frac{1}{2} m \dot{y}_B = m \dot{y}_B \\ \frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) &= \frac{d}{dt} m \dot{y}_B = m \ddot{y}_B \end{aligned}$$

$$\boxed{\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) = m \ddot{y}_B = m_{\text{red}} \ddot{y}_B}$$

$$2) \frac{\partial E}{\partial q} = \frac{\partial E}{\partial \dot{y}_B} = 0$$

③  $C_{\text{red}}$  meghatározása

$$U = U_1 + U_2 = \frac{1}{2} \frac{f_1^2}{C_1} + \frac{1}{2} \frac{f_2^2}{C_2} =$$



$$m_{\text{red}} \ddot{y}_B + \frac{1}{C_{\text{red}}} y_B = 0$$

$$= \frac{1}{2} \frac{\left( \frac{y_B}{2} \sin \varphi \right)^2}{C_1} + \frac{1}{2} \frac{y_B^2}{C_2} =$$

$$= \frac{1}{2} \left[ \frac{\sin^2 \varphi}{4C_1} + \frac{1}{C_2} \right] y_B^2$$

$$Q_C = - \frac{\partial U}{\partial q} = - \frac{\partial U}{\partial y_B} \frac{1}{2} \left[ \frac{\sin^2 \varphi}{4C_1} + \frac{1}{C_2} \right] y_B^2 =$$

$$= - \left[ \frac{\sin^2 \varphi}{4C_1} + \frac{1}{C_2} \right] y_B$$

$$\boxed{Q_C = - \left[ \frac{\sin^2 \varphi}{4C_1} + \frac{1}{C_2} \right] y_B = - \frac{1}{C_{\text{red}}} y_B}$$

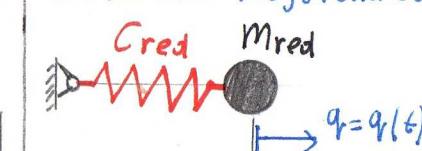
• behelyettesítve a Lagrange egyenletbe:

$$m \ddot{y}_B - 0 = - \left[ \frac{\sin^2 \varphi}{4C_1} + \frac{1}{C_2} \right] y_B$$

• átrendezve az egyenletet:

$$\boxed{m \ddot{y}_B + \left[ \frac{\sin^2 \varphi}{4C_1} + \frac{1}{C_2} \right] y_B = 0}$$

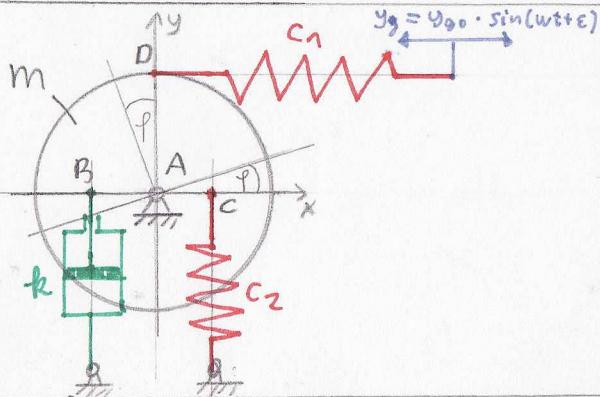
• el redukált rezgőrendszer:



$$m_{\text{red}} \ddot{y}_B + \frac{1}{C_{\text{red}}} y_B = 0$$

KK 1.5

## ÚTGERJESZTÉS



adott:  $m, R, h, c_1, c_2, y_g(t)$   
 $y_g(t) = y_{g0} \cdot \sin(\omega t + \varepsilon)$

az általános koordináták:  
 $\dot{\varphi} = \dot{\varphi}$     $\ddot{\varphi} = \ddot{\varphi}(w)$     $\ddot{\varphi} = \ddot{\varphi}(\varepsilon)$

Lagrange egyenlet

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$$

1.  $\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right)$  meghatároza

$$E = \frac{1}{2} \cdot \exists_a \cdot \omega^2 = \frac{1}{2} \cdot \left( \frac{1}{2} m R^2 \right) \dot{\varphi}^2$$

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) = \frac{1}{2} m R^2 \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$ 3. (4)  $Q_c$  és  $Q_g$  meghatároza

$$U = \frac{1}{2} \frac{\dot{x}_1^2}{c_1} + \frac{1}{2} \frac{\dot{x}_2^2}{c_2} = \\ = \frac{1}{2} \frac{(R\dot{\varphi} - y_g)^2}{c_1} + \frac{1}{2} \cdot \frac{(\frac{R}{2}\dot{\varphi})^2}{c_2}$$

$$Q_c + Q_g = - \frac{\partial U}{\partial \dot{\varphi}} = - \frac{R\dot{\varphi} - y_g}{c_1} \cdot R -$$

$$- \frac{R^2}{4c_2} \dot{\varphi} = - \frac{R^2 \dot{\varphi}}{c_1} + \frac{R y_g}{c_1} - \frac{R^2}{4c_2} \dot{\varphi}$$

$$= - \underbrace{\left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right)}_{Q_c} \dot{\varphi} + \underbrace{\frac{R}{c_1} y_g}_{Q_g}$$

$$Q_c = - \left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right) \dot{\varphi}$$

$$Q_g = \frac{R}{c_1} y_g(t)$$

5.  $Q_k$  meghatározása

$$D = \frac{1}{2} h V_d^2 = \frac{1}{2} h \cdot V_B^2 = \\ = \frac{1}{2} h \cdot \left( \frac{R}{2} \dot{\varphi} \right)^2 = \frac{1}{2} h \cdot \frac{R^2}{4} \dot{\varphi}^2$$

$$Q_k = - \frac{\partial D}{\partial \dot{\varphi}} = - h \frac{R^2}{4} \dot{\varphi}$$

$$Q_k = - k \frac{R^2}{4} \dot{\varphi}$$

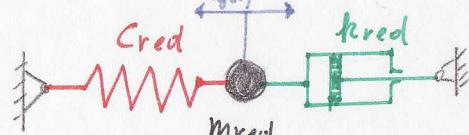
• lehetségtérítés Lagrange egyenlete

$$\frac{1}{2} M R^2 \ddot{\varphi} - 0 = - \left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right) \dot{\varphi} - h \frac{R^2}{4} \dot{\varphi} +$$

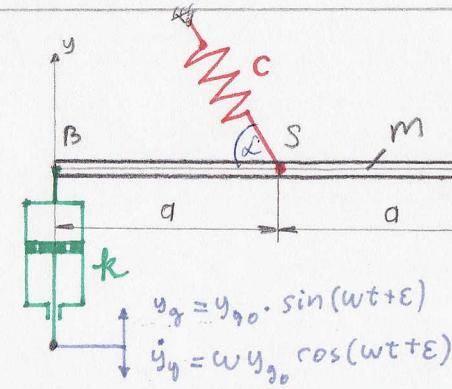
$$+ \frac{R}{c_1} y_g(t)$$

• rendelkez a legyenletet:

$$\frac{1}{2} M R^2 \ddot{\varphi} + \underbrace{k \frac{R^2}{4} \dot{\varphi}}_{k_{red}} + \underbrace{\left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right) \dot{\varphi}}_{\frac{1}{C_{red}}} = \underbrace{\frac{R}{c_1} y_g(t)}_{Q_g(t)}$$



$$m_{red} \ddot{\varphi} + k_r \dot{\varphi} + \frac{1}{C_{red}} \varphi = Q_g(t)$$



$$y_g = y_{g0} \cdot \sin(\omega t + \varepsilon)$$

$$\dot{y}_g = \omega y_{g0} \cos(\omega t + \varepsilon)$$

adott:  $m, a, R, k, c, y_g(t), \alpha$   
 az általános koordináta

$$\varphi = \varphi \quad \dot{\varphi} = \dot{\varphi}(w) \quad \ddot{\varphi} = \ddot{\varphi}(\varepsilon)$$

Lagrange egyenlet:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$$

1. (5)  $Q_k$  és  $Q_g$  meghatározása

$$D = \frac{1}{2} h (V_B - V_g)^2 = \\ = \frac{1}{2} h (2a \dot{\varphi} - \dot{y}_g)^2$$

$$Q_k + Q_g = - \frac{\partial D}{\partial \dot{\varphi}} = - h (2a \dot{\varphi} - \dot{y}_g) \cdot 2a$$

$$= - \underbrace{h 4a^2 \dot{\varphi}}_{Q_k} + \underbrace{2a h \dot{y}_g}_{Q_g}$$

$$Q_k = - 4a^2 h \dot{\varphi}$$

$$Q_g = 2a h \dot{y}_g(t)$$

lehetlennírás Lagrange egyenlete:

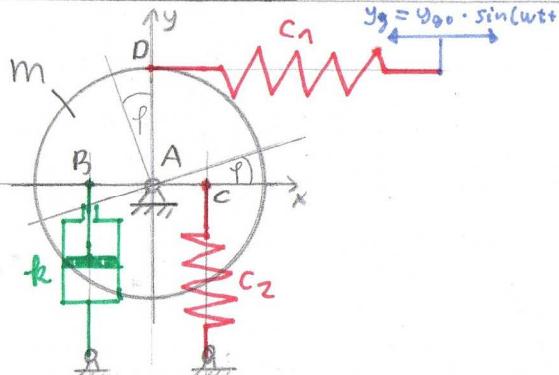
$$\frac{4}{3} m a^2 \ddot{\varphi} - 0 = - \frac{a^2 \cos^2 \beta}{c} \dot{\varphi} - 4a^2 h \dot{\varphi} + 2a h y_g$$

rendelkez a legyenletet:

$$\frac{4}{3} m a^2 \ddot{\varphi} + \underbrace{4a^2 h \dot{\varphi}}_{m_{red}} + \underbrace{\frac{a^2 \cos^2 \beta}{c} \dot{\varphi}}_{k_{red}} = \underbrace{2a h y_g(t)}_{Q_g(t)}$$

KK 1.5

# ÚTGERJESZTÉS



adott:  $m, R, h, c_1, c_2, y_g(t)$   
 $y_g(t) = y_{g0} \cdot \sin(\omega t + \epsilon)$

az általános koordináta:  
 $q = \varphi \quad \dot{q} = \dot{\varphi}(t) \quad \ddot{q} = \ddot{\varphi}(t)$

Lagrange egyenlet

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_C + Q_K + Q_g$$

1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} \cdot 3a \cdot \omega^2 = \frac{1}{2} \cdot \left( \frac{1}{2} mR^2 \right) \dot{\varphi}^2$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \frac{1}{2} mR^2 \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$

3. (4)  $Q_C$  és  $Q_g$  meghatározása

$$U = \frac{1}{2} \frac{f_1^2}{c_1} + \frac{1}{2} \frac{f_2^2}{c_2} = \frac{1}{2} \frac{(R\dot{\varphi} - y_g)^2}{c_1} + \frac{1}{2} \cdot \frac{\left( \frac{R}{2}\dot{\varphi} \right)^2}{c_2}$$

$$Q_C + Q_g = -\frac{\partial U}{\partial \dot{q}} = -\frac{R\dot{\varphi} - y_g}{c_1} \cdot R - \frac{R^2}{4c_2} \dot{\varphi} = -\frac{R^2 \dot{\varphi}}{c_1} + \frac{Ry_g}{c_1} - \frac{R^2}{4c_2} \dot{\varphi} = -\underbrace{\left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right) \dot{\varphi}}_{Q_C} + \underbrace{\frac{R}{c_1} y_g(t)}_{Q_g}$$

$$Q_C = -\left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right) \dot{\varphi}$$

$$Q_g = \frac{R}{c_1} y_g(t)$$

5.  $Q_K$  meghatározása

$$Q_K = \vec{F}_h \cdot \vec{P}_B \\ \vec{F}_h = -k \cdot \vec{v}_{dyg} = -k \cdot \left( -\frac{R}{2} \cdot \dot{\varphi} \right) \vec{j} \\ \vec{P}_B = \frac{\partial \vec{r}_B}{\partial \dot{q}} = \frac{\partial \left( -\frac{R}{2} \cdot \dot{\varphi} \vec{j} \right)}{\partial \dot{q}} = -\frac{R}{2} \vec{j} \\ Q_K = -k \cdot \left( -\frac{R}{2} \cdot \dot{\varphi} \right) \vec{j} \cdot \left( -\frac{R}{2} \vec{j} \right) = -k \frac{R^2}{4} \dot{\varphi}$$

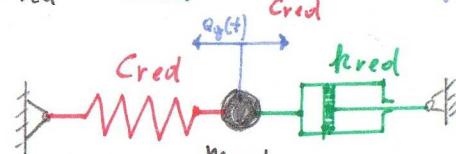
$$Q_K = -k \frac{R^2}{4} \dot{\varphi}$$

• lehelyettesítés Lagrange egyenlethez

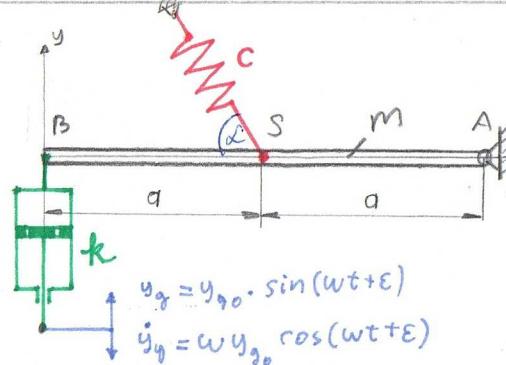
$$\frac{1}{2} m R^2 \ddot{\varphi} - 0 = -\left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right) \dot{\varphi} - k \frac{R^2}{4} \dot{\varphi} + \frac{R}{c_1} y_g(t)$$

• rendelkezésre álló Lagrange egyenletek:

$$\frac{1}{2} m R^2 \ddot{\varphi} + k \frac{R^2}{4} \dot{\varphi} + \left( \frac{R^2}{c_1} + \frac{R^2}{4c_2} \right) \dot{\varphi} = \frac{R}{c_1} y_g(t)$$



$$m_{red} \ddot{q} + k_{red} \dot{q} + \frac{1}{c_{red}} q = Q_g(t)$$



adott:  $m, a, R, k, c, y_g(t), \epsilon$

az általános koordináta:

$$q = \varphi \quad \dot{q} = \dot{\varphi}(t) \quad \ddot{q} = \ddot{\varphi}(t)$$

Lagrange egyenlet:

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] - \frac{\partial E}{\partial q} = Q_C + Q_K + Q_g$$

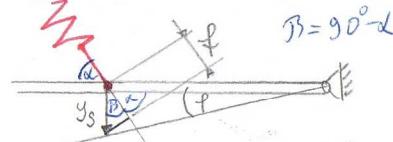
1.  $\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right]$  meghatározása

$$E = \frac{1}{2} \cdot 3a \cdot \omega^2 = \frac{1}{2} \cdot \left( \frac{1}{3} m(2a)^2 \right) \dot{\varphi}^2$$

$$\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{q}} \right] = \frac{4}{3} m a^2 \ddot{\varphi}$$

2.  $\frac{\partial E}{\partial q} = 0$

3.  $Q_C$  meghatározása



$$U = \frac{1}{2} \cdot \frac{f^2}{c} = \frac{1}{2} \cdot \frac{(a \cos \beta)^2}{c} = \frac{1}{2} \cdot \frac{a^2 \cos^2 \beta}{c} \dot{\varphi}^2$$

$$Q_C = -\frac{\partial U}{\partial \dot{q}} = -\frac{a^2 \cos^2 \beta}{c} \dot{\varphi}$$

$$Q_C = -\frac{a^2 \cos^2 \beta}{c} \dot{\varphi}$$

4. (5)  $Q_K$  és  $Q_g$  meghatározása

$$Q_K + Q_g = \vec{F}_h \cdot \vec{P}_B$$

$$\vec{F}_h = -k \cdot \vec{v}_{dyg} = -k \cdot (y - y_g) \vec{j} = -k(2a\dot{\varphi} - y_g) \vec{j}$$

$$\vec{P}_B = \frac{\partial \vec{r}_B}{\partial \dot{q}} = \frac{\partial (2a\dot{\varphi} \vec{j})}{\partial \dot{q}} = 2a \vec{j}$$

$$Q_K + Q_g = (-k2a\dot{\varphi} + y_g k) \vec{j} \cdot 2a \vec{j} = -4a^2 k \dot{\varphi} + 2ak y_g(t)$$

$$Q_K = -4a^2 k \dot{\varphi}$$

$$Q_g = 2ak y_g(t)$$

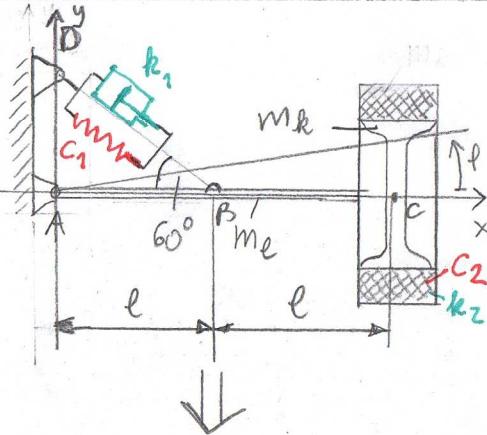
• lehelyettesítés Lagrange egyenlethez:

$$\frac{4}{3} m a^2 \ddot{\varphi} - 0 = -\frac{a^2 \cos^2 \beta}{c} \dot{\varphi} - 4a^2 k \dot{\varphi} + 2ak y_g$$

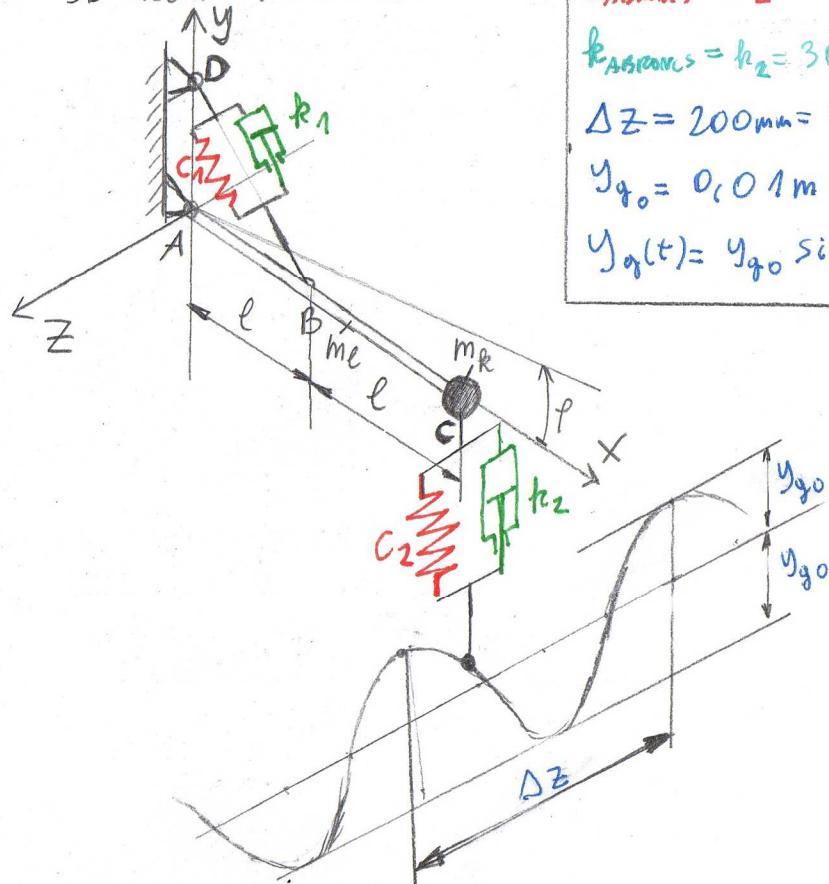
rendelkezésre álló Lagrange egyenletek:

$$\frac{4}{3} m a^2 \ddot{\varphi} + 4a^2 k \dot{\varphi} + \frac{a^2 \cos^2 \beta}{c} \dot{\varphi} = 2ak y_g(t)$$

$$m_{red} \ddot{q} + k_{red} \dot{q} + \frac{1}{c_{red}} q = Q_g(t)$$



3D mechanikai modell



adott:

$$V_{\text{Autó}} = 36 \frac{\text{km}}{\text{h}} = 10 \frac{\text{m}}{\text{s}}$$

$$M_{\text{LENGÖKKEK}} = M_e = 9 \text{ kg}$$

$$M_{\text{KÉRÉK}} = M_k = 20 \text{ kg}$$

$$C_1 = 4 \cdot 10^{-4} \frac{\text{m}}{\text{N}}$$

$$k_1 = 200 \frac{\text{Ns}}{\text{m}}$$

$$C_{\text{ABRÉMS}} = C_2 = 10^{-4} \frac{\text{m}}{\text{N}}$$

$$k_{\text{ABRÉMS}} = k_2 = 30 \frac{\text{Ns}}{\text{m}}$$

$$\Delta Z = 200 \text{ mm} = 0,2 \text{ m}$$

$$y_{g_0} = 0,01 \text{ m}$$

$$y_{g(t)} = y_{g_0} \sin(\omega_g t)$$

• Általános koordináta:  $q = \varphi$ ;  $\dot{q} = \dot{\varphi} = \omega$ ;  $\ddot{q} = \ddot{\varphi} = \varepsilon$

• gerjesztő függvény felírása:

- Ha a jármű  $V_{\text{Autó}}$  sebességgel halad, akkor  $T = \frac{\Delta z}{V_{\text{Autó}}}$  idő alatt teszi meg a két szomszédos emelkedés közötti utat, tehát a gerjesztés periodusideje:

$$T = \frac{\Delta z}{V_{\text{Autó}}} = \frac{0,2}{10} = 0,02 \text{ s}$$

- ebből a gerjesztés frekvenciája:

$$f = \frac{1}{T} = \frac{1}{0,02} = 50 \text{ Hz}$$

- innen a gerjesztés körfrekvenciája:

$$\omega_g = 2\pi f = 2\pi \cdot 50 = 314,159 \frac{\text{rad}}{\text{s}}$$

- Rgya a gerjesztés elmozdulás függvénye:

$$y_{g(t)} = y_{g_0} \sin(\omega_g t) = 0,01 \sin(314,159 t)$$

• A Lagrange-féle műsorral fajt mozdásegyenlet:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) - \underbrace{\frac{\partial E}{\partial q}}_{=0} = Q_c + Q_k + Q_g$$

$$\text{Ütgerjesztés esetén: } Q_g = Q_{gc} + Q_{gh}$$

gerjesztés a rugón  
keveréktől

gerjesztés a csillapítón  
keveréktől

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{\varphi}} \right) = Q_c + Q_{gc} + Q_k + Q_{gk}$$

$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{\varphi}} \right)$  meghatározása

$$E = \frac{1}{2} J_a \omega^2 + \frac{1}{2} m_k \overset{V_c = \dot{y}_c}{\overbrace{V_c^2}} =$$

$$= \frac{1}{2} \left[ \frac{1}{3} m_e (2e)^2 \right] \dot{\varphi}^2 + \frac{1}{2} m_k (2e \dot{\varphi})^2 =$$

$$= \frac{1}{2} \left[ \frac{1}{3} m_e 4e^2 + m_k 4e^2 \right] \dot{\varphi} =$$

$m_{real}$

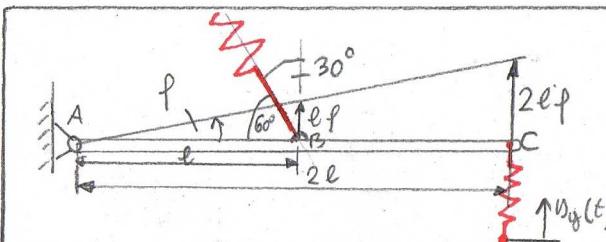
$$= \frac{1}{2} m_{real} \dot{\varphi}$$

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{\varphi}} \right) = M_{real} \ddot{\varphi}$$

$Q_c + Q_{gc}$  meghatározása

$$U = U_1 + U_2 = \frac{1}{2} \frac{\dot{\varphi}_1^2}{C_1} + \frac{1}{2} \frac{\dot{\varphi}_2^2}{C_2} =$$

$$= \frac{1}{2} \frac{(e \dot{\varphi} \cos 30^\circ)^2}{C_1} + \frac{1}{2} \frac{(2e \dot{\varphi} - y_g(t))^2}{C_2} =$$



$$(A-B)^2 = A^2 - 2AB + B^2$$

$$= \frac{1}{2} \frac{e^2 \dot{\varphi}^2 \cos^2 30^\circ}{C_1} + \frac{1}{2} \frac{4e^2 \dot{\varphi}^2 - 4e \dot{\varphi} y_g(t) + y_g(t)^2}{C_2} =$$

$$Q_c + Q_{gc} = - \frac{\partial U}{\partial \dot{\varphi}} = - \frac{1}{2} \cdot \frac{2e^2 \dot{\varphi} \cos^2 30^\circ}{C_1} -$$

$$- \frac{1}{2} \frac{8e^2 \dot{\varphi} - 4e y_g(t) + 0}{C_2} =$$

$$= - \left[ \frac{e^2 \cos^2 30^\circ}{C_1} + \frac{4e^2}{C_2} \right] \dot{\varphi} + \frac{2e}{C_2} y_g(t)$$

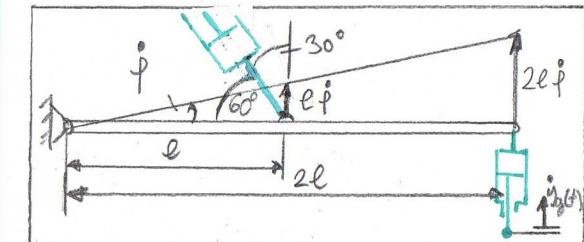
$Q_c$

$Q_{gc}$

$Q_k + Q_{gk}$  meghatározása

$$D = D_1 + D_2 = \frac{1}{2} k_1 V_{d1}^2 + \frac{1}{2} k_2 V_{d2}^2 =$$

$$= \frac{1}{2} k_1 (e \dot{\varphi} \cos 30^\circ)^2 + \frac{1}{2} k_2 (2e \dot{\varphi} - y_g(t))^2 =$$



$$(A-B)^2 = A^2 - 2AB + B^2$$

$$= \frac{1}{2} k_1 (e^2 \dot{\varphi}^2 \cos^2 30^\circ) + \frac{1}{2} k_2 (4e^2 \dot{\varphi}^2 - 4e \dot{\varphi} y_g(t) + y_g(t)^2)$$

$$Q_k + Q_{gk} = - \frac{\partial D}{\partial \dot{\varphi}} = - \frac{1}{2} \left[ (2e^2 \dot{\varphi} \cos^2 30^\circ) - \right.$$

$$\left. - \frac{1}{2} k_2 (8e^2 \dot{\varphi}^2 - 4e \dot{\varphi} y_g(t) + 0) \right] =$$

$$= - \left[ k_1 e^2 \cos^2 30^\circ + k_2 4e^2 \dot{\varphi}^2 \right] \dot{\varphi} + k_2 2e \dot{\varphi} y_g(t)$$

$k_{real}$

$Q_{gk}$

$Q_k$

• Vissza helyettesítés a Lagrange egyenletekbe:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{f}} \right) = Q_c + Q_{gc} + Q_k + Q_{gk}$$

$$\left[ \frac{1}{3} m_e 4l^2 + M_k 4l^2 \right] \ddot{f} = - \left[ \frac{e^2 \cos^2 30^\circ}{c_1} + \frac{4e^2}{c_2} \right] \dot{f} + \frac{2l}{c_2} y_g(t) - \left[ k_1 l^2 \cos^2 30^\circ + k_2 4e^2 \right] f + k_2 2l \dot{y}_g(t)$$

• átrendezve:

$$\left[ \frac{1}{3} m_e 4l^2 + M_k 4l^2 \right] \ddot{f} + \underbrace{\left[ k_1 l^2 \cos^2 30^\circ + 4k_2 e^2 \right] \dot{f}}_{k_{red}} + \underbrace{\left[ \frac{e^2 \cos^2 30^\circ}{c_1} + \frac{4e^2}{c_2} \right] f}_{\frac{1}{C_{red}}} = \underbrace{2k_2 l \dot{y}_g(t)}_{Q_g} + \underbrace{\frac{2l}{c_2} y_g(t)}_{Q_{gc}}$$

• a redukált rendszer jellemzői:

$$m_{red} = \frac{1}{3} m_e 4l^2 + M_k 4l^2 = \frac{1}{3} \cdot 9 \cdot 4 \cdot 0,5^2 + 20 \cdot 4 \cdot 0,5^2 = 23 \text{ kgm}^2$$

$$f_{red} = k_1 l^2 \cos^2 30^\circ + 4k_2 e^2 = 200 \cdot 0,5^2 \cos^2 30^\circ + 4 \cdot 30 \cdot 0,5^2 = 67,5 \text{ Nsm}$$

$$\frac{1}{C_{red}} = \frac{l^2 \cos^2 30^\circ}{c_1} + \frac{4e^2}{c_2} = \frac{0,5^2 \cos^2 30^\circ}{4 \cdot 10^{-4}} + \frac{4 \cdot 0,5^2}{10^{-4}} = 10468,75 \text{ Nm}$$

$$C_{red} = 10468,75^{-1} = 9,59 \cdot 10^{-5} \frac{\text{Nm}}{\text{rad}}$$

• behelyettesítve a Lagrange egyenletekbe:

$$23 \ddot{f} + 67,5 \dot{f} + 10468,75 f = 94,2 \cos(314,159 t) + 100 \sin(314,159 t)$$

• az általános gyorsító erő:

$$Q_y = 2k_2 l \frac{d}{dt} y_{g0} \sin(\omega_g t) + \frac{2l}{c_2} y_{g0} \sin(\omega_g t) =$$

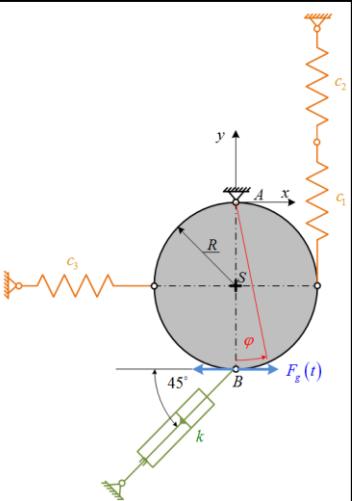
$$= 2k_2 l \omega_g y_{g0} \cos(\omega_g t) + \frac{2l}{c_2} y_{g0} \sin(\omega_g t) =$$

$$= 2l y_{g0} (k_2 \omega_g \cos(\omega_g t) + \frac{1}{c_2} \sin(\omega_g t)) =$$

$$= 2 \cdot 0,5 \cdot 0,01 (30 \cdot 314,159 \cdot \cos(314,159 t) +$$

$$+ \frac{1}{10^{-4}} \cdot \sin(314,159 t)) = 94,2 \cos(314,159 t) +$$

$$+ 100 \sin(314,159 t)$$



adott:

$R = 0,5\text{m}$   
 $m = 20\text{kg}$   
 $c_1 = 2 \cdot 10^{-5} \text{ m / N}$   
 $c_2 = 4 \cdot 10^{-5} \text{ m / N}$   
 $c_3 = 5 \cdot 10^{-5} \text{ m / N}$   
 $k = 500 \text{ Ns / m}$   
 $F_g(t) = F_{g0} \sin \omega t$   
 $F_{g0} = 200\text{N}$   
 $\omega = 10 \text{ rad / s}$   
 $q = \varphi$

általános koordináta:

$q = \varphi$   
 $\dot{q} = \dot{\varphi} = \omega$   
 $\ddot{q} = \ddot{\varphi} = \varepsilon$

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right)$$

$$+ \frac{\partial E}{\partial q} = Q_c$$

$$Q_k$$

$$Q_g$$



$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) \text{ meghatározása}$

$$E = \frac{1}{2} J_a \dot{\varphi}^2 = \frac{1}{2} J_a \varphi^2 = \frac{1}{2} \left( \frac{3}{2} m R^2 \right) \dot{\varphi}^2$$

$$\frac{\partial E}{\partial q} = \frac{\partial}{\partial \dot{\varphi}} \frac{1}{2} \left( \frac{3}{2} m R^2 \right) \dot{\varphi}^2 = \frac{3}{2} m R^2 \dot{\varphi}$$

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) = \frac{d}{dt} \frac{3}{2} m R^2 \dot{\varphi} = \frac{3}{2} m R^2 \ddot{\varphi}$$

$$\boxed{\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}} \right) = \frac{3}{2} m R^2 \ddot{\varphi} = m_{red} \ddot{\varphi}}$$

$Q_c$  meghatározása

$$U = U_{12} + U_3 = \frac{1}{2} \frac{f_{12}^2}{c_1 + c_2} + \frac{1}{2} \frac{f_3^2}{c_3}$$

**rugók hosszváltozása**

$$f_{12} = \sqrt{2}R\phi \cos 45^\circ = \sqrt{2}R\phi \frac{\sqrt{2}}{2} = R\phi$$

$$f_3 = \sqrt{2}R\phi \cos 45^\circ = \sqrt{2}R\phi \frac{\sqrt{2}}{2} = R\phi$$

$$U = \frac{1}{2} \frac{(R\phi)^2}{c_1 + c_2} + \frac{1}{2} \frac{(R\phi)^2}{c_3} =$$

$$= \frac{1}{2} \left( \frac{R^2}{c_1 + c_2} + \frac{R^2}{c_3} \right) \dot{\varphi}^2$$

$$Q_c = -\frac{\partial U}{\partial q} = -\frac{\partial U}{\partial \dot{\varphi}} \frac{1}{2} \left( \frac{R^2}{c_1 + c_2} + \frac{R^2}{c_3} \right) \dot{\varphi}^2 =$$

$$= -\left( \frac{R^2}{c_1 + c_2} + \frac{R^2}{c_3} \right) \dot{\varphi}$$

$$\boxed{Q_c = -\left( \frac{R^2}{c_1 + c_2} + \frac{R^2}{c_3} \right) \dot{\varphi} = -\frac{1}{c_{red}} \dot{\varphi}}$$

$Q_k$  meghatározása

dugattyú sebessége

$$v_B = 2R\dot{\varphi}$$

$$\vec{v}_B = 2R\dot{\varphi}\vec{i}$$

$$v_d = 2R\dot{\varphi} \cos 45^\circ = 2R\dot{\varphi} \frac{\sqrt{2}}{2} = \sqrt{2}R\dot{\varphi}$$

$$\vec{v}_d = v_d \vec{e}_d$$

$$\vec{v}_d = \sqrt{2}R\dot{\varphi} (\cos 45^\circ \vec{i} + \sin 45^\circ \vec{j})$$

$$\vec{v}_d = \sqrt{2}R\dot{\varphi} \left( \frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j} \right)$$

$$\vec{v}_d = R\dot{\varphi}\vec{i} + R\dot{\varphi}\vec{j}$$

$$Q_k = \vec{F}_k \cdot \vec{v}_d \text{ ahol:}$$

$\vec{F}_k = -k\vec{v}_d =$	$\vec{v}_d = \frac{\partial \vec{v}_B}{\partial \dot{\varphi}} =$
$= -k(R\dot{\varphi}\vec{i} + R\dot{\varphi}\vec{j})$	$= \frac{\partial \vec{v}_B}{\partial \dot{\varphi}} =$
	$= \frac{\partial 2R\dot{\varphi}\vec{i}}{\partial \dot{\varphi}} = 2R\vec{i}$

$$Q_k = \vec{F}_k \cdot \vec{v}_d = -k(R\dot{\varphi}\vec{i} + R\dot{\varphi}\vec{j}) \cdot 2R\vec{i} =$$

$$= -kR\dot{\varphi}2R = -2R^2k\dot{\varphi}$$

$$\boxed{Q_k = -2R^2k\dot{\varphi} = -k_{red}\dot{\varphi}}$$

$Q_g$  meghatározása

$\vec{Q}_g = \vec{F}_g \cdot \vec{v}_B$  ahol:

$\vec{F}_g = F_{g0} \sin(\omega t) \vec{i}$	$\vec{v}_B = \frac{\partial \vec{v}_B}{\partial \dot{\varphi}} =$
	$= \frac{\partial 2R\dot{\varphi}\vec{i}}{\partial \dot{\varphi}} = 2R\vec{i}$

$$Q_g = \vec{F}_g \cdot \vec{v}_B = F_{g0} \sin(\omega t) \vec{i} \cdot 2R\vec{i} =$$

$$\boxed{Q_g = 2RF_{g0} \sin(\omega t)}$$

Numerikus eredmények :

$$m_{red} = \frac{3}{2} m R^2 =$$

$$= \frac{3}{2} \cdot 20 \cdot 0,5^2 = 7,5 \text{ kgm}^2$$

$$\frac{1}{c_{red}} = \frac{R^2}{c_1 + c_2} + \frac{R^2}{c_3} =$$

$$= \frac{0,5^2}{(2+4) \cdot 10^{-5}} + \frac{0,5^2}{5 \cdot 10^{-5}} =$$

$$= 9166,6 \text{ Nm}$$

$$k_{red} = 2R^2 k =$$

$$= 2 \cdot 0,5^2 \cdot 500 = 250 \text{ Nsm}$$

$$Q_g = 2RF_{g0} \sin(\omega t) =$$

$$= 2 \cdot 0,5 \cdot 200 \cdot \sin(10t) =$$

$$= 200 \cdot \sin(10t)$$

$$\frac{3}{2} m R^2 \ddot{\varphi}$$

$$+ 0 = -\left( \frac{R^2}{c_1 + c_2} + \frac{R^2}{c_3} \right) \dot{\varphi}$$

$$+ -2R^2k\dot{\varphi} + 2RF_{g0} \sin(\omega t)$$

$$\frac{3}{2} m R^2 \ddot{\varphi}$$

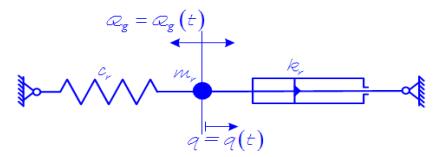
$$+ 2R^2k\dot{\varphi} + \left( \frac{R^2}{c_1 + c_2} + \frac{R^2}{c_3} \right) \dot{\varphi} = 2RF_{g0} \sin(\omega t)$$

$$m_{red} \ddot{\varphi}$$

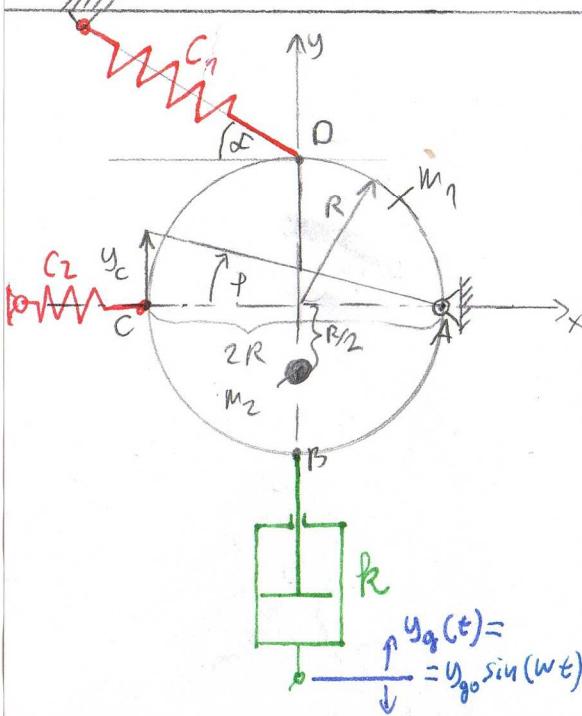
$$+ k_{red} \dot{\varphi} + \frac{1}{c_{red}} \dot{\varphi} = 2RF_{g0} \sin(\omega t)$$

$$7,5 \ddot{\varphi}$$

$$+ 250 \dot{\varphi} + 9166,6 \dot{\varphi} = 200 \sin(10t)$$







$$m_1 = 20 \text{ kg}$$

$$m_2 = 12 \text{ kg}$$

$$R = 2 \text{ m}$$

$$C_1 = 5 \cdot 10^4 \frac{\text{N}}{\text{m}}$$

$$k = 1000 \frac{\text{Ns}}{\text{m}}$$

$$y_{g0} = 15 \text{ mm}$$

$$\omega = 50 \frac{\text{rad}}{\text{s}}$$

$$\varphi = 30^\circ$$

Lagrange II. módszerével:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{y}_c} \right) - \frac{\partial E}{\partial y_c} = Q_c + Q_k + Q_g$$

①

②

③

④

a redukált vezetőrendszerven:

①  $\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{y}_c} \right)$  meghatározása:

$$\begin{aligned} E &= \frac{1}{2} \frac{1}{2} I_a \dot{\varphi}^2 + \frac{1}{2} m_2 v_2^2 = \\ &= \frac{1}{2} \left( \frac{3}{2} m_1 R^2 \right) \dot{\varphi}^2 + \frac{1}{2} m_2 \left( \sqrt{R^2 + \frac{R^2}{4}} \dot{y}_c \right)^2 \\ &= \frac{1}{2} \left( \frac{3}{2} m_1 R^2 \right) \left( \frac{\dot{y}_c}{2R} \right)^2 + \frac{1}{2} m_2 \left( \sqrt{\frac{R^2 + R^2}{4}} \frac{\dot{y}_c}{2R} \right)^2 \\ &= \frac{1}{2} \left( \frac{3}{2} m_1 R^2 \frac{1}{4R^2} \left( \frac{R^2 + R^2}{4} \right) \frac{1}{4R^2} \right) \dot{y}_c^2 = \\ &= \frac{1}{2} \left( \frac{3}{8} m_1 + \left( \frac{1}{4} + \frac{1}{16} \right) m_2 \right) \dot{y}_c^2 \\ \frac{d}{dt} \frac{\partial E}{\partial \dot{y}_c} &= \left( \frac{3}{8} m_1 + \frac{5}{16} m_2 \right) \ddot{y}_c \end{aligned}$$

M<sub>red</sub>

$$\begin{aligned} M_{red} &= \frac{3}{8} m_1 + \frac{5}{16} m_2 = \frac{3}{8} \cdot 20 + \frac{5}{16} \cdot 12 = \\ &= 11,25 \text{ kg} \end{aligned}$$

$$② \frac{\partial E}{\partial y_c} = 0$$

visszahelyettesítve és átrendezve:

$$\left( \frac{3}{8} m_1 + \frac{5}{16} m_2 \right) \ddot{y}_c + \frac{1}{4} k \dot{y}_c + \frac{\cos^2 \beta}{2C_1} y_c = \frac{1}{2} k w y_{g0} \cos(\omega t)$$

$$M_{red} \ddot{y}_c + k_{red} \dot{y}_c + \frac{\cos^2 \beta}{2C_1} y_c = Q_g(t)$$

$$11,25 \ddot{y}_c + 250 \dot{y}_c + 67 y_c = 75 \cos(50t)$$

$$M_{red} \ddot{y}_c + k_{red} \dot{y}_c + \frac{1}{2} k y_c = Q_g(t)$$

③ Q<sub>c</sub> meghatározása

$$\begin{aligned} U &= U_1 + U_2 = \frac{1}{2} \frac{f_1^2}{C_1} + \frac{1}{2} \frac{f_2^2}{C_2} = \\ &= \frac{1}{2} \frac{(12R \rho \cos \beta)^2}{C_1} + \frac{1}{2} \frac{(2R \rho \cos 2\varphi)^2}{C_2} = \\ &= \frac{1}{2} \frac{(12R \frac{y_c}{2R} \cos \beta)^2}{C_1} + \frac{1}{2} \frac{0}{C_2} = \\ &= \frac{1}{2} \left( \frac{1}{2} \cos^2 \beta \right) y_c^2 = \frac{1}{2} \left( \frac{\cos^2 \beta}{2C_1} \right) y_{g0}^2 \\ Q_c &= - \frac{\partial U}{\partial y_c} = - \frac{\cos^2 \beta}{2C_1} y_c \\ \frac{1}{C_{red}} &= \frac{\cos^2 \beta}{2C_1} = \frac{\cos^2 75^\circ}{2 \cdot 5 \cdot 10^{-4}} = 67 \frac{\text{N}}{\text{m}} \\ C_{red} &= 67^{-1} = 0,015 \frac{\text{m}}{\text{N}} \end{aligned}$$

④ Q<sub>k</sub> + Q<sub>g</sub> meghatározása

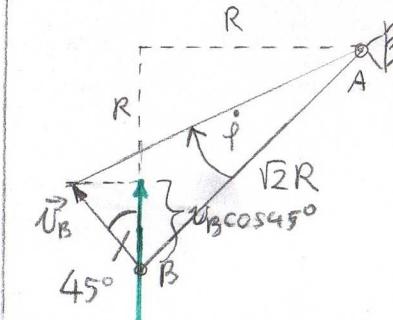
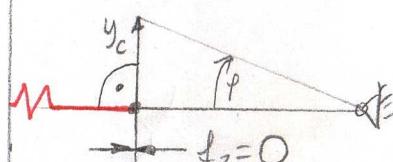
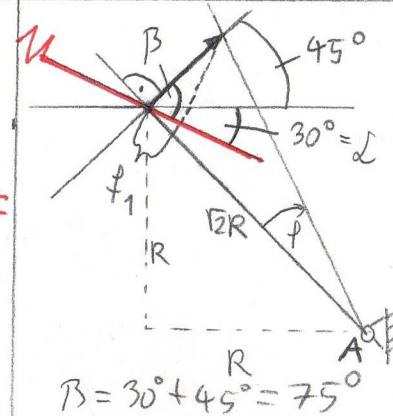
$$\begin{aligned} V_d &= \sqrt{2} R \rho \cos 45^\circ - \dot{y}_g(t) = \\ &= \sqrt{2} R \frac{\dot{y}_c}{2R} \cdot \frac{\sqrt{2}}{2} - \dot{y}_g(t) = \frac{1}{2} \dot{y}_c - \dot{y}_g(t) \end{aligned}$$

$$D = \frac{1}{2} k V_d^2 = \frac{1}{2} k \left( \frac{1}{2} \dot{y}_c - \dot{y}_g(t) \right)^2$$

$$Q_k + Q_g = - \frac{\partial D}{\partial \dot{y}_c} = - k \left( \frac{1}{2} \dot{y}_c - \dot{y}_g(t) \right) \cdot \frac{1}{2} =$$

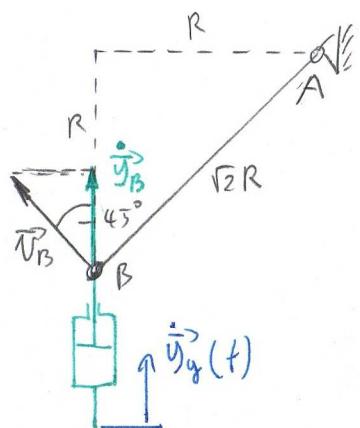
$$= - \frac{1}{4} k \dot{y}_c + \frac{1}{2} k \dot{y}_g(t)$$

$$k_{red} = \frac{1}{4} k = \frac{1}{4} \cdot 10000 = 250 \frac{\text{Ns}}{\text{m}}$$



$$\begin{aligned} Q_g &= \frac{1}{2} k w y_{g0} \cos(\omega t) = \\ &= \frac{1}{2} \cdot 1000 \cdot 50 \cdot 0,015 \cos(50t) \\ &= 375 \cos(50t) \text{ N} \end{aligned}$$

④  $Q_h + Q_g$  meghatározása (műsik módszer)



$$\begin{aligned} V_B &= \sqrt{2}R\dot{i} = \sqrt{2}R \frac{\dot{y}_c}{2R} = \frac{\sqrt{2}}{2} \dot{y}_c \\ \vec{V}_B &= V_B (-\sin 45^\circ \vec{i} + \cos 45^\circ \vec{j}) = \\ &= \frac{\sqrt{2}}{2} \dot{y}_c \left( -\frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j} \right) = \\ &= -\underbrace{\frac{1}{2} \dot{y}_c \vec{i}}_{\vec{x}_B} + \underbrace{\frac{1}{2} \dot{y}_c \vec{j}}_{\vec{y}_B} \\ \vec{V}_d &= (\vec{y}_B - \vec{y}_g(t)) \end{aligned}$$

$$\begin{aligned} \vec{V}_B &= \vec{V}_A + \vec{\omega} + \vec{v}_{AB} = \vec{0} + (-\dot{i} \vec{k}) \times (-R \vec{i} - R \vec{j}) = \\ &= \dot{i} R (\vec{k} \times \vec{i}) + \dot{i} R (\underbrace{\vec{k} \times \vec{j}}_{-\vec{i}}) = \\ &= -\dot{i} R \vec{i} + \dot{i} R \vec{j} = -\frac{\dot{y}_c}{2R} R \vec{i} + \frac{\dot{y}_c}{2R} R \vec{j} = \\ &= -\underbrace{\frac{1}{2} \dot{y}_c \vec{i}}_{\vec{x}_B} + \underbrace{\frac{1}{2} \dot{y}_c \vec{j}}_{\vec{y}_B} \\ \vec{V}_d &= (\vec{y}_B - \vec{y}_g(t)) \end{aligned}$$

$$Q_h = \vec{F}_h \cdot \vec{p}_B$$

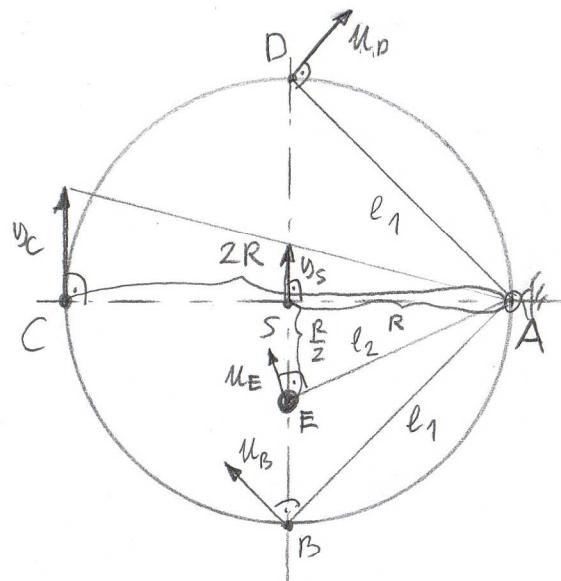
$$\begin{aligned} \vec{F}_h &= -k \vec{V}_d = -k (\vec{y}_B - \vec{y}_g(t)) = \\ &= -k \left( \frac{1}{2} \dot{y}_c \vec{j} - \vec{y}_g(t) \vec{j} \right) = \\ &= -k \cdot \left( \frac{1}{2} \dot{y}_c - \vec{y}_g(t) \right) \vec{j} \end{aligned}$$

$$\begin{aligned} \vec{p}_B &= \frac{\partial \vec{V}_B}{\partial \dot{y}_c} = \frac{\partial \left( -\frac{1}{2} \dot{y}_c \vec{i} + \frac{1}{2} \dot{y}_c \vec{j} \right)}{\partial \dot{y}_c} = \\ &= -\frac{1}{2} \vec{i} + \frac{1}{2} \vec{j} \end{aligned}$$

$$Q_h = \vec{F}_h \cdot \vec{p}_B = -k \left( \frac{1}{2} \dot{y}_c - \vec{y}_g(t) \right) \vec{j} \cdot \left( -\frac{1}{2} \vec{i} + \frac{1}{2} \vec{j} \right) = -\underbrace{\frac{1}{2} k \dot{y}_c}_{Q_h} + \underbrace{\frac{1}{2} k \vec{y}_g(t)}_{Q_g}$$

Megjegyzés:  $\ell$  nélkül arányosság alapján:

A forgásponttól távolodva az elmozdulás (és annak deriváltjai) lineárisan nőnek



ált koord:  $y_C$

$\Downarrow$

$$y_S = \frac{R}{2R} y_C = \frac{1}{2} y_C$$

$$\begin{aligned} u_D &= \frac{l_1}{2R} y_C = \frac{\sqrt{2}R}{2R} y_C = \frac{\sqrt{2}}{2} y_C \Rightarrow U = \frac{1}{2} \frac{\dot{u}_D^2}{c_1} = \frac{1}{2} \frac{(u_D \cos \beta)^2}{c_1} = \frac{1}{2} \frac{\left(\frac{\sqrt{2}}{2} y_C \cos \beta\right)^2}{c_1} = \\ &= \frac{1}{2} \left(\frac{\frac{1}{2} \cos^2 \beta}{c_1}\right) y_C^2 \end{aligned}$$

$$\begin{aligned} U_B &= \dot{u}_B = \frac{l_1}{2R} \dot{y}_C = \frac{\sqrt{2}R}{2R} \dot{y}_C = \frac{\sqrt{2}}{2} \dot{y}_C \Rightarrow D = \frac{1}{2} k \left( \dot{u}_B \cos 45^\circ - \dot{y}_g(t) \right)^2 = \\ &= \frac{1}{2} k \left( \frac{\sqrt{2}}{2} \dot{y}_C \frac{\sqrt{2}}{2} - \dot{y}_g(t) \right)^2 = \frac{1}{2} k \left( \frac{1}{2} \dot{y}_C - \dot{y}_g(t) \right)^2 \end{aligned}$$

$$U_2 = U_E = \dot{u}_E = \frac{l_2}{2R} \dot{y}_C = \frac{\sqrt{R^2 + \frac{R^2}{4}}}{2R} \dot{y}_C = \sqrt{\frac{R^2 + \frac{R^2}{4}}{4R^2}} \dot{y}_C = \sqrt{\frac{1}{4} + \frac{1}{16}} \dot{y}_C = \sqrt{\frac{5}{16}} \dot{y}_C$$

$$\Rightarrow E_2 = \frac{1}{2} m_2 U_2^2 = \frac{1}{2} m_2 \left( \sqrt{\frac{5}{16}} \dot{y}_C \right)^2 = \frac{1}{2} \left( \frac{5}{16} m_2 \right) \dot{y}_C^2$$