1.0 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE

• általános koordináták

$q = y_B$ - B pont elmozdulása

$\dot{q} = \dot{y}_B \ (v_B)$ - B pont sebessége

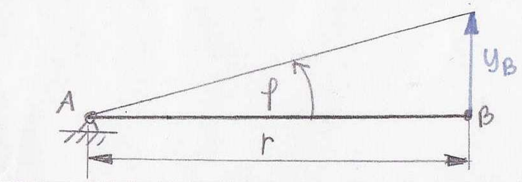
$\ddot{q} = \ddot{y}_B \ (a_B)$ - B pont gyorsulása

$q = \varphi$ szögelfordulás

$\dot{q} = \dot{\varphi} \ (\omega)$ szögsebesség

$\ddot{q} = \ddot{\varphi} \ (\varepsilon)$ szöggyorsulás

• összefüggés általános koordináták között



$y_B = r \cdot \varphi$

$\dot{y}_B = r \cdot \dot{\varphi}$

$\ddot{y}_B = r \cdot \ddot{\varphi}$

• Lagrange egyenlet: $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right] - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right]$ meghatározása

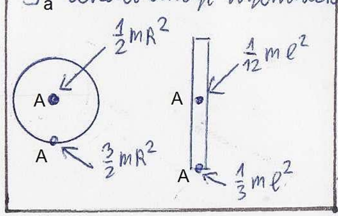
1. lépés: rendszer összenergiája

	$\dot{q} = \dot{y}_B$	$\dot{q} = \dot{\varphi}$
a) tömegpont: $E = \frac{1}{2} m v^2$	$\frac{1}{2} m \dot{y}_B^2$	$\frac{1}{2} m (r\dot{\varphi})^2$
b) merev test: $E = \frac{1}{2} J_a \omega^2$	$\frac{1}{2} J_a \left(\frac{\dot{y}_B}{r}\right)^2$	$\frac{1}{2} J_a \dot{\varphi}^2$

az energia általános alakja: $E = \frac{1}{2} \cdot m_{red} \cdot \dot{q}^2$

2. lépés: deriválás: $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right] = m_{red} \cdot \ddot{q}$

J_a tehetetlenségi nyomaték



2. $\frac{\partial E}{\partial q}$ meghatározása

$\frac{\partial E}{\partial q} = 0$

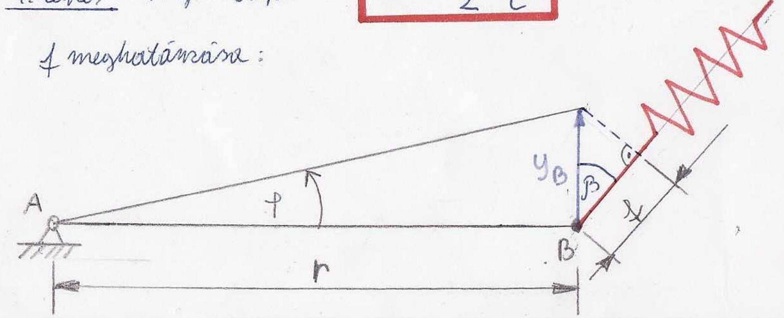
3. Q_c meghatározása

1. lépés: rugóenergia: $\boxed{U = \frac{1}{2} \frac{f^2}{c}}$

f = rugó hosszváltozása

c = rugóállandó

f meghatározása:



$f = y_B \cdot \cos\beta$

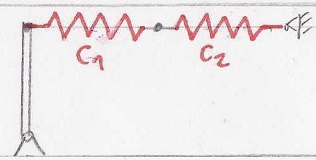
$y_B = r \cdot \varphi$

$f = r \cdot \varphi \cdot \cos\beta$

2. lépés: rugóerő

$$\boxed{Q_c = -\frac{\partial U}{\partial q} = -\frac{1}{c_{red}} \cdot q}$$

soros kapcsolt rugók: $U = \frac{1}{2} \cdot \frac{f^2}{c_1 + c_2}$



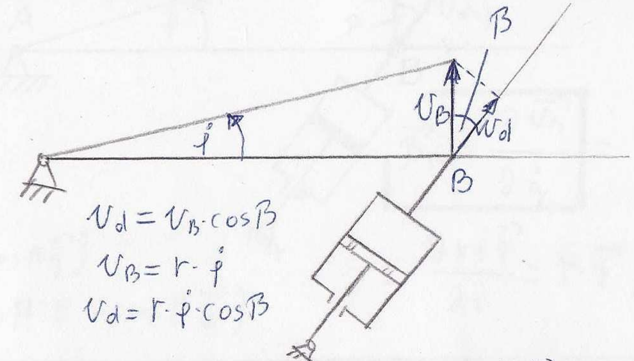
4. Q_k meghatározása

1. lépés: csillapítási energia

$$\boxed{D = \frac{1}{2} k v_d^2}$$

2. lépés: csillapító erő

$$\boxed{Q_k = -\frac{\partial D}{\partial \dot{q}} = -k_{red} \cdot \dot{q}}$$



$v_d = v_B \cdot \cos\beta$

$v_B = r \cdot \dot{\varphi}$

$v_d = r \cdot \dot{\varphi} \cdot \cos\beta$

5. Q_g meghatározása

$$\boxed{Q_g = \vec{F}_g(t) \cdot \vec{\beta}_B} \quad \text{vagy} \quad \boxed{Q_g = \vec{M}_g(t) \cdot \vec{b}}$$

$\vec{\beta}_B = \frac{\partial \vec{v}_B}{\partial \dot{q}}$

$\vec{b} = \frac{\partial \vec{\omega}}{\partial \dot{q}}$

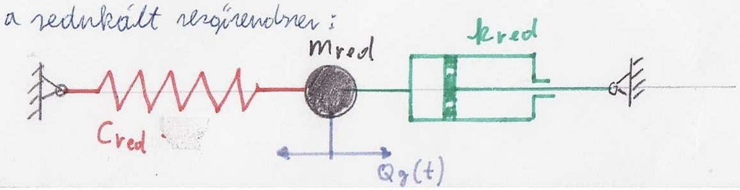
• behelyettesítés Lagrange egyenletbe

$$m_{red} \cdot \ddot{q} - 0 = -\frac{1}{c_{red}} \cdot q - k_{red} \cdot \dot{q} + Q_g(t)$$

• rendezze az egyenletet:

$$m_{red} \ddot{q} + k_{red} \dot{q} + \frac{1}{c_{red}} q = Q_g(t)$$

• a redukált rezgőrendszer:

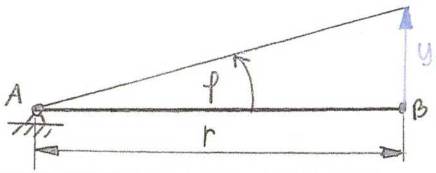


4.0 EGYSZABADSÁGFOKÚ REZGŐRENDSZEREK MOZGÁSEGYENLETE

- általános koordináták

$q = y_B$ – B pont elmozdulása | $q = \varphi$ szögelfordulás
$\dot{q} = \dot{y}_B\ (v_B)$ – B pont sebessége | $\dot{q} = \dot{\varphi}\ (\omega)$ szögsebesség
$\ddot{q} = \ddot{y}_B\ (a_B)$ – B pont gyorsulása | $\ddot{q} = \ddot{\varphi}\ (\varepsilon)$ szöggyorsulás

- összefüggés általános koordináták között



$y_B = r\cdot\varphi$
$\dot{y}_B = r\cdot\dot{\varphi}$
$\ddot{y}_B = r\cdot\ddot{\varphi}$

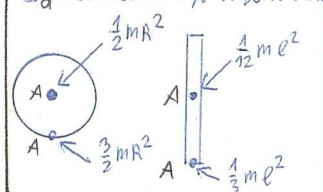
- Lagrange egyenlet: $\dfrac{d}{dt}\left[\dfrac{\partial E}{\partial \dot{q}}\right] - \dfrac{\partial E}{\partial q} = Q_c + Q_k + Q_g$

(1.) $\dfrac{d}{dt}\left[\dfrac{\partial E}{\partial \dot{q}}\right]$ meghatározása

1. lépés: rendszer összenergiája

		$q = y_B$	$q = \varphi$
a) tömegpont:	$E = \frac{1}{2} m v^2$	$\frac{1}{2} m \dot{y}_B^2$	$\frac{1}{2} m (r\cdot\dot{\varphi})^2$
b) merev test:	$E = \frac{1}{2} J_a \omega^2$	$\frac{1}{2} J_a \left(\frac{\dot{y}_B}{r}\right)^2$	$\frac{1}{2} J_a \dot{\varphi}^2$

J_a tehetetlenségi nyomaték



az energia általános alakja: $E = \frac{1}{2}\cdot m_{red}\cdot \dot{q}^2$

2. lépés: deriválás: $\dfrac{d}{dt}\left[\dfrac{\partial E}{\partial \dot{q}}\right] = m_{red}\cdot\ddot{q}$

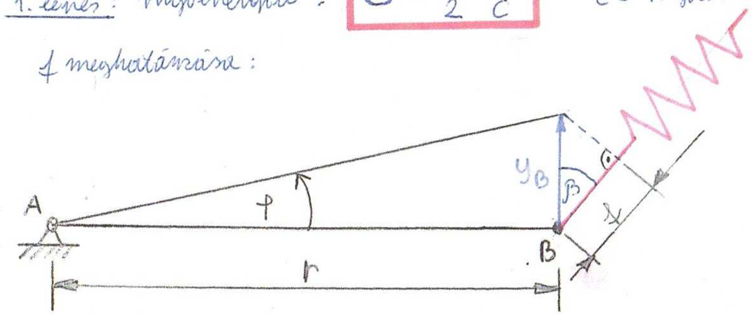
(2.) $\dfrac{\partial E}{\partial q}$ meghatározása $\qquad \dfrac{\partial E}{\partial q} = 0$

(3.) Q_c meghatározása

1. lépés: rugóenergia: $\boxed{U = \frac{1}{2}\frac{f^2}{c}}$

f = rugó hosszváltozása
c = rugóállandó

f meghatározása:



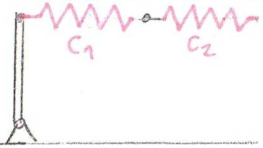
$f = y_B\cdot\cos\beta$
$y_B = r\cdot\varphi$
$f = r\cdot\varphi\cdot\cos\beta$

2. lépés: rugóerő

$$\boxed{Q_c = -\frac{\partial U}{\partial q} = -\frac{1}{c_{red}}\cdot q}$$

sorba kapcsolt rugók:

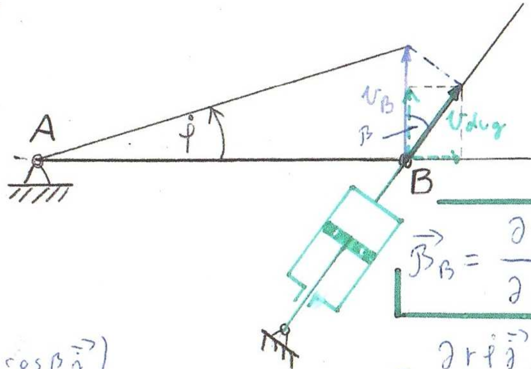
$U = \frac{1}{2}\cdot\frac{f^2}{c_1+c_2}$



(4.) Q_k meghatározása

$$\boxed{Q_k = \vec{F}_k\cdot\vec{\beta}_B}$$

$$\boxed{F_k = -k\cdot\vec{v}_{dug}}$$



$v_{dug} = v_B\cdot\cos\beta$
$v_B = r\cdot\dot{\varphi}$
$v_{dug} = r\cdot\dot{\varphi}\cdot\cos\beta$
$\vec{v}_{dug} = v_{dug}\,(\sin\beta\,\vec{i} + \cos\beta\,\vec{j})$
$\vec{v}_{dug} = r\cdot\dot{\varphi}\cdot\cos\beta\cdot(\sin\beta\,\vec{i} + \cos\beta\,\vec{j})$

$$\boxed{\vec{\beta}_B = \frac{\partial \vec{v}_B}{\partial \dot{q}}} = \frac{\partial\, r\dot{\varphi}\vec{j}}{\partial \dot{\varphi}} = r\cdot\vec{j}$$

(5.) Q_g meghatározása

$$\boxed{Q_g = \vec{F}_g(t)\cdot\vec{\beta}_B} \quad \text{vagy} \quad \boxed{Q_g = \vec{M}_g(t)\cdot\vec{b}}$$

$\vec{\beta}_B = \dfrac{\partial \vec{v}_B}{\partial \dot{q}}$

$\vec{b} = \dfrac{\partial \vec{\omega}}{\partial \dot{q}}$

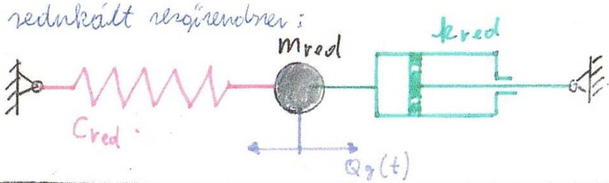
- behelyettesítés Lagrange egyenletbe

$m_{red}\cdot\ddot{q} - 0 = -\dfrac{1}{c_{red}}\cdot q - k_{red}\cdot\dot{q} + Q_g(t)$

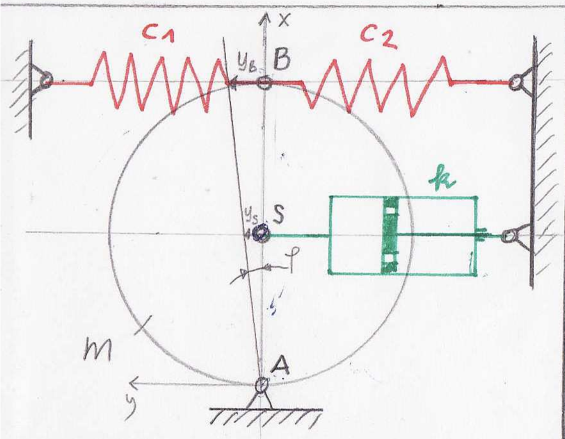
- rendezze az egyenletet:

$m_{red}\,\ddot{q} + k_{red}\,\dot{q} + \dfrac{1}{c_{red}}\,q = Q_g(t)$

- a redukált rezgőrendszer:



1.1 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



a)

(1.) $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E=\frac{1}{2}J_a\cdot\dot\varphi^2=\frac{1}{2}\left(\frac{3}{2}mR^2\right)\dot\varphi^2$$

$$\frac{\partial E}{\partial\dot\varphi}=\frac{3}{2}mR^2\dot\varphi$$

$$\frac{d}{dt}\frac{3}{2}mR^2\dot\varphi=\frac{3}{2}mR^2\ddot\varphi$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]=\frac{3}{2}mR^2\ddot\varphi}$$

(2.) $\frac{\partial E}{\partial q}=0$

(3.) Q_c meghatározása

$$U=U_1+U_2=\frac{1}{2}\cdot\frac{y_B^2}{c_1}+\frac{1}{2}\cdot\frac{y_B^2}{c_2}=$$

$$=\frac{1}{2}\cdot\frac{(2R\varphi)^2}{c_1}+\frac{1}{2}\cdot\frac{(2R\varphi)^2}{c_2}=$$

$$=\frac{1}{2}\cdot\left(\frac{4R^2\varphi^2}{c_1}+\frac{4R^2\varphi^2}{c_2}\right)=\frac{2R^2\varphi^2}{c_1}+\frac{2R^2\varphi^2}{c_2}$$

$$Q_c=-\frac{\partial U}{\partial\varphi}=-\frac{\partial}{\partial\varphi}\left(\frac{2R^2\varphi^2}{c_1}+\frac{2R^2\varphi^2}{c_2}\right)=$$

adott: c_1, c_2, k, m, R

• az általános koordináta:

a) $q=\varphi$, $\dot q=\dot\varphi\,(\omega)$, $\ddot q=\ddot\varphi\,(\varepsilon)$

b) $q=y_B$, $\dot q=\dot y_B\,(v_B)$, $\ddot q=\ddot y_B\,(a_B)$

• Lagrange egyenlet:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial\dot q}\right)-\frac{\partial E}{\partial q}=Q_c+Q_k$$

$$=-\left(\frac{4R^2\varphi}{c_1}+\frac{4R^2\varphi}{c_2}\right)=-\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)\cdot\varphi$$

$$\boxed{Q_c=-\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)\cdot\varphi}$$

(4.) Q_k meghatározása

$$D=\frac{1}{2}k\,v_d^2=\frac{1}{2}k(R\cdot\dot\varphi)^2=$$

$$=\frac{1}{2}k\cdot R^2\dot\varphi^2$$

$$Q_k=-\frac{\partial D}{\partial\dot\varphi}=-kR^2\dot\varphi$$

$$\boxed{Q_k=-kR^2\dot\varphi}$$

• behelyettesítés Lagrange egyenletbe

$$\frac{3}{2}mR^2\ddot\varphi-0=-\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)\cdot\varphi-kR^2\dot\varphi$$

• rendezve az egyenletet:

$$\underbrace{\frac{3}{2}mR^2}_{m_{red}}\cdot\ddot\varphi+\underbrace{k\cdot R^2}_{k_{red}}\cdot\dot\varphi+\underbrace{\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)}_{\frac{1}{c_{red}}}\cdot\varphi=0$$

• a redukált rezgőrendszer:

$$m_{red}\cdot\ddot q+k_{red}\cdot\dot q+\frac{1}{c_{red}}\cdot q=0$$

b)

(1.) $\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]$ meghatározása

$$E=\frac{1}{2}J_a\omega^2=\frac{1}{2}\left(\frac{3}{2}mR^2\right)\cdot\omega^2=$$

$$=\frac{1}{2}\left(\frac{3}{2}mR^2\right)\cdot\left(\frac{\dot y_B}{2R}\right)^2=\frac{1}{2}\cdot\left(\frac{3}{8}m\right)\cdot\dot y_B^2$$

$$\frac{\partial E}{\partial\dot y_B}=\frac{3}{8}m\dot y_B$$

$$\frac{d}{dt}\left[\frac{\partial E}{\partial\dot y_B}\right]=\frac{3}{8}m\ddot y_B$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]=\frac{3}{8}m\ddot y_B}$$

(2.) $\frac{\partial E}{\partial q}=0$

(3.) Q_c meghatározása

$$U=U_1+U_2=\frac{1}{2}\cdot\frac{y_B^2}{c_1}+\frac{1}{2}\cdot\frac{y_B^2}{c_2}=$$

$$=\frac{1}{2}\cdot\left(\frac{1}{c_1}+\frac{1}{c_2}\right)y_B^2$$

$$Q_c=-\frac{\partial U}{\partial y_B}=-\left(\frac{1}{c_1}+\frac{1}{c_2}\right)y_B$$

$$\boxed{Q_c=-\left(\frac{1}{c_1}+\frac{1}{c_2}\right)y_B}$$

(4.) Q_k meghatározása

$$D=\frac{1}{2}k\,v_d^2=\frac{1}{2}k\cdot\left(\frac{\dot y_B}{2}\right)^2=$$

$$=\frac{1}{2}k\,\frac{1}{4}\cdot\dot y_B^2$$

$$Q_k=-\frac{\partial D}{\partial\dot y_B}=-k\cdot\frac{1}{4}\cdot\dot y_B$$

$$\boxed{Q_k=-k\cdot\frac{1}{4}\cdot\dot y_B}$$

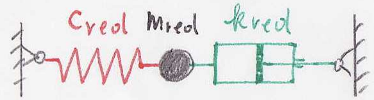
• behelyettesítés Lagrange egyenletbe

$$\frac{3}{8}m\ddot y_B-0=-\left(\frac{1}{c_1}+\frac{1}{c_2}\right)y_B-\frac{k}{4}\cdot\dot y_B$$

• rendezve az egyenletet

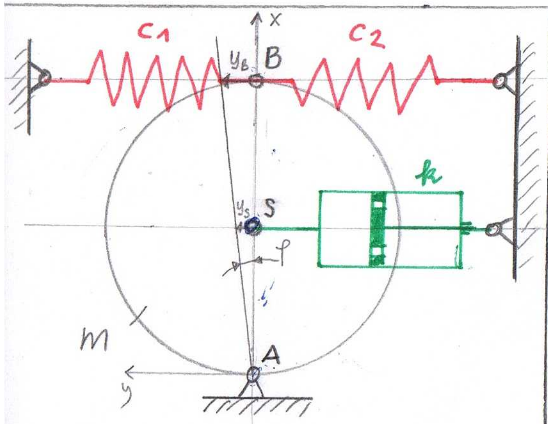
$$\underbrace{\frac{3}{8}m}_{m_{red}}\ddot y_B+\underbrace{\frac{k}{4}}_{k_{red}}\cdot\dot y_B+\underbrace{\left(\frac{1}{c_1}+\frac{1}{c_2}\right)}_{\frac{1}{c_{red}}}y_B=0$$

• a redukált rezgőrendszer:



$$m_{red}\,\ddot q+k_{red}\,\dot q+\frac{1}{c_{red}}\cdot q=0$$

1.1 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



a)

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E=\frac12 J_a\cdot\dot\varphi^2=\frac12\left(\frac32 mR^2\right)\dot\varphi^2$$

$$\frac{\partial E}{\partial\dot\varphi}=\frac32 mR^2\dot\varphi$$

$$\frac{d}{dt}\frac32 mR^2\dot\varphi=\frac32 mR^2\ddot\varphi$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]=\frac32 mR^2\ddot\varphi}$$

2. $\frac{\partial E}{\partial q}=0$

3. Q_c meghatározása

$$U=U_1+U_2=\frac12\cdot\frac{y_B^2}{c_1}+\frac12\cdot\frac{y_B^2}{c_2}=$$

$$=\frac12\cdot\frac{(2R\varphi)^2}{c_1}+\frac12\cdot\frac{(2R\varphi)^2}{c_2}=$$

$$=\frac12\cdot\left(\frac{4R^2\varphi^2}{c_1}+\frac{4R^2\varphi^2}{c_2}\right)=\frac{2R^2\varphi^2}{c_1}+\frac{2R^2\varphi^2}{c_2}$$

$$Q_c=-\frac{\partial U}{\partial\varphi}=-\frac{\partial}{\partial\varphi}\left(\frac{2R^2\varphi^2}{c_1}+\frac{2R^2\varphi^2}{c_2}\right)=$$

adott: c_1, c_2, k, m, R

• az általános koordináta:

a) $q=\varphi$, $\dot q=\dot\varphi(\omega)$, $\ddot q=\ddot\varphi(\varepsilon)$

b) $q=y_B$, $\dot q=\dot y_B(v_B)$, $\ddot q=\ddot y_B(a_B)$

• Lagrange egyenlet:

$$\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]-\frac{\partial E}{\partial q}=Q_c+Q_k$$

$$=-\left(\frac{4R^2\varphi}{c_1}+\frac{4R^2\varphi}{c_2}\right)=-\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)\cdot\varphi$$

$$\boxed{Q_c=-\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)\cdot\varphi}$$

4. Q_k meghatározása

$$Q_k=\vec F_k\cdot\vec\beta_S$$

$$\vec F_k=-k\cdot\vec v_{din}=-k\cdot\vec v_S=-k\cdot(R\cdot\dot\varphi)\vec j$$

$$\vec\beta_S=\frac{\partial\vec v_S}{\partial\dot\varphi}=\frac{\partial(R\cdot\dot\varphi\,\vec j)}{\partial\dot\varphi}=R\cdot\vec j$$

$$Q_k=-k\cdot R\cdot\dot\varphi\cdot\vec j\cdot R\cdot\vec j=-kR^2\dot\varphi$$

$$\boxed{Q_k=-kR^2\dot\varphi}$$

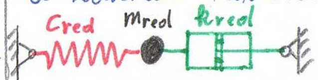
• behelyettesítés Lagrange egyenletbe

$$\frac32 mR^2\ddot\varphi-0=-\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)\cdot\varphi-kR^2\dot\varphi$$

• rendezve az egyenletet:

$$\underbrace{\frac32 mR^2}_{m_{red}}\cdot\ddot\varphi+\underbrace{k\cdot R^2}_{k_{red}}\cdot\dot\varphi+\underbrace{\left(\frac{4R^2}{c_1}+\frac{4R^2}{c_2}\right)}_{\frac{1}{c_{red}}}\cdot\varphi=0$$

• a redukált rezgőrendszer:



$$m_{red}\cdot\ddot q+k_{red}\cdot\dot q+\frac{1}{c_{red}}\cdot q=0$$

b)

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E=\frac12 J_a\omega^2=\frac12\left(\frac32 mR^2\right)\cdot\omega^2=$$

$$=\frac12\left(\frac32 mR^2\right)\cdot\left(\frac{\dot y_B}{2R}\right)^2=\frac12\cdot\left(\frac38 m\right)\cdot\dot y_B^2$$

$$\frac{\partial E}{\partial\dot y_B}=\frac38 m\dot y_B$$

$$\frac{d}{dt}\left[\frac{\partial E}{\partial\dot y_B}\right]=\frac38 m\ddot y_B$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]=\frac38 m\ddot y_B}$$

2. $\frac{\partial E}{\partial q}=0$

3. Q_c meghatározása

$$U=U_1+U_2=\frac12\frac{y_B^2}{c_1}+\frac12\cdot\frac{y_B^2}{c_2}=$$

$$=\frac12\cdot\left(\frac1{c_1}+\frac1{c_2}\right)y_B^2$$

$$Q_c=-\frac{\partial U}{\partial y_B}=-\left(\frac1{c_1}+\frac1{c_2}\right)y_B$$

$$\boxed{Q_c=-\left(\frac1{c_1}+\frac1{c_2}\right)y_B}$$

4. Q_k meghatározása

$$Q_k=\vec F_k\cdot\vec\beta_S$$

$$\vec F_k=-k\cdot\vec v_{din}=-k\cdot\vec v_S=$$

$$=-k\left(\frac{\dot y_B}{2}\right)\vec j$$

$$\vec\beta_S=\frac{\partial\vec v_S}{\partial\dot y_B}=\frac{\partial\left(\frac{\dot y_B}{2}\vec j\right)}{\partial\dot y_B}=\frac12\vec j$$

$$Q_k=-k\frac{\dot y_B}{2}\vec j\cdot\frac12\cdot\vec j=-k\cdot\frac14\cdot\dot y_B$$

$$\boxed{Q_k=-k\cdot\frac14\cdot\dot y_B}$$

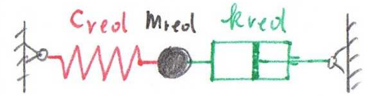
• behelyettesítés Lagrange egyenletbe

$$\frac38 m\ddot y_B-0=-\left(\frac1{c_1}+\frac1{c_2}\right)y_B-\frac{k}{4}\cdot\dot y_B$$

• rendezve az egyenletet

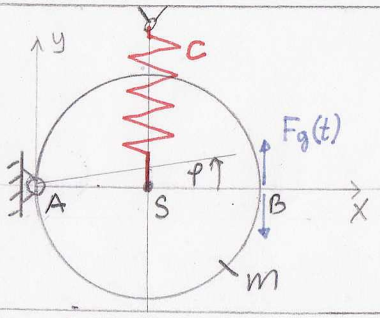
$$\underbrace{\frac38 m}_{m_{red}}\ddot y_B+\underbrace{\frac{k}{4}}_{k_{red}}\cdot\dot y_B+\underbrace{\left(\frac1{c_1}+\frac1{c_2}\right)}_{\frac1{c_{red}}}y_B=0$$

• a redukált rezgőrendszer:



$$m_{red}\ddot q+k_{red}\cdot\dot q+\frac1{c_{red}}\cdot q=0$$

1.2 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



adott: $c, m, R, F_g(t) = F_{g0} \cdot \sin(\omega t + \varepsilon)$

• az általános koordináta:

$q = \varphi \quad \dot q = \dot\varphi\,(\omega) \quad \ddot q = \ddot\varphi\,(\varepsilon)$

• Lagrange egyenlet

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c + Q_g$$

(1.) $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2} J_a \omega^2 = \frac{1}{2}\left(\frac{3}{2} m R^2\right) \dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = \frac{3}{2} m R^2 \ddot\varphi}$$

(2.) $\frac{\partial E}{\partial q} = 0$

(3.) Q_c meghatározása

$$U = \frac{1}{2} \cdot \frac{(R\varphi)^2}{c} = \frac{1}{2} \frac{R^2}{c} \cdot \varphi^2$$

$$Q_c = -\frac{\partial U}{\partial \varphi} = -\frac{R^2}{c} \cdot \varphi$$

$$\boxed{Q_c = -\frac{R^2}{c} \cdot \varphi}$$

(4.) Q_g meghatározása

$$Q_g = \vec F_g \cdot \vec\beta_B$$

$$\vec F_g = F_{g0} \cdot \sin(\omega t + \varepsilon)\, \vec j$$

$$\vec\beta_B = \frac{\partial \vec v_B}{\partial \dot\varphi} = \frac{\partial (2R\dot\varphi \cdot \vec j)}{\partial \dot\varphi} = 2R\vec j$$

$$Q_g = \vec F_g \cdot \vec\beta_B = 2R F_g(t)$$

$$\boxed{Q_g = 2R F_g(t)}$$

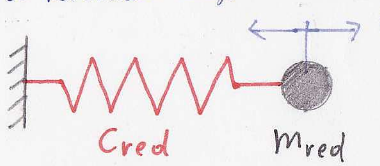
• behelyettesítve Lagrange egyenletbe:

$$\frac{3}{2} m R^2 \ddot\varphi - 0 = -\frac{R^2}{c} \cdot \varphi + 2R F_g(t)$$

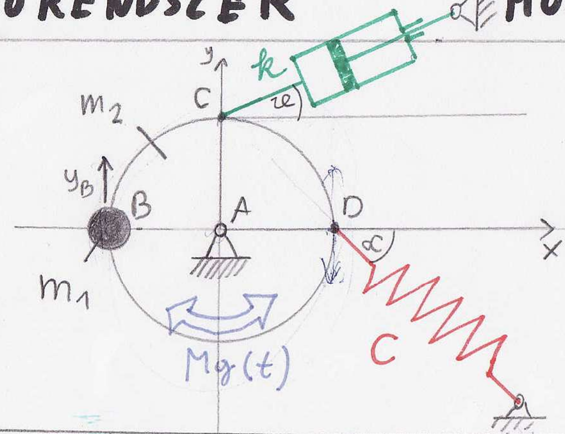
• rendezve az egyenletet:

$$\underbrace{\frac{3}{2} m R^2}_{m_{red}} \ddot\varphi + \underbrace{\frac{R^2}{c}}_{\frac{1}{c_{red}}} \cdot \varphi = \underbrace{2R F_g(t)}_{Q_g(t)}$$

• a redukált rezgőrendszer:



$$m_{red} \cdot \ddot q + \frac{1}{c_{red}} \cdot q = Q_g(t)$$



adott: $m_1, m_2, c, k, \alpha, \vartheta, R,$
$M_g(t) = M_{g0} \cdot \sin(\omega t + \varepsilon)$

az általános koordináta:

$q = y_B \quad \dot q = \dot y_B\,(v_B) \quad \ddot q = \ddot y_B\,(a_B)$

• Lagrange egyenlet

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - 0 = Q_c + Q_k + Q_g$$

(1.) $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = E_1 + E_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} J_a \omega^2 =$$

$$= \frac{1}{2} m_1 \dot y_B^2 + \frac{1}{2}\left(\frac{1}{2} m_2 R^2\right) \cdot \left(\frac{\dot y_B}{R}\right)^2 =$$

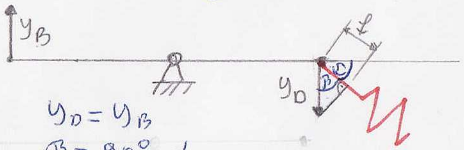
$$= \frac{1}{2}\left(m_1 + \frac{1}{2} m_2 \cancel{R^2} \frac{1}{\cancel{R^2}}\right) \cdot \dot y_B^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = \left(m_1 + \frac{1}{2} m_2\right) \ddot y_B}$$

(2.) $\frac{\partial E}{\partial q} = 0$

(3.) Q_c meghatározása

$$U = \frac{1}{2} \cdot \frac{(y_B \cdot \cos\beta)^2}{c} = \frac{1}{2} \cdot \frac{\cos^2\beta}{c} \cdot y_B^2$$



$y_D = y_B$

$\beta = 90° - \alpha$

$$Q_c = -\frac{\partial U}{\partial y_B} = -\frac{\cos^2\beta}{c} \cdot y_B$$

$$\boxed{Q_c = -\frac{\cos^2\beta}{c} \cdot y_B}$$

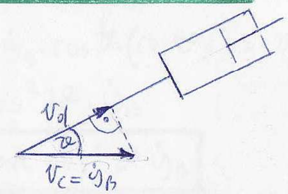
(4.) Q_k meghatározása

$$D = \frac{1}{2} k v_d^2 = \frac{1}{2} k \left(\dot y_B \cos\vartheta\right)^2 =$$

$$= \frac{1}{2} k \cdot \dot y_B^2 \cdot \cos^2\vartheta$$

$$Q_k = -\frac{\partial D}{\partial \dot y_B} = -k \cdot \cos^2\vartheta\, \dot y_B$$

$$\boxed{Q_k = -k \cdot \cos^2\vartheta \cdot \dot y_B}$$



(5.) Q_g meghatározása

$$Q_g = \vec M_g(t) \cdot \vec b$$

$$\vec M_g(t) = M_g(t)\,\vec k = M_{g0} \sin(\omega t + \varepsilon)\,\vec k$$

$$\vec b = \frac{\partial \vec\omega}{\partial \dot y_B} = \frac{\partial\left(\frac{\dot y_B}{R}\vec k\right)}{\partial \dot y_B} = \frac{1}{R}\vec k$$

$$\boxed{Q_g = \frac{1}{R} M_g(t)}$$

• behelyettesítés Lagrange egyenletbe

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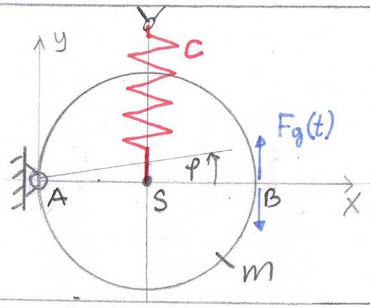
1.2 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



adott: $c, m, R, F_g(t) = F_{g0} \cdot \sin(\omega t + \varepsilon)$

- az általános koordináta:

$$q = \varphi \quad \dot{q} = \dot{\varphi}\,(\omega) \quad \ddot{q} = \ddot{\varphi}\,(\varepsilon)$$

- Lagrange egyenlet

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right] - \frac{\partial E}{\partial q} = Q_c + Q_g$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right]$ meghatározása

$$E = \frac{1}{2} J_a \omega^2 = \frac{1}{2}\left(\frac{3}{2} m R^2\right) \dot{\varphi}^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right] = \frac{3}{2} m R^2 \ddot{\varphi}}$$

2. $\frac{\partial E}{\partial q} = 0$

3. Q_c meghatározása

$$U = \frac{1}{2} \cdot \frac{(R\varphi)^2}{c} = \frac{1}{2}\frac{R^2}{c} \cdot \varphi^2$$

$$Q_c = -\frac{\partial U}{\partial \varphi} = -\frac{R^2}{c} \cdot \varphi$$

$$\boxed{Q_c = -\frac{R^2}{c} \cdot \varphi}$$

4. Q_g meghatározása

$$Q_g = \vec{F}_g \cdot \vec{\beta}_B$$

$$\vec{F}_g = F_{g0} \cdot \sin(\omega t + \varepsilon)\,\vec{j}$$

$$\vec{\beta}_B = \frac{\partial \vec{v}_B}{\partial \dot{\varphi}} = \frac{\partial (2R\dot{\varphi}\,\vec{j})}{\partial \dot{\varphi}} = 2R\vec{j}$$

$$Q_g = \vec{F}_g \cdot \vec{\beta}_B = 2R F_g(t)$$

$$\boxed{Q_g = 2R F_g(t)}$$

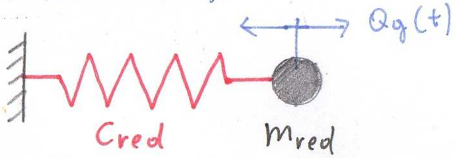
- behelyettesítve Lagrange egyenletbe:

$$\frac{3}{2} m R^2 \ddot{\varphi} - 0 = -\frac{R^2}{c} \cdot \varphi + 2R F_g(t)$$

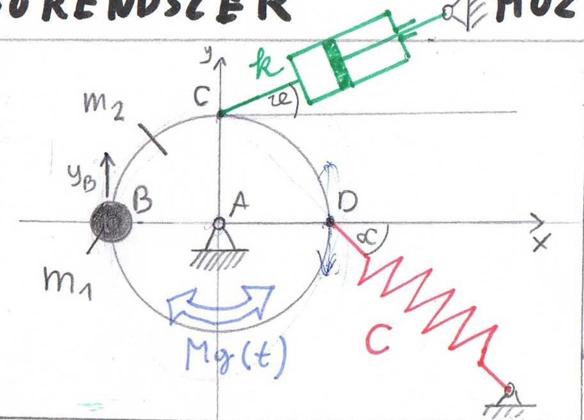
- rendezve az egyenletet:

$$\underbrace{\frac{3}{2} m R^2}_{m_{red}} \ddot{\varphi} + \underbrace{\frac{R^2}{c}}_{\frac{1}{c_{red}}} \cdot \varphi = \underbrace{2R F_g(t)}_{Q_g(t)}$$

- a redukált rezgőrendszer:



$$m_{red} \cdot \ddot{q} + \frac{1}{c_{red}} \cdot q = Q_g(t)$$



adott: $m_1, m_2, c, k, \alpha, \vartheta, R,$
$M_g(t) = M_{g0} \cdot \sin(\omega t + \varepsilon)$

az általános koordináta:

$$q = y_B \quad \dot{q} = \dot{y}_B\,(v_B) \quad \ddot{q} = \ddot{y}_B\,(a_B)$$

- Lagrange egyenlet

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right] - 0 = Q_c + Q_k + Q_g$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right]$ meghatározása

$$E = E_1 + E_2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} J_a \omega^2 =$$

$$= \frac{1}{2} m_1 \dot{y}_B^2 + \frac{1}{2}\left(\frac{1}{2} m_2 R^2\right) \cdot \left(\frac{\dot{y}_B}{R}\right)^2 =$$

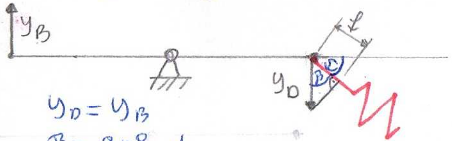
$$= \frac{1}{2}\left(m_1 + \frac{1}{2} m_2 \cancel{R^2} \frac{1}{\cancel{R^2}}\right) \cdot \dot{y}_B^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot{q}}\right] = \left(m_1 + \frac{1}{2} m_2\right) \ddot{y}_B}$$

2. $\frac{\partial E}{\partial q} = 0$

3. Q_c meghatározása

$$U = \frac{1}{2} \cdot \frac{(y_B \cdot \cos\beta)^2}{c} = \frac{1}{2} \cdot \frac{\cos^2\beta}{c} \cdot y_B^2$$



$y_D = y_B$

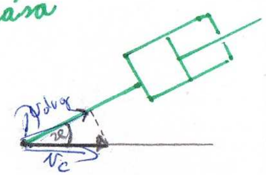
$\beta = 90° - \alpha$

$$Q_c = -\frac{\partial U}{\partial y_B} = -\frac{\cos^2\beta}{c} \cdot y_B$$

$$\boxed{Q_c = -\frac{\cos^2\beta}{c} \cdot y_B}$$

4. Q_k meghatározása

$$Q_k = \vec{F}_k \cdot \vec{\beta}_C$$



$$\vec{F}_k = -k \cdot \vec{v}_{dug}$$

$$|v_C| = |v_B| = |\dot{y}_B|$$

$$v_{dug} = v_C \cdot \cos\vartheta = \dot{y}_B \cdot \cos\vartheta$$

$$\vec{v}_{dug} = v_{dug} \cdot (\cos\vartheta\,\vec{i} + \sin\vartheta\,\vec{j})$$

$$\vec{\beta}_C = \frac{\partial \vec{v}_C}{\partial \dot{y}_B} = \frac{\partial (\dot{y}_B \vec{i})}{\partial \dot{y}_B} = \vec{i}$$

$$Q_k = -k \cdot \dot{y}_B \cdot \cos\vartheta \cdot (\cos\vartheta\,\vec{i} + \sin\vartheta\,\vec{j}) \cdot \vec{i} =$$

$$= -k \cdot \cos^2\vartheta \cdot \dot{y}_B$$

$$\boxed{Q_k = -k \cos^2\vartheta\, \dot{y}_B}$$

5. Q_g meghatározása

$$Q_g = \vec{M}_g(t) \cdot \vec{b}$$

$$\vec{M}_g(t) = M_g(t)\,\vec{k} = M_{g0} \sin(\omega t + \varepsilon)\,\vec{k}$$

$$\vec{b} = \frac{\partial \vec{\omega}}{\partial \dot{y}_B} = \frac{\partial \left(\frac{\dot{y}_B}{R}\vec{k}\right)}{\partial \dot{y}_B} = \frac{1}{R}\vec{k}$$

$$\boxed{Q_g = \frac{1}{R} M_g(t)}$$

- behelyettesítés Lagrange egyenletbe

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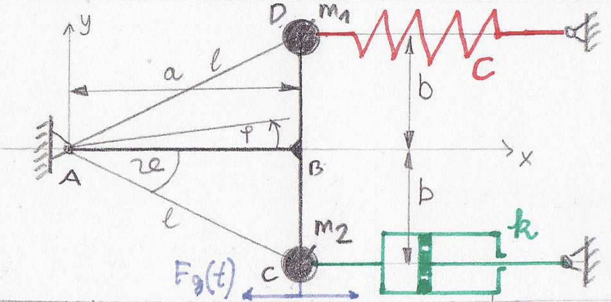
1.3 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



adott: $m_1, m_2, a, b, c, k,$
$F_g(t)=F_{g0}\cdot\sin(\omega t+\varepsilon)$

az általános koordináta:
$q=\varphi \quad \dot q=\dot\varphi(\omega) \quad \ddot q=\ddot\varphi(\varepsilon)$

Lagrange egyenlet:
$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]-\frac{\partial E}{\partial q}=Q_c+Q_k+Q_g$$

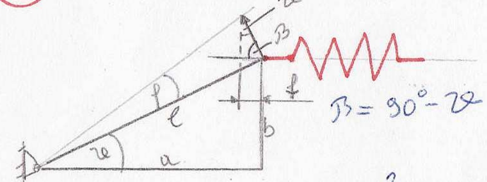
1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E=\frac{1}{2}m_1 v_1^2+\frac{1}{2}m_2 v_2^2=$$
$$=\frac{1}{2}m_1(\ell\dot\varphi)^2+\frac{1}{2}m_2(\ell\dot\varphi)^2=$$
$$=\frac{1}{2}m_1\left((a^2+b^2)\dot\varphi^2\right)+\frac{1}{2}m_2\left((a^2+b^2)\dot\varphi^2\right)=$$
$$=\frac{1}{2}(m_1+m_2)(a^2+b^2)\,\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]=(m_1+m_2)(a^2+b^2)\,\ddot\varphi}$$

2. $\frac{\partial E}{\partial q}=0$

3. Q_c meghatározása



$\beta=90^\circ-\vartheta$

$$U=\frac{1}{2}\frac{f^2}{c}=\frac{1}{2}\cdot\frac{(\ell\cdot\varphi\cdot\cos\beta)^2}{c}=$$
$$=\frac{1}{2}\cdot\frac{(a^2+b^2)\cdot\varphi^2\cdot\cos^2\beta}{c}=$$
$$=\frac{1}{2}\cdot\frac{(a^2+b^2)\cdot\cos^2\beta}{c}\cdot\varphi^2$$

$$Q_c=-\frac{\partial U}{\partial\varphi}=-\frac{(a^2+b^2)\cos^2\beta}{c}\cdot\varphi$$

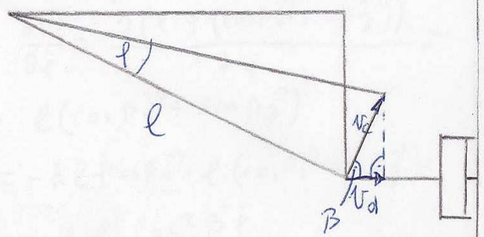
$$\boxed{Q_c=-\frac{(a^2+b^2)\cos^2\beta}{c}\cdot\varphi}$$

4. Q_k meghatározása

$$D=\frac{1}{2}k\,v_d^2=\frac{1}{2}k\cdot(v_c\cos\beta)^2=$$
$$=\frac{1}{2}k\cdot(\dot\varphi\,\ell\cdot\cos\beta)^2=$$
$$=\frac{1}{2}k\cdot\dot\varphi^2\ell^2\cdot\cos^2\beta$$

$$Q_k=-\frac{\partial D}{\partial\dot\varphi}=-k\cdot\ell^2\cdot\cos^2\beta\cdot\dot\varphi$$

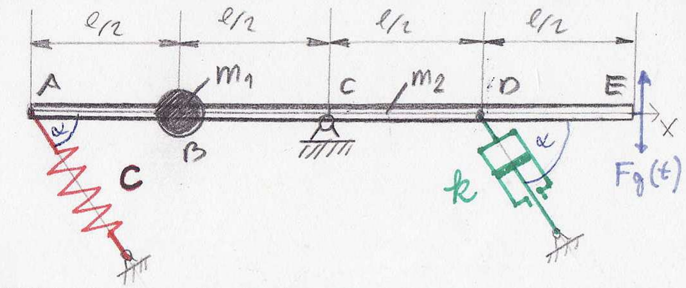
$$\boxed{Q_k=-k\cdot\ell^2\cdot\cos^2\beta\cdot\dot\varphi}$$



5. Q_g meghatározása

$$Q_g=\vec F_g(t)\cdot\vec\beta_C$$
$$\vec F_g(t)=F_g(t)\,\vec i=F_{g0}\sin(\omega t+\varepsilon)\,\vec i$$
$$\vec\beta_C=\frac{\partial\vec v_c}{\partial\dot\varphi}=\ell(\cos\beta\,\vec i+\sin\beta\,\vec j)$$

$$\boxed{Q_g=F_g(t)\cdot\ell\cdot\cos\beta}$$



adott: $m_1, m_2, \ell, c, k, \alpha,$
$F_g(t)=F_{g0}\sin(\omega t+\varepsilon)$

az általános koordináta:
$q=\varphi \quad \dot q=\dot\varphi(\omega) \quad \ddot q=\ddot\varphi(\varepsilon)$

Lagrange egyenlet
$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]-\frac{\partial E}{\partial q}=Q_c+Q_k+Q_g$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

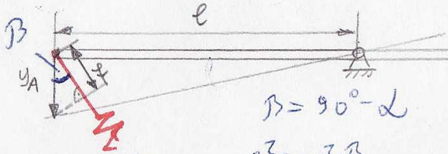
$$E=E_1+E_2=\frac{1}{2}m_1 v_1^2+\frac{1}{2}J_c\,\omega^2=$$
$$=\frac{1}{2}m_1\cdot\left(\frac{\ell}{2}\cdot\dot\varphi\right)^2+\frac{1}{2}\left(\frac{1}{12}m_2(2\ell)^2\right)\dot\varphi^2=$$
$$=\frac{1}{2}\cdot\left(\frac{m_1}{4}+\frac{m_2}{3}\right)\cdot\ell^2\cdot\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]=\left(\frac{m_1}{4}+\frac{m_2}{3}\right)\ell^2\cdot\ddot\varphi}$$

2. $\frac{\partial E}{\partial q}=0$

3. Q_c meghatározása

$$U=\frac{1}{2}\frac{f^2}{c}=\frac{1}{2}\cdot\frac{(\ell\,\varphi\cos\beta)^2}{c}=$$
$$=\frac{1}{2}\cdot\frac{\ell^2\cos^2\beta}{c}\cdot\varphi^2$$



$\beta=90^\circ-\alpha$

$$Q_c=-\frac{\partial U}{\partial\varphi}=-\frac{\ell^2\cos^2\beta}{c}\cdot\varphi$$

$$\boxed{Q_c=-\frac{\ell^2\cos^2\beta}{c}\cdot\varphi}$$

4. Q_k meghatározása

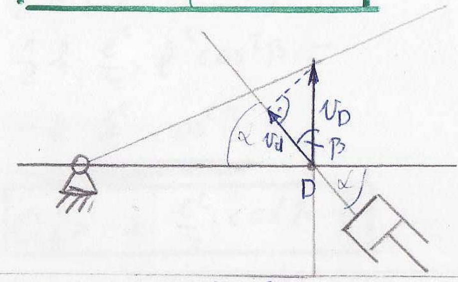
$\cancel{Q_k=\vec F_k\cdot\vec\beta_D}$

$\cancel{\vec F_k=-k\cdot\vec v_{dug}}$

$$D=\frac{1}{2}k\cdot v_d^2=\frac{1}{2}k\cdot(v_D\cdot\cos\beta)^2=$$
$$=\frac{1}{2}k\cdot\left(\frac{\ell}{2}\cdot\dot\varphi\cos\beta\right)^2=\frac{1}{2}k\cdot\frac{\ell^2}{4}\cdot\dot\varphi^2\cos^2\beta$$

$$Q_k=-\frac{\partial D}{\partial\dot\varphi}=-k\cdot\frac{\ell^2}{4}\cdot\cos^2\beta\cdot\dot\varphi$$

$$\boxed{Q_k=-k\cdot\frac{\ell^2}{4}\cdot\cos^2\beta\cdot\dot\varphi}$$

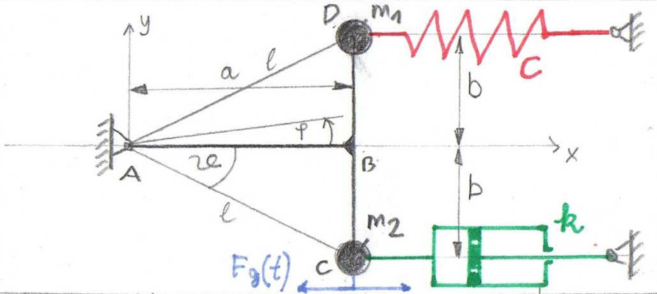


5. Q_g meghatározása

$$Q_g=\vec F_g\cdot\vec\beta_E$$
$$\vec F_g=F_g(t)\,\vec j=F_{g0}\sin(\omega t+\varepsilon)\,\vec j$$
$$\vec\beta_E=\frac{\partial\vec v_E}{\partial\dot\varphi}=\frac{\partial(\ell\cdot\dot\varphi\,\vec j)}{\partial\dot\varphi}=\ell\cdot\vec j$$
$$Q_g=F_g(t)\,\vec j\cdot\ell\cdot\vec j=F_g(t)\cdot\ell$$

$$\boxed{Q_g=F_g(t)\cdot\ell}$$

1.3 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



adott: $m_1, m_2, a, b, c, k,$
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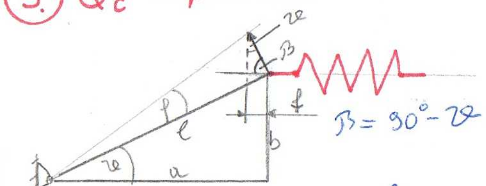
$$=\frac{1}{2}m_1(\ell\dot\varphi)^2+\frac{1}{2}m_2(\ell\dot\varphi)^2=$$
$$=\frac{1}{2}m_1\left((a^2+b^2)\dot\varphi^2\right)+\frac{1}{2}m_2\left((a^2+b^2)\dot\varphi^2\right)=$$
$$=\frac{1}{2}(m_1+m_2)(a^2+b^2)\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]=(m_1+m_2)(a^2+b^2)\ddot\varphi}$$

2. $\frac{\partial E}{\partial q}=0$

3. Q_c meghatározása



$\beta=90°-\vartheta$

$$U=\frac{1}{2}\frac{f^2}{c}=\frac{1}{2}\cdot\frac{(\ell\cdot\varphi\cdot\cos\beta)^2}{c}=$$
$$=\frac{1}{2}\cdot\frac{(a^2+b^2)\cdot\varphi^2\cdot\cos^2\beta}{c}=$$
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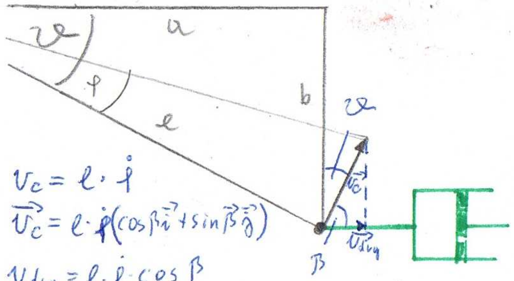
$$Q_c=-\frac{\partial U}{\partial\varphi}=-\frac{(a^2+b^2)\cos^2\beta}{c}\cdot\varphi$$

$$\boxed{Q_c=-\frac{(a^2+b^2)\cos^2\beta}{c}\cdot\varphi}$$

4. Q_k meghatározása

$$Q_k=\vec F_k\cdot\vec\beta_C$$



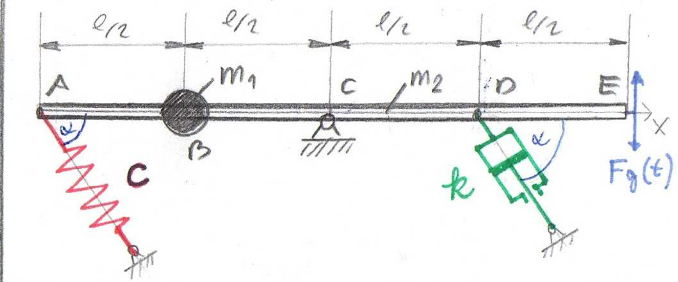
$$v_C=\ell\cdot\dot\varphi$$
$$\vec v_C=\ell\cdot\dot\varphi(\cos\beta\vec i+\sin\beta\vec j)$$
$$v_{dug}=\ell\cdot\dot\varphi\cdot\cos\beta$$
$$\vec v_{dug}=\ell\cdot\dot\varphi\cos\beta\cdot\vec i$$
$$\vec F_k=-k\cdot\vec v_{dug}=-k\cdot\ell\cdot\dot\varphi\cdot\cos\beta\cdot\vec i$$
$$\vec\beta_C=\frac{\partial\vec v_C}{\partial\dot\varphi}=\frac{\partial\left(\ell\cdot\dot\varphi\cdot(\cos\beta\vec i+\sin\beta\vec j)\right)}{\partial\dot\varphi}=$$
$$=\ell(\cos\beta\vec i+\sin\beta\vec j)$$
$$Q_k=-k\ell\dot\varphi\cos\beta\vec i\cdot\ell\cdot(\cos\beta\vec i+\sin\beta\vec j)=$$
$$=-k\ell^2\cos^2\beta\dot\varphi$$

$$\boxed{Q_k=-k\ell^2\cos^2\beta\dot\varphi}$$

5. Q_g meghatározása

$$Q_g=\vec F_g(t)\cdot\vec\beta_C$$
$$\vec F_g(t)=F_g(t)\vec i=F_{g0}\sin(\omega t+\varepsilon)\vec i$$
$$\vec\beta_C=\frac{\partial\vec v_C}{\partial\dot\varphi}=\ell(\cos\beta\vec i+\sin\beta\vec j)$$

$$\boxed{Q_g=F_g(t)\cdot\ell\cdot\cos\beta}$$



adott: $m_1, m_2, \ell, c, k, \alpha,$
$F_g(t)=F_{g0}\sin(\omega t+\varepsilon)$

az általános koordináta:
$q=\varphi \quad \dot q=\dot\varphi(\omega)\quad \ddot q=\ddot\varphi(\varepsilon)$

Lagrange egyenlet
$$\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]-\frac{\partial E}{\partial q}=Q_c+Q_k+Q_g$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]$ meghatározása

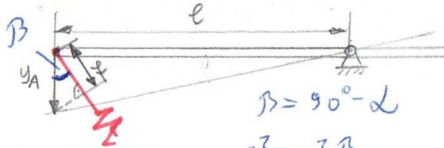
$$E=E_1+E_2=\frac{1}{2}m_1v_1^2+\frac{1}{2}J_C\omega^2=$$
$$=\frac{1}{2}m_1\cdot\left(\frac{\ell}{2}\cdot\dot\varphi\right)^2+\frac{1}{2}\left(\frac{1}{12}m_2(2\ell)^2\right)\dot\varphi^2=$$
$$=\frac{1}{2}\cdot\left(\frac{m_1}{4}+\frac{m_2}{3}\right)\cdot\ell^2\cdot\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial\dot q}\right]=\left(\frac{m_1}{4}+\frac{m_2}{3}\right)\ell^2\cdot\ddot\varphi}$$

2. $\frac{\partial E}{\partial q}=0$

3. Q_c meghatározása

$$U=\frac{1}{2}\frac{f^2}{c}=\frac{1}{2}\cdot\frac{(\ell\varphi\cos\beta)^2}{c}=$$
$$=\frac{1}{2}\cdot\frac{\ell^2\cos^2\beta}{c}\cdot\varphi^2$$



$\beta=90°-\alpha$

$$Q_c=-\frac{\partial U}{\partial\varphi}=-\frac{\ell^2\cos^2\beta}{c}\cdot\varphi$$

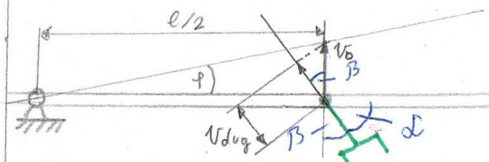
$$\boxed{Q_c=-\frac{\ell^2\cos^2\beta}{c}\cdot\varphi}$$

4. Q_k meghatározása

$\cancel{Q_k=\vec F_k\cdot\vec\beta_D}$

$\cancel{\vec F_k=-k\cdot\vec v_{dug}}$

$$v_{dug}=\frac{\ell}{2}\cdot\dot\varphi\cdot\cos\beta$$



$$\vec v_{dug}=v_{dug}\cdot(\sin\beta\vec i+\cos\beta\vec j)$$
$$\vec F_k=-k\cdot\vec v_{dug}=-k\cdot\frac{\ell}{2}\cdot\dot\varphi\cdot\cos\beta\cdot(\sin\beta\vec i+\cos\beta\vec j)$$
$$\vec\beta_D=\frac{\partial\vec v_D}{\partial\dot\varphi}=\frac{\partial\left(\frac{\ell}{2}\cdot\dot\varphi\cdot\vec j\right)}{\partial\dot\varphi}=\frac{\ell}{2}\cdot\vec j$$
$$Q_k=-k\cdot\frac{\ell}{2}\cdot\dot\varphi\cdot\cos\beta\cdot(\sin\beta\vec i+\cos\beta\vec j)\cdot\frac{\ell}{2}\cdot\vec j=$$
$$=\cancel{-k\frac{\ell^2}{2}\cos^2\beta\dot\varphi}$$

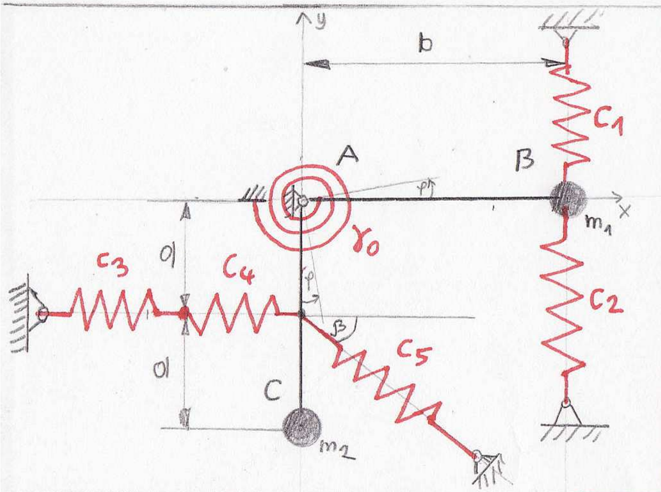
$$\boxed{Q_k=-k\cdot\frac{\ell^2}{4}\cdot\cos^2\beta\cdot\dot\varphi}$$

5. Q_g meghatározása

$$Q_g=\vec F_g\cdot\vec\beta_E$$
$$\vec F_g=F_g(t)\vec j=F_{g0}\sin(\omega t+\varepsilon)\vec j$$
$$\vec\beta_E=\frac{\partial\vec v_E}{\partial\dot\varphi}=\frac{\partial(\ell\cdot\dot\varphi\vec j)}{\partial\dot\varphi}=\ell\cdot\vec j$$
$$Q_g=F_g(t)\vec j\cdot\ell\cdot\vec j=F_g(t)\cdot\ell$$

$$\boxed{Q_g=F_g(t)\cdot\ell}$$

1.4 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



adott: $m_1, m_2, a, b, c_1, c_2, c_3, c_4, c_5, \gamma_0, \beta$

- az általános koordináta:
  $q = \varphi$ (szögelfordulás)
  $\dot q = \dot\varphi\,(\omega)$ (szögsebesség)
  $\ddot q = \ddot\varphi\,(\varepsilon)$ (szöggyorsulás)

- Lagrange egyenlet:

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1(b\dot\varphi)^2 + \frac{1}{2}m_2(2a\dot\varphi)^2 = \frac{1}{2}(m_1 b^2 + m_2 4a^2)\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = (m_1 b^2 + m_2 4a^2)\ddot\varphi}$$

2. $\frac{\partial E}{\partial q} = 0$

3. Q_c meghatározása

$$U = U_1 + U_2 + U_3 + U_4 + U_5 + U_6 = \frac{1}{2}\cdot\frac{(b\varphi)^2}{c_1} + \frac{1}{2}\cdot\frac{(b\varphi)^2}{c_2} + \frac{1}{2}\cdot\frac{(a\varphi)^2}{c_3+c_4} + \frac{1}{2}\cdot\frac{(a\varphi\cos\beta)^2}{c_5} + \frac{1}{2}\cdot\frac{\varphi^2}{\gamma_0} =$$

$$= \frac{1}{2}\cdot\left[\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right]\varphi^2$$

$$Q_c = -\frac{\partial U}{\partial \varphi}$$

$$\boxed{Q_c = -\left(\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right)\varphi}$$

- behelyettesítés a Lagrange egyenletbe

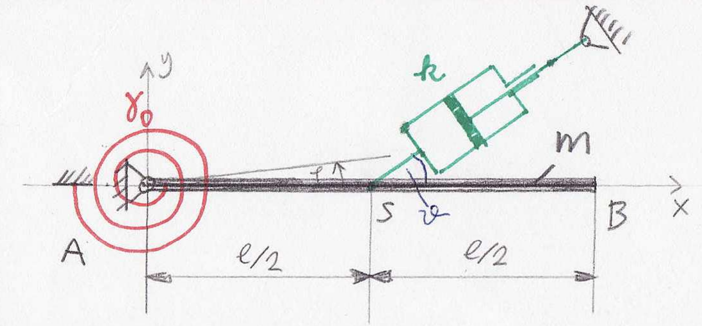
$$(m_1 b^2 + m_2 4a^2)\ddot\varphi - 0 = -\left(\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right)\varphi$$

- rendezve az egyenletet

$$\underbrace{(m_1 b^2 + m_2 4a^2)}_{m_{red}}\ddot\varphi + \underbrace{\left(\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right)}_{\frac{1}{c_{red}}}\varphi = 0$$

- a redukált rezgőrendszer:

$$m_{red}\ddot q + \frac{1}{c_{red}} q = 0$$



adott: $m, \ell, k, \gamma_0, \vartheta$

- az általános koordináta
  $q = \varphi \quad \dot q = \dot\varphi(\omega) \quad \ddot q = \ddot\varphi(\varepsilon)$

- Lagrange egyenlet:

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c + Q_k$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2}J_a\omega^2 = \frac{1}{2}\left(\frac{1}{3}m\ell^2\right)\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = \frac{1}{3}m\ell^2\ddot\varphi}$$

2. $\frac{\partial E}{\partial q} = 0$

3. Q_c meghatározása

$$U = \frac{1}{2}\cdot\frac{\varphi^2}{\gamma_0} \qquad Q_c = -\frac{\partial U}{\partial \varphi}$$

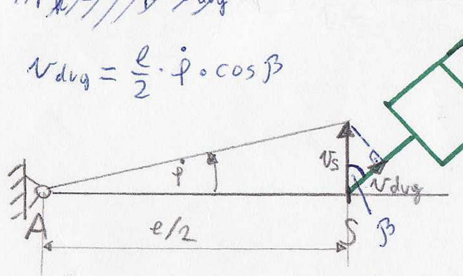
$$\boxed{Q_c = -\frac{1}{\gamma_0}\cdot\varphi}$$

4. Q_k meghatározása

$$\cancel{Q_k = \vec F_k \cdot \vec\beta}$$

$$\cancel{\vec F_k = -k\cdot\vec v_{dug}}$$

$$v_{dug} = \frac{\ell}{2}\cdot\dot\varphi\cdot\cos\beta$$



$$\cancel{\vec v_{dug} = v_{dug}(\sin\beta\,\vec i + \cos\beta\,\vec j)}$$

$$D = \frac{1}{2}k v_d^2 = \frac{1}{2}k(v_S\cdot\cos\beta)^2 = \frac{1}{2}k\cdot\left(\frac{\ell}{2}\dot\varphi\cos\beta\right)^2 = \frac{1}{2}k\cdot\frac{\ell^2}{4}\cdot\dot\varphi^2\cos^2\beta$$

$$Q_k = -\frac{\partial D}{\partial \dot\varphi} = -k\cdot\frac{\ell^2}{4}\cdot\cos^2\beta\cdot\dot\varphi$$

$$\boxed{Q_k = -k\frac{\ell^2}{4}\cdot\cos^2\beta\cdot\dot\varphi}$$

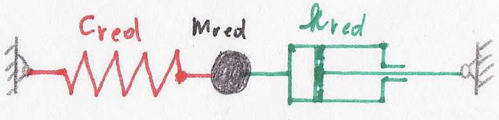
- behelyettesítés a Lagrange egyenletbe

$$\frac{1}{3}m\ell^2\ddot\varphi - 0 = -\frac{1}{\gamma_0}\cdot\varphi - k\frac{\ell^2}{4}\cos^2\beta\cdot\dot\varphi$$

- rendezve az egyenletet

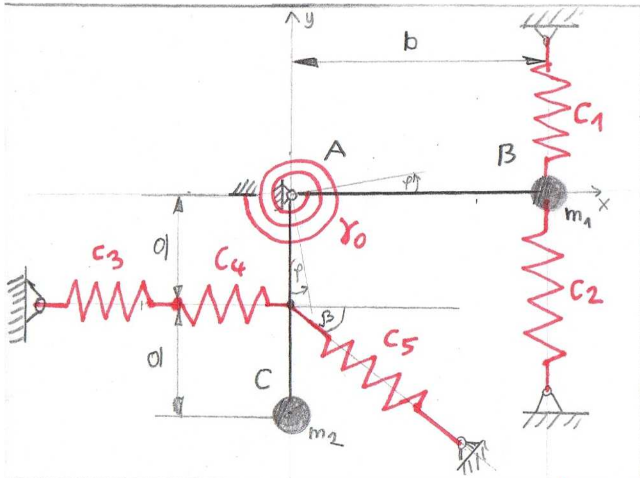
$$\underbrace{\frac{1}{3}m\ell^2}_{m_{red}}\ddot\varphi + \underbrace{k\frac{\ell^2}{4}\cos^2\beta}_{k_{red}}\dot\varphi + \underbrace{\frac{1}{\gamma_0}}_{\frac{1}{c_{red}}}\cdot\varphi = 0$$

- a redukált rezgőrendszer



$$m_{red}\ddot\varphi + k_{red}\dot\varphi + \frac{1}{c_{red}}\varphi = 0$$

1.4 EGYSZABADSÁGFOKÚ REZGŐRENDSZER MOZGÁSEGYENLETE



adott: $m_1, m_2, a, b, c_1, c_2, c_3, c_4, c_5, \gamma_0, \beta$

• az általános koordináta:

$q = \varphi$ (szögelfordulás)

$\dot q = \dot\varphi(\omega)$ (szögsebesség)

$\ddot q = \ddot\varphi(\varepsilon)$ (szöggyorsulás)

• Lagrange egyenlet:

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1(b\dot\varphi)^2 + \frac{1}{2}m_2(2a\dot\varphi)^2 = \frac{1}{2}\left(m_1 b^2 + m_2 4a^2\right)\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = \left(m_1 b^2 + m_2 4a^2\right)\ddot\varphi}$$

2. $\frac{\partial E}{\partial q} = 0$

3. Q_c meghatározása

$$U = U_1 + U_2 + U_3 + U_4 + U_5 + U_6 = \frac{1}{2}\cdot\frac{(b\varphi)^2}{c_1} + \frac{1}{2}\cdot\frac{(b\varphi)^2}{c_2} + \frac{1}{2}\cdot\frac{(a\varphi)^2}{c_3+c_4} + \frac{1}{2}\cdot\frac{(a\varphi\cos\beta)^2}{c_5} + \frac{1}{2}\cdot\frac{\varphi^2}{\gamma_0} =$$

$$= \frac{1}{2}\cdot\left[\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right]\varphi^2$$

$$Q_c = -\frac{\partial U}{\partial \varphi}$$

$$\boxed{Q_c = -\left(\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right)\varphi}$$

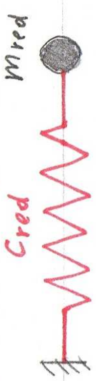
• behelyettesítés a Lagrange egyenletbe

$$\left(m_1 b^2 + m_2 4a^2\right)\ddot\varphi - 0 = -\left(\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right)\varphi$$

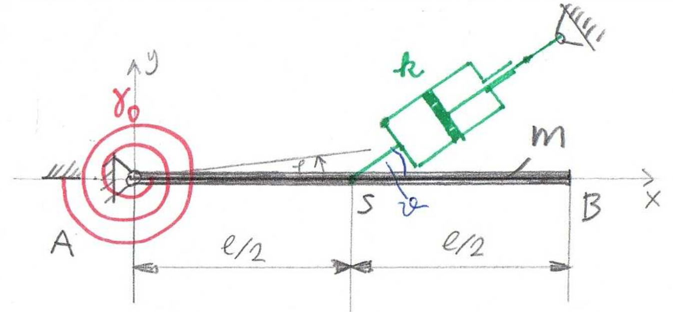
• rendezve az egyenletet

$$\underbrace{\left(m_1 b^2 + m_2 4a^2\right)}_{m_{red}}\ddot\varphi + \underbrace{\left(\frac{b^2}{c_1} + \frac{b^2}{c_2} + \frac{a^2}{c_3+c_4} + \frac{a^2\cos^2\beta}{c_5} + \frac{1}{\gamma_0}\right)}_{\frac{1}{c_{red}}}\varphi = 0$$

• a redukált rezgőrendszer:



$$m_{red}\,\ddot q + \frac{1}{c_{red}}\,q = 0$$



adott: $m, l, k, \gamma_0, \vartheta$

• az általános koordináta

$q = \varphi$ $\quad \dot q = \dot\varphi(\omega)$ $\quad \ddot q = \ddot\varphi(\varepsilon)$

• Lagrange egyenlet:

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c + Q_k$$

1. $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2}J_a\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = \frac{1}{3}ml^2\ddot\varphi}$$

2. $\frac{\partial E}{\partial q} = 0$

3. Q_c meghatározása

$$U = \frac{1}{2}\cdot\frac{\varphi^2}{\gamma_0} \qquad Q_c = -\frac{\partial U}{\partial \varphi}$$

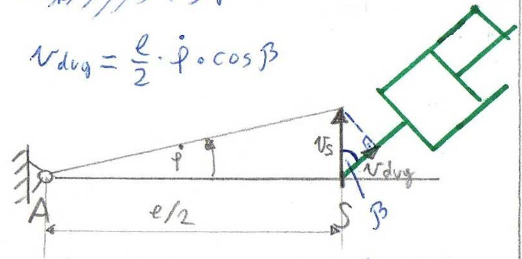
$$\boxed{Q_c = -\frac{1}{\gamma_0}\cdot\varphi}$$

4. Q_k meghatározása

$\cancel{Q_k = \vec F_k\cdot\vec\beta_s}$

$\cancel{\vec F_k = -k\cdot\vec v_{dug}}$

$$v_{dug} = \frac{l}{2}\cdot\dot\varphi\cdot\cos\beta$$



$\cancel{\vec v_{dug} = v_{dug}(\sin\beta\,\vec i + \cos\beta\,\vec j)}$

$$\vec F_k = -k\cdot\vec v_{dug} = -k\cdot\frac{l}{2}\cdot\dot\varphi\cdot\cos\beta\cdot(\sin\beta\,\vec i + \cos\beta\,\vec j) =$$

$$= -k\cdot\frac{l}{2}\cdot\dot\varphi\left(\cos\beta\sin\beta\,\vec i + \cos^2\beta\,\vec j\right) =$$

$$\vec\beta_s = \frac{\partial \vec v_s}{\partial \dot\varphi} = \frac{\partial\left(\frac{l}{2}\cdot\dot\varphi\,\vec j\right)}{\partial \dot\varphi} = \frac{l}{2}\vec j$$

$$Q_k = -k\cdot\frac{l}{2}\cdot\dot\varphi\left(\cos\beta\sin\beta\,\vec i + \cos^2\beta\,\vec j\right)\cdot\frac{l}{2}\vec j =$$

$$= -k\cdot\frac{l}{2}\cdot\dot\varphi\cdot\cos^2\beta\cdot\frac{l}{2} = -k\cdot\frac{l^2}{4}\cdot\cos^2\beta\cdot\dot\varphi$$

$$\boxed{Q_k = -k\,\frac{l^2}{4}\cdot\cos^2\beta\cdot\dot\varphi}$$

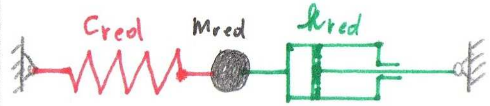
• behelyettesítés a Lagrange egyenletbe

$$\frac{1}{3}ml^2\ddot\varphi - 0 = -\frac{1}{\gamma_0}\varphi - k\,\frac{l^2}{4}\cos^2\beta\cdot\dot\varphi$$

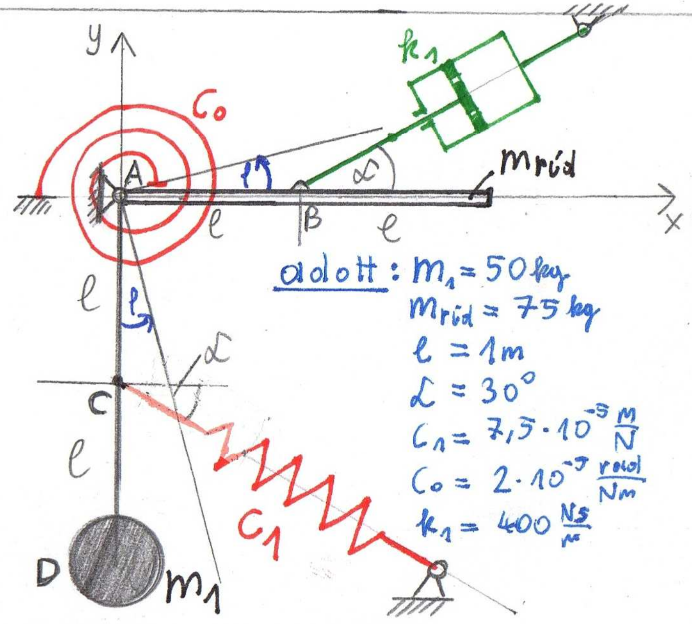
• rendezve az egyenletet

$$\underbrace{\frac{1}{3}ml^2}_{M_{red}}\ddot\varphi + \underbrace{k\,\frac{l^2}{4}\cdot\cos^2\beta}_{k_{red}}\dot\varphi + \underbrace{\frac{1}{\gamma_0}}_{\frac{1}{c_{red}}}\cdot\varphi = 0$$

• a redukált rezgőrendszer



$$m_{red}\,\ddot\varphi + k_{red}\,\dot\varphi + \frac{1}{c_{red}}\,\varphi = 0$$



adott: $m_1 = 50\,kg$
$m_{rúd} = 75\,kg$
$\ell = 1\,m$
$\alpha = 30°$
$c_1 = 7{,}5\cdot 10^{-5}\,\frac{m}{N}$
$c_0 = 2\cdot 10^{-5}\,\frac{rad}{Nm}$
$k_1 = 400\,\frac{Ns}{m}$

- általános koordináta:

$q = \varphi$ – szögelfordulás
$\dot q = \dot\varphi = \omega$ – szögsebesség
$\ddot q = \ddot\varphi = \varepsilon$ – szöggyorsulás

- Lagrange egyenlet:

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c + Q_k$$

- Vissza helyettesítve a Lagrange egyenletbe:

$$m_{red}\,\ddot\varphi - 0 = -\frac{1}{c_{red}}\varphi - k_{red}\,\dot\varphi$$

$$m_{red}\,\ddot\varphi + k_{red}\,\dot\varphi + \frac{1}{c_{red}}\varphi = 0$$

$$300\,\ddot\varphi + 100\,\dot\varphi + 60000\,\varphi = 0$$

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c + Q_k$$

$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2} m_1 v_D^2 + \frac{1}{2} J_a \omega^2$$

$$E = \frac{1}{2} m_1 \dot x_D^2 + \frac{1}{2}\left[\frac{1}{3} m_{rúd} (2\ell)^2\right]\dot\varphi^2$$

$$E = \frac{1}{2} m_1 (2\ell\dot\varphi)^2 + \frac{1}{2}\left[\frac{1}{3} m_{rúd}\, 4\ell^2\right]\dot\varphi^2$$

$$E = \frac{1}{2}\left[m_1 4\ell^2 + \frac{4}{3} m_{rúd}\,\ell^2\right]\dot\varphi^2$$

$$\frac{\partial E}{\partial \dot q} = \frac{\partial}{\partial \dot\varphi}\frac{1}{2}\left[m_1 4\ell^2 + \frac{4}{3} m_{rúd}\,\ell^2\right]\dot\varphi^2 = \left[m_1 4\ell^2 + \frac{4}{3} m_{rúd}\,\ell^2\right]\dot\varphi$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot q}\right) = \underbrace{\left[m_1 4\ell^2 + \frac{4}{3} m_{rúd}\,\ell^2\right]}_{m_{red}}\ddot\varphi}$$

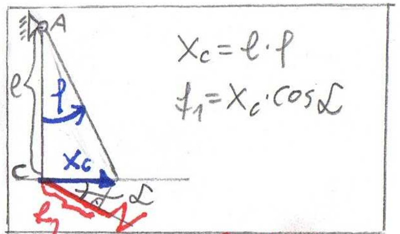
$$m_{red} = m_1 4\ell^2 + \frac{4}{3} m_{rúd}\,\ell^2 = 50\cdot 4\cdot 1^2 + \frac{4}{3}75\cdot 1^2 = 300\,kgm^2$$

Q_c meghatározása

$$U = U_0 + U_1$$

$$U = \frac{1}{2}\frac{\varphi^2}{c_0} + \frac{1}{2}\frac{f_1^2}{c_1}$$

$$U = \frac{1}{2}\frac{\varphi^2}{c_0} + \frac{1}{2}\frac{(\ell\varphi\cos\alpha)^2}{c_1}$$



$x_c = \ell\cdot\varphi$
$f_1 = x_c\cdot\cos\alpha$

$$U = \frac{1}{2}\left(\frac{1}{c_0} + \frac{\ell^2\cos^2\alpha}{c_1}\right)\varphi^2$$

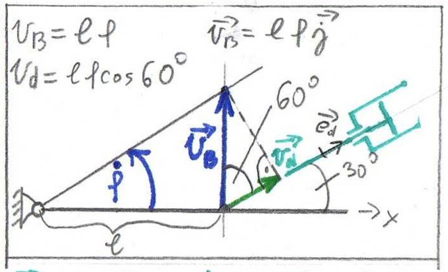
$$Q_c = -\frac{\partial U}{\partial q} = -\frac{\partial U}{\partial \varphi} = -\left(\frac{1}{c_0} + \frac{\ell^2\cos^2\alpha}{c_1}\right)\varphi$$

$$\boxed{Q_c = -\underbrace{\left(\frac{1}{c_0} + \frac{\ell^2\cos^2\alpha}{c_1}\right)}_{\frac{1}{c_{red}}}\varphi}$$

$$\frac{1}{c_{red}} = \frac{1}{c_0} + \frac{\ell^2\cos^2\alpha}{c_1} = \frac{1}{2\cdot 10^{-5}} + \frac{1^2\cos^2 30°}{7{,}5\cdot 10^{-5}} = 6\cdot 10^5\,Nm$$

$$c_{red} = \frac{1}{6\cdot 10^5} = 1{,}6\cdot 10^{-6}\,\frac{1}{Nm}$$

Q_k meghatározása



$$\vec v_d = v_d\,\vec e_d = \ell\dot\varphi\cos 60°(\cos 30°\,\vec i + \sin 30°\,\vec j)$$

$$\vec v_d = \ell\dot\varphi\frac{1}{2}\left(\frac{\sqrt3}{2}\vec i + \frac{1}{2}\vec j\right)$$

$$\vec v_d = \frac{\sqrt3}{4}\ell\dot\varphi\,\vec i + \frac{1}{4}\ell\dot\varphi\,\vec j$$

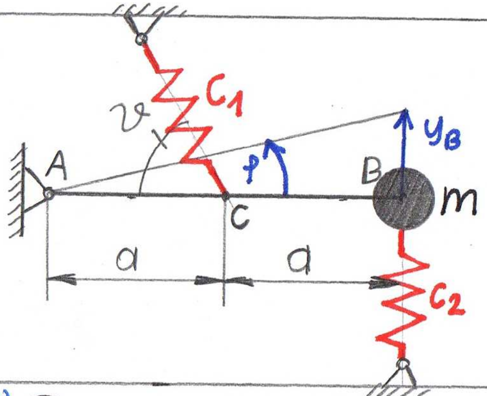
$$\vec F_k = -k\,\vec v_d = -k\left(\frac{\sqrt3}{4}\ell\vec i + \frac{1}{4}\ell\vec j\right)\dot\varphi$$

$$\vec\beta_B = \frac{\partial \vec v_B}{\partial\dot\varphi} = \frac{\partial\,\ell\dot\varphi\vec j}{\partial\dot\varphi} = \ell\vec j$$

$$Q_k = \vec F_k\,\vec\beta_B = -k\left(\frac{\sqrt3}{4}\ell\vec i + \frac{1}{4}\ell\vec j\right)\dot\varphi\cdot\ell\vec j = -k\frac{1}{4}\ell^2\dot\varphi$$

$$\boxed{Q_k = -\underbrace{\frac{1}{4}k\ell^2}_{k_{red}}\dot\varphi}$$

$$k_{red} = -\frac{1}{4}k\ell^2 = -\frac{1}{4}\cdot 400\cdot 1^2 = 100\,Nms$$



a) 1. m_{red} meghatározása

$$E=\frac{1}{2}m v_B^2=\frac{1}{2}m\dot{y}_B^2=$$
$$=\frac{1}{2}m(2a\dot{\varphi})^2=\frac{1}{2}m4a^2\dot{\varphi}^2$$

$$\frac{\partial E}{\partial \dot{q}}=\frac{\partial E}{\partial \dot{\varphi}}=\cancel{2}\cdot\frac{1}{\cancel{2}}4ma^2\dot{\varphi}=4ma^2\dot{\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)=\frac{d}{dt}4ma^2\dot{\varphi}=4ma^2\ddot{\varphi}$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)=4ma^2\ddot{\varphi}=m_{red}\ddot{\varphi}}$$

2. $\frac{\partial E}{\partial q}=\frac{\partial E}{\partial \varphi}=0$

3. C_{red} meghatározása

$$U=U_1+U_2=\frac{1}{2}\frac{f_1^2}{C_1}+\frac{1}{2}\frac{f_2^2}{C_2}=$$

• adott: $a, \vartheta, m, C_1, C_2$

• általános koordináta:

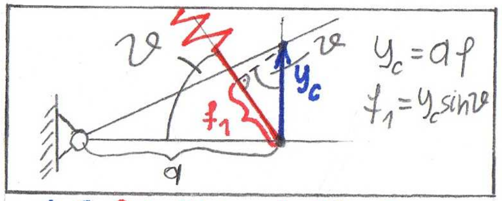
a) $q=\varphi$, $\dot{q}=\dot{\varphi}=\omega$, $\ddot{q}=\ddot{\varphi}=\varepsilon$

b) $q=y_B$, $\dot{q}=\dot{y}_B=v_B$, $\ddot{q}=\ddot{y}_B=a_B$

• Lagrange egyenlet:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)-\frac{\partial E}{\partial q}=Q_c$$

$$=\frac{1}{2}\frac{(a\varphi\sin\vartheta)^2}{C_1}+\frac{1}{2}\frac{(2a\varphi)^2}{C_2}=$$



$$=\frac{1}{2}\left[\frac{a^2\sin^2\vartheta}{C_1}+\frac{4a^2}{C_2}\right]\varphi^2$$

$$Q_c=-\frac{\partial U}{\partial q}=-\frac{\partial U}{\partial \varphi}\frac{1}{2}\left(\frac{a^2\sin^2\vartheta}{C_1}+\frac{4a^2}{C_2}\right)\varphi^2=$$
$$=-\left(\frac{a^2\sin^2\vartheta}{C_1}+\frac{4a^2}{C_2}\right)\varphi$$

$$\boxed{Q_c=-\left(\frac{a^2\sin^2\vartheta}{C_1}+\frac{4a^2}{C_2}\right)\varphi=-\frac{1}{C_{red}}\varphi}$$

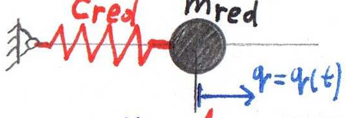
• behelyettesítve a Lagrange egyenletbe:

$$4ma^2\ddot{\varphi}-0=-\left(\frac{a^2\sin^2\vartheta}{C_1}+\frac{4a^2}{C_2}\right)\varphi$$

• átrendezve az egyenletet:

$$\underbrace{4ma^2}_{m_{red}}\ddot{\varphi}+\underbrace{\left(\frac{a^2\sin^2\vartheta}{C_1}+\frac{4a^2}{C_2}\right)}_{\frac{1}{C_{red}}}\varphi=0$$

• a redukált rezgőrendszer:



$$m_{red}\ddot{q}+\frac{1}{C_{red}}q=0$$

b) 1. m_{red} meghatározása

$$E=\frac{1}{2}m v_B^2=\frac{1}{2}m\dot{y}_B^2$$

$$\frac{\partial E}{\partial \dot{q}}=\frac{\partial E}{\partial \dot{y}_B}=2\cdot\frac{1}{2}m\dot{y}_B=m\dot{y}_B$$

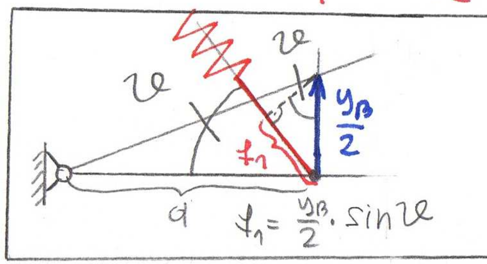
$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)=\frac{d}{dt}m\dot{y}_B=m\ddot{y}_B$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)=m\ddot{y}_B=m_{red}\ddot{y}_B}$$

2. $\frac{\partial E}{\partial q}=\frac{\partial E}{\partial y_B}=0$

3. C_{red} meghatározása

$$U=U_1+U_2=\frac{1}{2}\frac{f_1^2}{C_1}+\frac{1}{2}\frac{f_2^2}{C_2}=$$



$$=\frac{1}{2}\frac{\left(\frac{y_B}{2}\cdot\sin\vartheta\right)^2}{C_1}+\frac{1}{2}\frac{y_B^2}{C_2}=$$

$$=\frac{1}{2}\left[\frac{\sin^2\vartheta}{4C_1}+\frac{1}{C_2}\right]y_B^2$$

$$Q_c=-\frac{\partial U}{\partial q}=-\frac{\partial U}{\partial y_B}\frac{1}{2}\left[\frac{\sin^2\vartheta}{4C_1}+\frac{1}{C_2}\right]y_B^2=$$
$$=-\left[\frac{\sin^2\vartheta}{4C_1}+\frac{1}{C_2}\right]y_B$$

$$\boxed{Q_c=-\left[\frac{\sin^2\vartheta}{4C_1}+\frac{1}{C_2}\right]y_B=-\frac{1}{C_{red}}y_B}$$

• behelyettesítve a Lagrange egyenletbe:

$$m\ddot{y}_B-0=-\left[\frac{\sin^2\vartheta}{4C_1}+\frac{1}{C_2}\right]y_B$$

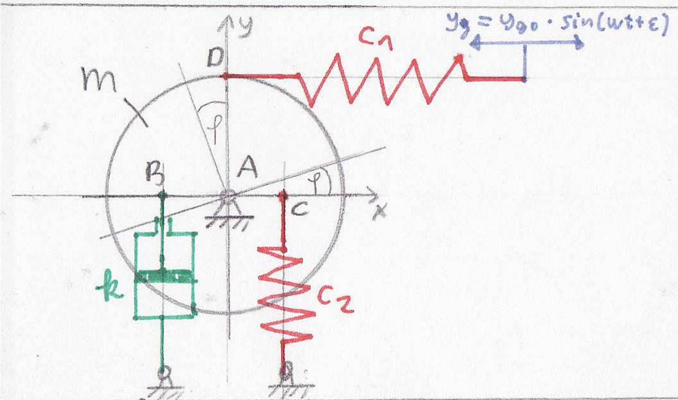
• átrendezve az egyenletet:

$$\underbrace{m}_{m_{red}}\ddot{y}_B+\underbrace{\left[\frac{\sin^2\vartheta}{4C_1}+\frac{1}{C_2}\right]}_{\frac{1}{C_{red}}}y_B=0$$

• a redukált rezgőrendszer:

$$m_{red}\ddot{q}+\frac{1}{C_{red}}q=0$$

~~KH~~ 1.5 ÚTGERJESZTÉS



adott: $m, R, k, c_1, c_2, y_g(t)$

$y_g(t) = y_{g0} \cdot \sin(\omega t + \varepsilon)$

az általános koordináta:

$q = \varphi \quad \dot q = \dot\varphi(\omega) \quad \ddot q = \ddot\varphi(\varepsilon)$

Lagrange egyenlet

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$$

① $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2} \cdot J_a \cdot \omega^2 = \frac{1}{2} \cdot \left(\frac{1}{2} m R^2\right) \dot\varphi^2$$

$$\boxed{\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = \frac{1}{2} m R^2 \ddot\varphi}$$

② $\frac{\partial E}{\partial q} = 0$

③ ④ Q_c és Q_g meghatározása

$$U = \frac{1}{2}\frac{f_1^2}{c_1} + \frac{1}{2}\frac{f_2^2}{c_2} =$$

$$= \frac{1}{2}\frac{(R\varphi - y_g)^2}{c_1} + \frac{1}{2} \cdot \frac{\left(\frac{R}{2}\varphi\right)^2}{c_2}$$

$$Q_c + Q_g = -\frac{\partial U}{\partial \varphi} = -\frac{R\varphi - y_g}{c_1} \cdot R - \frac{R^2}{4c_2}\varphi = -\frac{R^2\varphi}{c_1} + \frac{R y_g}{c_1} - \frac{R^2}{4c_2}\varphi$$

$$= \underbrace{-\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)\varphi}_{Q_c} + \underbrace{\frac{R}{c_1} y_g(t)}_{Q_g}$$

$$\boxed{Q_c = -\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)\varphi}$$

$$\boxed{Q_g = \frac{R}{c_1} y_g(t)}$$

⑤ Q_k meghatározása

$$D = \frac{1}{2} k v_d^2 = \frac{1}{2} k \cdot v_B^2 =$$

$$= \frac{1}{2} k \cdot \left(\frac{R}{2} \cdot \dot\varphi\right)^2 = \frac{1}{2} k \cdot \frac{R^2}{4} \cdot \dot\varphi^2$$

$$Q_k = -\frac{\partial D}{\partial \dot\varphi} = -k \frac{R^2}{4} \cdot \dot\varphi$$

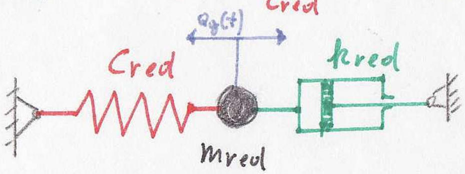
$$\boxed{Q_k = -k \frac{R^2}{4} \dot\varphi}$$

• behelyettesítés Lagrange egyenletbe

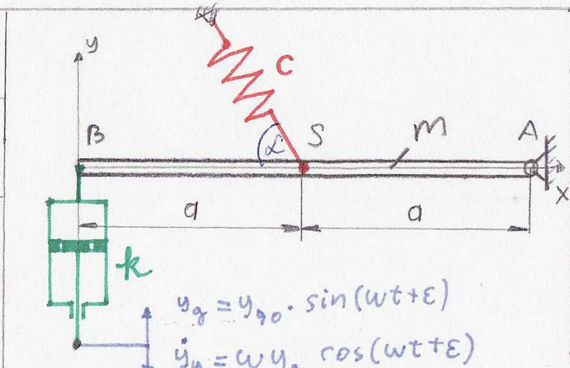
$$\frac{1}{2} m R^2 \ddot\varphi - 0 = -\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)\varphi - k\frac{R^2}{4}\dot\varphi + \frac{R}{c_1} y_g(t)$$

• rendezzük az egyenletet:

$$\underbrace{\frac{1}{2} m R^2}_{m_{red}} \ddot\varphi + \underbrace{k \frac{R^2}{4}}_{k_{red}} \cdot \dot\varphi + \underbrace{\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)}_{\frac{1}{c_{red}}} \varphi = \underbrace{\frac{R}{c_1} y_g(t)}_{Q_g(t)}$$



$$m_{red}\ddot q + k_r \dot q + \frac{1}{c_{rd}} q = Q_g(t)$$



$$y_g = y_{g0} \cdot \sin(\omega t + \varepsilon)$$

$$\dot y_g = \omega y_{g0} \cos(\omega t + \varepsilon)$$

adott: $m, a, R, k, c, y_g(t), \alpha$

az általános koordináta

$q = \varphi \quad \dot q = \dot\varphi(\omega) \quad \ddot q = \ddot\varphi(\varepsilon)$

Lagrange egyenlet:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot q}\right) - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$$

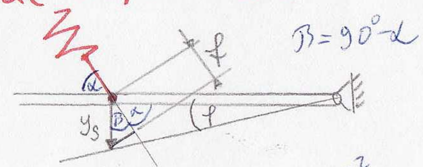
① $\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right]$ meghatározása

$$E = \frac{1}{2} J_a \omega^2 = \frac{1}{2} \cdot \left(\frac{1}{3} m (2a)^2\right) \dot\varphi^2$$

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot q}\right] = \frac{4}{3} m a^2 \ddot\varphi$$

② $\frac{\partial E}{\partial q} = 0$

③ Q_c meghatározása



$\beta = 90° - \alpha$

$$U = \frac{1}{2} \cdot \frac{f^2}{c} = \frac{1}{2} \cdot \frac{(a\varphi \cos\beta)^2}{c} =$$

$$= \frac{1}{2} \cdot \frac{a^2 \cos^2\beta}{c} \varphi^2$$

$$Q_c = -\frac{\partial U}{\partial \varphi} = -\frac{a^2\cos^2\beta}{c}\varphi$$

$$\boxed{Q_c = -\frac{a^2\cos^2\beta}{c}\varphi}$$

④ ⑤ Q_k és Q_g meghatározása

$$D = \frac{1}{2} k (v_B - v_g)^2 =$$

$$= \frac{1}{2} k (2a\dot\varphi - \dot y_g)^2$$

$$Q_k + Q_g = -\frac{\partial D}{\partial \dot\varphi} = -k(2a\dot\varphi - \dot y_g) \cdot 2a$$

$$= \underbrace{-k 4a^2 \dot\varphi}_{Q_k} + \underbrace{2ak\dot y_g}_{Q_g}$$

$$\boxed{Q_k = -4a^2 k \dot\varphi}$$

$$\boxed{Q_g = 2ak\dot y_g(t)}$$

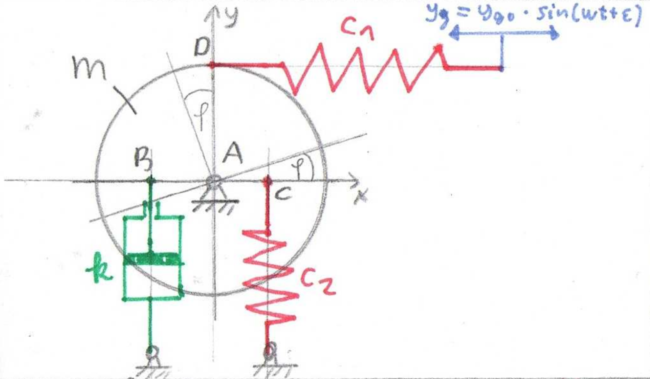
behelyettesítés Lagrange egyenletbe:

$$\frac{4}{3} m a^2 \ddot\varphi - 0 = -\frac{a^2\cos^2\beta}{c}\varphi - 4a^2 k\dot\varphi + 2ak\dot y_g$$

rendezzük az egyenletet:

$$\underbrace{\frac{4}{3} m a^2}_{m_{red}} \ddot\varphi + \underbrace{4a^2 k}_{k_{red}} \dot\varphi + \underbrace{\frac{a^2\cos^2\beta}{c}}_{\frac{1}{c_{red}}} \varphi = \underbrace{2ak\dot y_g}_{Q_g(t)}$$

1.5 ÚTGERJESZTÉS



adott: $m, R, k, c_1, c_2, y_g(t)$

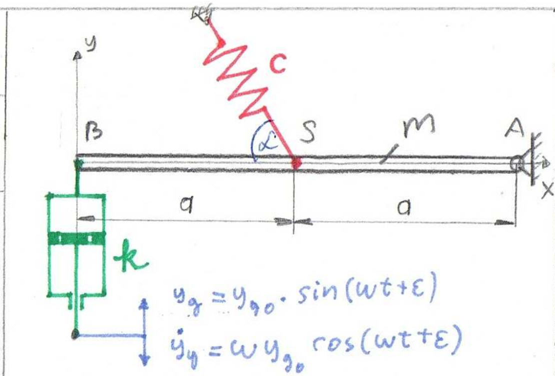
$y_g(t) = y_{g_0} \cdot \sin(\omega t + \varepsilon)$

az általános koordináta:

$q = \varphi \quad \dot{q} = \dot{\varphi}(\omega) \quad \ddot{q} = \ddot{\varphi}(\varepsilon)$

Lagrange egyenlet

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right) - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$$



adott: $m, a, R, k, c, y_g(t), \alpha$

az általános koordináta

$q = \varphi \quad \dot{q} = \dot{\varphi}(\omega) \quad \ddot{q} = \ddot{\varphi}(\varepsilon)$

Lagrange egyenlet:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right) - \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$$

1. $\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)$ meghatározása

$$E = \frac{1}{2} \cdot J_a \cdot \omega^2 = \frac{1}{2} \cdot \left(\frac{1}{2} m R^2\right) \dot{\varphi}^2$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right) = \frac{1}{2} m R^2 \ddot{\varphi}}$$

2. $\frac{\partial E}{\partial q} = 0$

3. 4. Q_c és Q_g meghatározása

$$U = \frac{1}{2}\frac{f_1^2}{c_1} + \frac{1}{2}\frac{f_2^2}{c_2} =$$

$$= \frac{1}{2}\frac{(R\varphi - y_g)^2}{c_1} + \frac{1}{2} \cdot \frac{\left(\frac{R}{2}\varphi\right)^2}{c_2}$$

$$Q_c + Q_g = -\frac{\partial U}{\partial \varphi} = -\frac{R\varphi - y_g}{c_1} \cdot R - \frac{R^2}{4c_2}\varphi = -\frac{R^2\varphi}{c_1} + \frac{R y_g}{c_1} - \frac{R^2}{4c_2}\varphi$$

$$= \underbrace{-\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)\varphi}_{Q_c} + \underbrace{\frac{R}{c_1} y_g(t)}_{Q_g}$$

$$\boxed{Q_c = -\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)\varphi}$$

$$\boxed{Q_g = \frac{R}{c_1} y_g(t)}$$

5. Q_k meghatározása

$$Q_k = \vec{F}_k \cdot \vec{\beta}_B$$

$$\vec{F}_k = -k \cdot \vec{v}_{dug} = -k \cdot \left(-\frac{R}{2}\dot{\varphi}\right)\vec{j}$$

$$\vec{\beta}_B = \frac{\partial \vec{v}_B}{\partial \dot{\varphi}} = \frac{\partial\left(-\frac{R}{2}\dot{\varphi}\vec{j}\right)}{\partial \dot{\varphi}} = -\frac{R}{2}\vec{j}$$

$$Q_k = -k\left(-\frac{R}{2}\dot{\varphi}\right)\vec{j}\left(-\frac{R}{2}\right)\vec{j} = -k\frac{R^2}{4}\dot{\varphi}$$

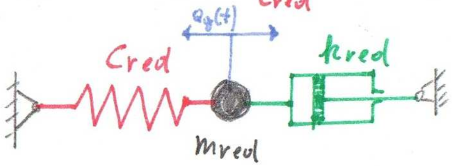
$$\boxed{Q_k = -k\frac{R^2}{4}\dot{\varphi}}$$

• behelyettesítés Lagrange egyenletbe

$$\frac{1}{2} m R^2 \ddot{\varphi} - 0 = -\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)\varphi - k\frac{R^2}{4}\dot{\varphi} + \frac{R}{c_1} y_g(t)$$

• rendezve az egyenletet:

$$\underbrace{\frac{1}{2} m R^2}_{m_{red}}\ddot{\varphi} + \underbrace{k\frac{R^2}{4}}_{k_{red}}\dot{\varphi} + \underbrace{\left(\frac{R^2}{c_1} + \frac{R^2}{4c_2}\right)}_{\frac{1}{c_{red}}}\varphi = \underbrace{\frac{R}{c_1} y_g(t)}_{Q_g(t)}$$



$$m_{red}\ddot{q} + k_r\dot{q} + \frac{1}{c_{rd}} q = Q_g(t)$$

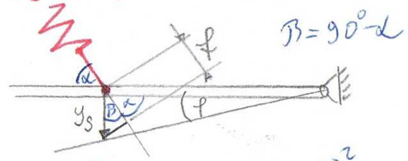
1. $\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right)$ meghatározása

$$E = \frac{1}{2} J_a \omega^2 = \frac{1}{2} \cdot \left(\frac{1}{3} m (2a)^2\right)\dot{\varphi}^2$$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}}\right) = \frac{4}{3} m a^2 \ddot{\varphi}$$

2. $\frac{\partial E}{\partial q} = 0$

3. Q_c meghatározása



$$U = \frac{1}{2} \cdot \frac{f^2}{c} = \frac{1}{2} \cdot \frac{(a\varphi\cos\beta)^2}{c} = \frac{1}{2} \cdot \frac{a^2\cos^2\beta}{c}\varphi^2$$

$$Q_c = -\frac{\partial U}{\partial \varphi} = -\frac{a^2\cos^2\beta}{c}\varphi$$

$$\boxed{Q_c = -\frac{a^2\cos^2\beta}{c}\varphi}$$

4. 5. Q_k és Q_g meghatározása

$$Q_k + Q_g = \vec{F}_k \cdot \vec{\beta}_B$$

$$F_k = -k \cdot \vec{v}_{dug} = -k(\dot{y} - \dot{y}_g)\vec{j} =$$

$$= -k(2a\dot{\varphi} - \dot{y}_g)\vec{j}$$

$$\vec{\beta}_B = \frac{\partial \vec{v}_B}{\partial \dot{\varphi}} = \frac{\partial(2a\dot{\varphi}\vec{j})}{\partial \dot{\varphi}} = 2a\vec{j}$$

$$Q_k + Q_g = (-k2a\dot{\varphi} + \dot{y}_g k)\vec{j} \cdot 2a\vec{j} =$$

$$= \underbrace{-4a^2k\dot{\varphi}}_{Q_k} + \underbrace{2ak\dot{y}_g(t)}_{Q_g}$$

$$\boxed{Q_k = -4a^2k\dot{\varphi}}$$

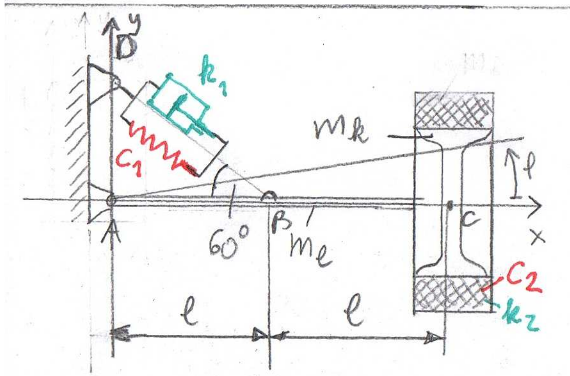
$$\boxed{Q_g = 2ak\dot{y}_g(t)}$$

behelyettesítés Lagrange egyenletbe:

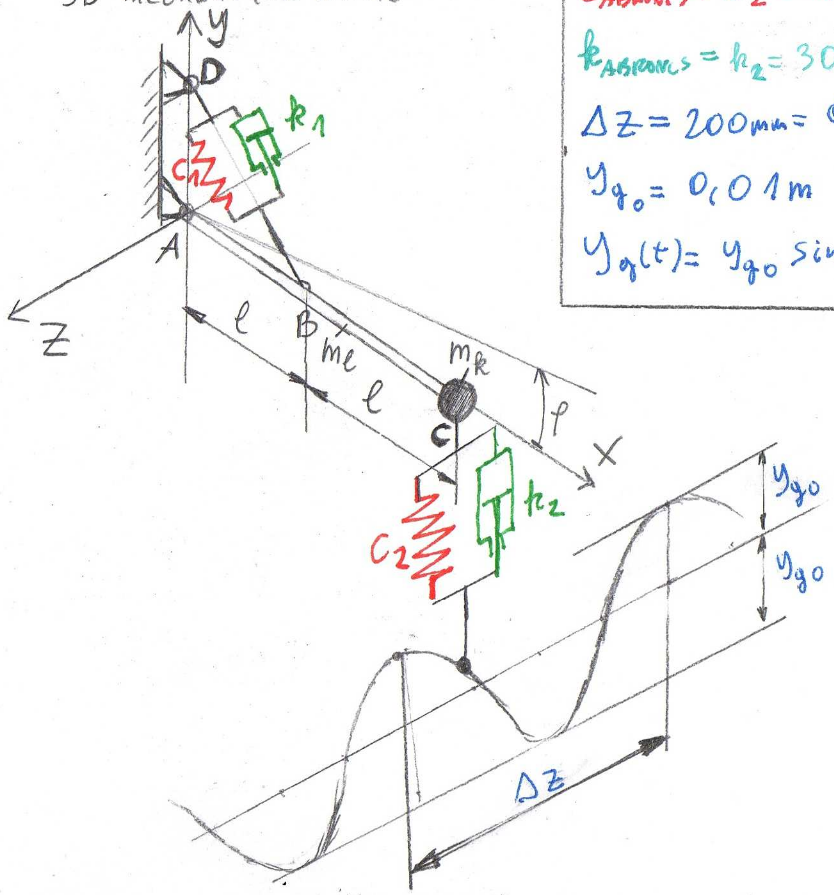
$$\frac{4}{3} m a^2 \ddot{\varphi} - 0 = -\frac{a^2\cos^2\beta}{c}\varphi - 4a^2k\dot{\varphi} + 2ak\dot{y}_g$$

rendezve az egyenletet:

$$\underbrace{\frac{4}{3} m a^2}_{m_{red}}\ddot{\varphi} + \underbrace{4a^2k}_{k_{red}}\dot{\varphi} + \underbrace{\frac{a^2\cos^2\beta}{c}}_{\frac{1}{c_{red}}}\varphi = \underbrace{2ak\dot{y}_g}_{Q_g(t)}$$



3D mechanikai modell



adott:

$v_{Autó} = 36 \frac{km}{h} = 10 \frac{m}{s}$

$m_{LENGŐKAR} = m_\ell = 9\ kg$

$m_{KERÉK} = m_k = 20\ kg$

$c_1 = 4 \cdot 10^{-4} \frac{m}{N}$

$k_1 = 200 \frac{Ns}{m}$

$c_{ABRONCS} = c_2 = 10^{-4} \frac{m}{N}$

$k_{ABRONCS} = k_2 = 30 \frac{Ns}{m}$

$\Delta z = 200 mm = 0,2 m$

$y_{g0} = 0,01 m$

$y_g(t) = y_{g0} \sin(\omega_g t)$

• általános koordináta: $q = \varphi$; $\dot{q} = \dot{\varphi} = \omega$; $\ddot{q} = \ddot{\varphi} = \varepsilon$

• gerjesztő függvény felírása:

– Ha a jármű $v_{Autó}$ sebességgel halad, akkor $T = \frac{\Delta z}{v_{Autó}}$ idő alatt teszi meg a két szomszédos emelkedés közötti utat, tehát a gerjesztés periódusideje:

$$T = \frac{\Delta z}{v_{Autó}} = \frac{0,2}{10} = 0,02 s$$

– ebből a gerjesztés frekvenciája:

$$f = \frac{1}{T} = \frac{1}{0,02} = 50 Hz$$

– innen a gerjesztés körfrekvenciája:

$$\omega_g = 2\pi f = 2\pi 50 = 314,159 \frac{rad}{s}$$

– így a gerjesztés elmozdulás függvénye:

$$y_g(t) = y_{g0} \sin(\omega_g t) = 0,01 \sin(314,159 t)$$

• A Lagrange-féle másodfajú mozgásegyenlet:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\varphi}}\right) - \underbrace{\frac{\partial E}{\partial \varphi}}_{=\emptyset} = Q_c + Q_k + Q_g$$

útgerjesztés esetén: $Q_g = Q_{gc} + Q_{gk}$

Q_{gc}: gerjesztés a rugón keresztül

Q_{gk}: gerjesztés a csillapítón keresztül

$$\underbrace{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot\varphi}\right)} = \underbrace{Q_c + Q_{yc}} + \underbrace{Q_k + Q_{yk}}$$

$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot\varphi}\right)$ meghatározása

$$E = \frac{1}{2} J_a \omega^2 + \frac{1}{2} m_k v_c^2 = \qquad v_c = \dot y_c$$

$$= \frac{1}{2}\left[\frac{1}{3} m_\ell (2\ell)^2\right]\dot\varphi^2 + \frac{1}{2} m_k (2\ell\dot\varphi)^2 =$$

$$= \frac{1}{2}\underbrace{\left\{\frac{1}{3} m_\ell 4\ell^2 + m_k 4\ell^2\right\}}_{m_{red}}\dot\varphi^2 =$$

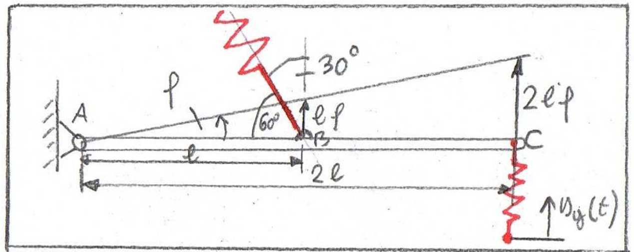
$$= \frac{1}{2} m_{red}\dot\varphi^2$$

$$\frac{d}{dt}\left[\frac{\partial E}{\partial \dot\varphi}\right] = m_{red}\ddot\varphi$$

$Q_c + Q_{yc}$ meghatározása

$$U = U_1 + U_2 = \frac{1}{2}\frac{f_1^2}{c_1} + \frac{1}{2}\frac{f_2^2}{c_2} =$$

$$= \frac{1}{2}\frac{(\ell\varphi\cos 30^\circ)^2}{c_1} + \frac{1}{2}\frac{(2\ell\varphi - y_g(t))^2}{c_2} =$$



$$(A-B)^2 = A^2 - 2AB + B^2$$

$$= \frac{1}{2}\frac{\ell^2\varphi^2\cos^2 30^\circ}{c_1} + \frac{1}{2}\frac{4\ell^2\varphi^2 - 4\ell\varphi y_g(t) + y_g(t)^2}{c_2} =$$

$$Q_c + Q_{yc} = -\frac{\partial U}{\partial \varphi} = -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}\ell^2\varphi\cos^2 30^\circ}{c_1} -$$

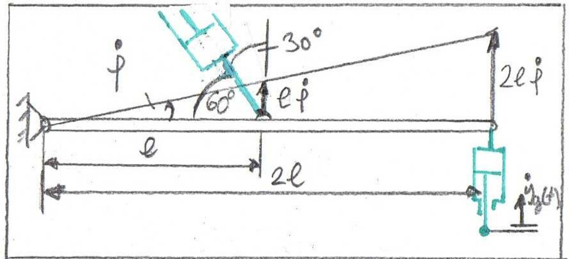
$$-\frac{1}{\cancel{2}}\frac{\cancel{8}^{4}\ell^2\varphi - \cancel{4}^{2}\ell y_g(t) + 0}{c_2} =$$

$$= \underbrace{-\underbrace{\left[\frac{\ell^2\cos^2 30^\circ}{c_1} + \frac{4\ell^2}{c_2}\right]}_{\frac{1}{c_{red}}}\varphi}_{Q_c} + \underbrace{\frac{2\ell}{c_2} y_g(t)}_{Q_{yc}}$$

$Q_k + Q_{yk}$ meghatározása

$$D = D_1 + D_2 = \frac{1}{2} k_1 v_{d1}^2 + \frac{1}{2} k_2 v_{d2}^2 =$$

$$= \frac{1}{2} k_1 (\ell\dot\varphi\cos 30^\circ)^2 + \frac{1}{2} k_2 (2\ell\dot\varphi - \dot y_g(t))^2 =$$



$$(A-B)^2 = A^2 - 2AB + B^2$$

$$= \frac{1}{2} k_1 (\ell^2\dot\varphi^2\cos^2 30^\circ) + \frac{1}{2} k_2 (4\ell^2\dot\varphi^2 - 4\ell\dot\varphi\dot y_g(t) + \dot y_g(t)^2)$$

$$Q_k + Q_{yk} = -\frac{\partial D}{\partial \dot\varphi} = -\frac{1}{\cancel{2}} k_1 (\cancel{2}\ell^2\dot\varphi\cos^2 30^\circ) -$$

$$-\frac{1}{\cancel{2}} k_2 (\cancel{8}^{4}\ell^2\dot\varphi - \cancel{4}^{2}\ell\dot y_g(t) + 0) =$$

$$= \underbrace{-\underbrace{\left[k_1\ell^2\cos^2 30^\circ + k_2\cdot 4\ell^2\right]}_{k_{red}}\dot\varphi}_{Q_k} + \underbrace{k_2 2\ell\dot y_g(t)}_{Q_{yk}}$$

• Vissza helyettesítés a Lagrange egyenletbe:

$$\underbrace{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot\varphi}\right)}_{\left[\frac{1}{3} m_e 4\ell^2 + m_k 4\ell^2\right]\ddot\varphi} = \overbrace{-\left(\frac{\ell^2\cos^2 30^\circ}{c_1}+\frac{4\ell^2}{c_2}\right)\varphi}^{Q_c} + \overbrace{\frac{2\ell}{c_2}y_g(t)}^{Q_{gc}} \overbrace{-\left[k_1\ell^2\cos^2 30^\circ + k_2 4\ell^2\right]\dot\varphi}^{Q_k} + \overbrace{k_2 2\ell\, \dot y_g(t)}^{Q_{gk}}$$

• átrendezve:

$$\underbrace{\left[\frac{1}{3} m_e 4\ell^2+m_k 4\ell^2\right]}_{m_{red}}\ddot\varphi + \underbrace{\left[k_1\ell^2\cos^2 30^\circ+4k_2\ell^2\right]}_{k_{red}}\dot\varphi + \underbrace{\left[\frac{\ell^2\cos^2 30^\circ}{c_1}+\frac{4\ell^2}{c_2}\right]}_{\frac{1}{c_{red}}}\varphi = \overbrace{\underbrace{2k_2\ell\,\dot y_g(t)}_{Q_{gk}} + \underbrace{\frac{2\ell}{c_2}y_g(t)}_{Q_{gc}}}^{Q_g}$$

• a redukált rendszer jellemzői:

$$m_{red} = \frac{1}{3} m_e 4\ell^2 + m_k 4\ell^2 = \frac{1}{3}\cdot 9\cdot 4\cdot 0{,}5^2 + 20\cdot 4\cdot 0{,}5^2 = 23\ \mathrm{kg\,m^2}$$

$$k_{red} = k_1\ell^2\cos^2 30^\circ + 4k_2\ell^2 = 200\cdot 0{,}5^2\cos^2 30^\circ + 4\cdot 30\cdot 0{,}5^2 = 67{,}5\ \mathrm{Nsm}$$

$$\frac{1}{c_{red}} = \frac{\ell^2\cos^2 30^\circ}{c_1} + \frac{4\ell^2}{c_2} = \frac{0{,}5^2\cos^2 30^\circ}{4\cdot 10^{-4}} + \frac{4\cdot 0{,}5^2}{10^{-4}} = 10468{,}75\ \mathrm{N\cdot m}$$

$$c_{red} = 10468{,}75^{-1} = 9{,}55\cdot 10^{-5}\ \frac{\mathrm{rad}}{\mathrm{N\cdot m}}$$

• az általános gerjesztőerő:

$$Q_g = 2k_2\ell \frac{d}{dt} y_{g0}\sin(\omega_g t) + \frac{2\ell}{c_2} y_{g0}\sin(\omega_g t) =$$

$$= 2k_2\ell\omega_g y_{g0}\cos(\omega_g t) + \frac{2\ell}{c_2} y_{g0}\sin(\omega_g t) =$$

$$= 2\ell y_{g0}\left(k_2\omega_g\cos(\omega_g t) + \frac{1}{c_2}\sin(\omega_g t)\right) =$$

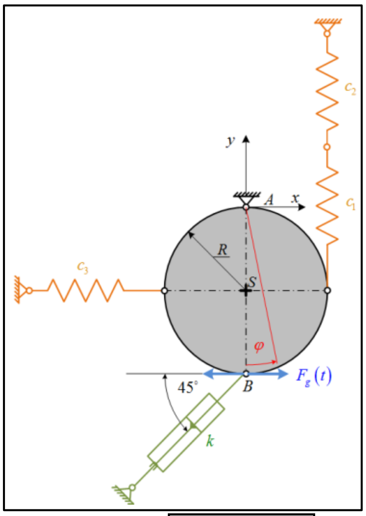
$$= 2\cdot 0{,}5\cdot 0{,}01\left(30\cdot 314{,}159\cdot\cos(314{,}159\,t) + \right.$$

$$\left. + \frac{1}{10^{-4}}\cdot\sin(314{,}159\,t)\right) = 94{,}2\cos(314{,}159\,t) +$$

$$+ 100\cdot\sin(314{,}159\,t)$$

• behelyettesítve Lagrange egyenletbe:

$$\boxed{23\ddot\varphi + 67{,}5\dot\varphi + 10468{,}75\varphi = 94{,}2\cos(314{,}159\,t) + 100\sin(314{,}159\,t)}$$



adott :

$R = 0{,}5m$

$m = 20kg$

$c1 = 2\cdot 10^{-5} m/N$

$c2 = 4\cdot 10^{-5} m/N$

$c3 = 5\cdot 10^{-5} m/N$

$k = 500 Ns/m$

$F_g(t) = F_{g0}\sin\omega t$

$F_{g0} = 200N$

$\omega = 10 rad/s$

$q = \varphi$

általános koordináta :

$q = \varphi$

$\dot q = \dot\varphi = \omega$

$\ddot q = \ddot\varphi = \varepsilon$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot q}\right) + \frac{\partial E}{\partial q} = Q_c + Q_k + Q_g$$

$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot q}\right)$ meghatározása

$$E = \frac{1}{2}J_a\omega^2 = \frac{1}{2}J_a\dot\varphi^2 = \frac{1}{2}\left(\frac{3}{2}mR^2\right)\dot\varphi^2$$

$$\frac{\partial E}{\partial \dot q} = \frac{\partial}{\partial\dot\varphi}\frac{1}{2}\left(\frac{3}{2}mR^2\right)\dot\varphi^2 = \frac{3}{2}mR^2\dot\varphi$$

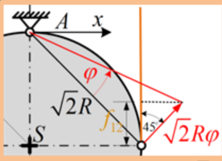
$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot q}\right) = \frac{d}{dt}\frac{3}{2}mR^2\dot\varphi = \frac{3}{2}mR^2\ddot\varphi$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot q}\right) = \frac{3}{2}mR^2\ddot\varphi = m_{red}\ddot\varphi}$$

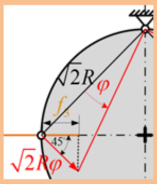
Q_c meghatározása

$$U = U_{12} + U_3 = \frac{1}{2}\frac{f_{12}^2}{c_1+c_2} + \frac{1}{2}\frac{f_3^2}{c_3}$$

rugók hosszváltozása



$$f_{12} = \sqrt{2}R\varphi\cos 45^\circ = \sqrt{2}R\varphi\frac{\sqrt{2}}{2} = R\varphi$$



$$f_3 = \sqrt{2}R\varphi\cos 45^\circ = \sqrt{2}R\varphi\frac{\sqrt{2}}{2} = R\varphi$$

$$U = \frac{1}{2}\frac{(R\varphi)^2}{c_1+c_2} + \frac{1}{2}\frac{(R\varphi)^2}{c_3} =$$

$$= \frac{1}{2}\left(\frac{R^2}{c_1+c_2} + \frac{R^2}{c_3}\right)\varphi^2$$

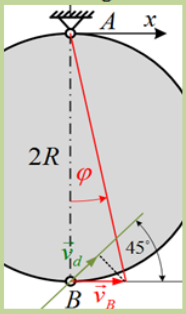
$$Q_c = -\frac{\partial U}{\partial q} = -\frac{\partial U}{\partial\varphi}\frac{1}{2}\left(\frac{R^2}{c_1+c_2} + \frac{R^2}{c_3}\right)\varphi^2 =$$

$$= -\left(\frac{R^2}{c_1+c_2} + \frac{R^2}{c_3}\right)\varphi$$

$$\boxed{Q_c = -\left(\frac{R^2}{c_1+c_2} + \frac{R^2}{c_3}\right)\varphi = -\frac{1}{c_{red}}\varphi}$$

Q_k meghatározása

dugattyú sebessége



$$v_B = 2R\dot\varphi \quad \vec v_B = 2R\dot\varphi\vec i$$

$$v_d = 2R\dot\varphi\cos 45^\circ = 2R\dot\varphi\frac{\sqrt{2}}{2} = \sqrt{2}R\dot\varphi$$

$$\vec v_d = v_d\vec e_d$$

$$\vec v_d = \sqrt{2}R\dot\varphi\left(\cos 45^\circ\,\vec i + \sin 45^\circ\,\vec j\right)$$

$$\vec v_d = \sqrt{2}R\dot\varphi\left(\frac{\sqrt{2}}{2}\vec i + \frac{\sqrt{2}}{2}\vec j\right)$$

$$\vec v_d = R\dot\varphi\vec i + R\dot\varphi\vec j$$

$Q_k = \vec F_k\cdot\vec\beta_B$ ahol:

$\vec F_k = -k\vec v_d =$ $= -k\left(R\dot\varphi\vec i + R\dot\varphi\vec j\right)$	$\vec\beta_B = \frac{\partial\vec v_B}{\partial\dot\varphi} =$ $= \frac{\partial 2R\dot\varphi\vec i}{\partial\dot\varphi} = 2R\vec i$

$$Q_k = \vec F_k\cdot\vec\beta_B = -k\left(R\dot\varphi\vec i + R\dot\varphi\vec j\right)2R\vec i =$$

$$= -kR\dot\varphi 2R = -2R^2k\dot\varphi$$

$$\boxed{Q_k = -2R^2k\dot\varphi = -k_{red}\dot\varphi}$$

Q_g meghatározása

$Q_g = \vec F_g\cdot\vec\beta_B$ ahol:

$\vec F_g = F_{g0}\sin(\omega t)\vec i$	$\vec\beta_B = \frac{\partial\vec v_B}{\partial\dot\varphi} =$ $= \frac{\partial 2R\dot\varphi\vec i}{\partial\dot\varphi} = 2R\vec i$

$$Q_g = \vec F_g\cdot\vec\beta_B = F_{g0}\sin(\omega t)\vec i\,2R\vec i$$

$$\boxed{Q_g = 2RF_{g0}\sin(\omega t)}$$

Numerikus eredmények :

$$m_{red} = \frac{3}{2}mR^2 =$$

$$= \frac{3}{2}\cdot 20\cdot 0{,}5^2 = 7{,}5\,\text{kgm}^2$$

$$\frac{1}{c_{red}} = \frac{R^2}{c_1+c_2} + \frac{R^2}{c_3} =$$

$$= \frac{0{,}5^2}{(2+4)\cdot 10^{-5}} + \frac{0{,}5^2}{5\cdot 10^{-5}} =$$

$$= 9166{,}\dot 6\,Nm$$

$$k_{red} = 2R^2k =$$

$$= 2\cdot 0{,}5^2\cdot 500 = 250\,Nsm$$

$$Q_g = 2RF_{g0}\sin(\omega t) =$$

$$= 2\cdot 0{,}5\cdot 200\cdot\sin(10t) =$$

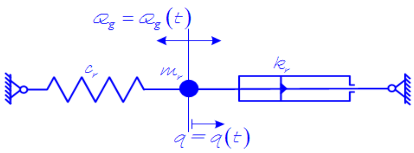
$$= 200\cdot\sin(10t)$$

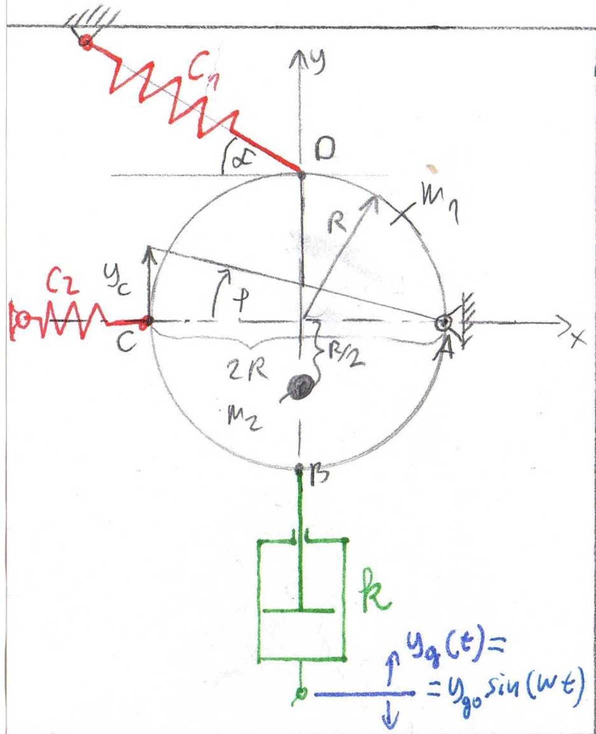
$$\frac{3}{2}mR^2\ddot\varphi + 0 = -\left(\frac{R^2}{c_1+c_2} + \frac{R^2}{c_3}\right)\varphi + (-2R^2k\dot\varphi) + 2RF_{g0}\sin(\omega t)$$

$$\frac{3}{2}mR^2\ddot\varphi + 2R^2k\dot\varphi + \left(\frac{R^2}{c_1+c_2} + \frac{R^2}{c_3}\right)\varphi = 2RF_{g0}\sin(\omega t)$$

$$m_{red}\ddot\varphi + k_{red}\dot\varphi + \frac{1}{c_{red}}\varphi = 2RF_{g0}\sin(\omega t)$$

$$7{,}5\ddot\varphi + 250\dot\varphi + 9166{,}\dot 6\varphi = 200\sin(10t)$$





$m_1 = 20$ kg
$m_2 = 12$ kg
$R = 2$ m
$c_1 = c_2 = 5 \cdot 10^{-4} \frac{m}{N}$
$k = 1000 \frac{Ns}{m}$
$y_{g0} = 15$ mm
$\omega = 50 \frac{rad}{s}$
$\alpha = 30°$

általános koordináta

$q = y_c, \dot{q} = \dot{y}_c, \ddot{q} = \ddot{y}_c$

$y_c = 2R\varphi \Rightarrow \varphi = \frac{y_c}{2R}$

$\dot{y}_c = 2R\dot{\varphi} \Rightarrow \dot{\varphi} = \frac{\dot{y}_c}{2R}$

$\ddot{y}_c = 2R\ddot{\varphi} \Rightarrow \ddot{\varphi} = \frac{\ddot{y}_c}{2R}$

Lagrange II. mozgásegyenlete

$$\underbrace{\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{y}_c}\right)}_{①} - \underbrace{\frac{\partial E}{\partial y_c}}_{②} = \underbrace{Q_c}_{③} + \underbrace{Q_k + Q_g}_{④}$$

a redukált rezgőrendszer:

① $\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{y}_c}\right)$ meghatározása:

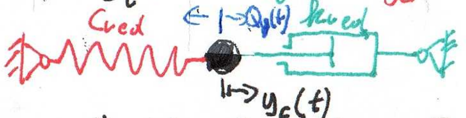
$$E = \frac{1}{2} J_A \omega^2 + \frac{1}{2} m_2 v_2^2 =$$
$$= \frac{1}{2}\left[\frac{3}{2} m_1 R^2\right]\dot{\varphi}^2 + \frac{1}{2} m_2 \left[\sqrt{R^2+\frac{R^2}{4}}\,\dot{\varphi}\right]^2$$
$$= \frac{1}{2}\left(\frac{3}{2} m_1 R^2\right)\left(\frac{\dot{y}_c}{2R}\right)^2 + \frac{1}{2} m_2 \left[\sqrt{R^2+\frac{R^2}{4}}\,\frac{\dot{y}_c}{2R}\right]^2$$
$$= \frac{1}{2}\left(\frac{3}{2} m_1 R^2 \frac{1}{4R^2} + \overset{m_2}{\downarrow}\left(R^2+\frac{R^2}{4}\right)\frac{1}{4R^2}\right)\dot{y}_c^2 =$$
$$= \frac{1}{2}\left[\frac{3}{8} m_1 + \left(\frac{1}{4}+\frac{1}{16}\right) m_2\right]\dot{y}_c^2$$
$$\frac{d}{dt}\frac{\partial E}{\partial \dot{y}_c} = \underbrace{\left[\frac{3}{8} m_1 + \frac{5}{16} m_2\right]}_{m_{red}}\ddot{y}_c$$
$$m_{red} = \frac{3}{8} m_1 + \frac{5}{16} m_2 = \frac{3}{8}\cdot 20 + \frac{5}{16}\cdot 12 =$$
$$= \underline{\underline{11{,}25 \text{ kg}}}$$

② $\frac{\partial E}{\partial y_c} = 0$

visszahelyettesítveés átrendezve:

$$\underbrace{\left(\frac{3}{8} m_1 + \frac{5}{16} m_2\right)}_{m_{red}}\ddot{y}_c + \underbrace{\frac{1}{4} k}_{k_{red}}\dot{y}_c + \underbrace{\frac{\cos^2\beta}{2c_1}}_{1/c_{red}} y_c = \underbrace{\frac{1}{2} k \omega y_{g0} \cos(\omega t)}_{Q_g(t)}$$

$$11{,}25\,\ddot{y}_c + 250\,\dot{y}_c + 67\,y_c = 375 \cos(50t)$$



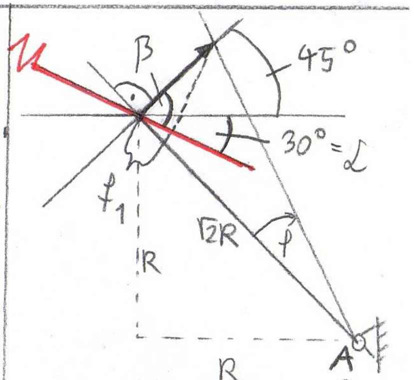
$$m_{red}\ddot{y}_c + k_{red}\dot{y}_c + \frac{1}{c_{red}} y = Q_g(t)$$

③ Q_c meghatározása

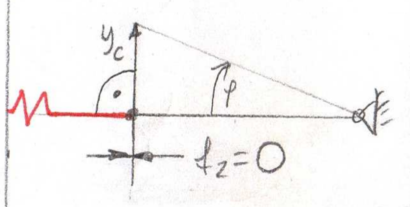
$$U = U_1 + U_2 = \frac{1}{2}\frac{f_1^2}{c_1} + \frac{1}{2}\frac{f_2^2}{c_2} =$$
$$= \frac{1}{2}\frac{(\sqrt{2}R\varphi\cos\beta)^2}{c_1} + \frac{1}{2}\frac{(2R\varphi\cos 30°)^2}{c_2} =$$
$$= \frac{1}{2}\frac{\left(\sqrt{2}R\frac{y_c}{2R}\cos\beta\right)^2}{c_1} + \frac{1}{2}\frac{0}{c_2} =$$
$$= \frac{1}{2}\left(\frac{\frac{1}{2}\cos^2\beta}{c_1}\right)y_c^2 = \frac{1}{2}\left(\frac{\cos^2\beta}{2c_1}\right)y_c^2$$
$$Q_c = -\frac{\partial U}{\partial y_c} = -\frac{\cos^2\beta}{2c_1}y_c$$
$$\frac{1}{c_{red}} = \frac{\cos^2\beta}{2c_1} = \frac{\cos^2 75°}{2\cdot 5\cdot 10^{-4}} = 67\frac{N}{m}$$
$$c_{red} = 67^{-1} = \underline{\underline{0{,}015\frac{m}{N}}}$$

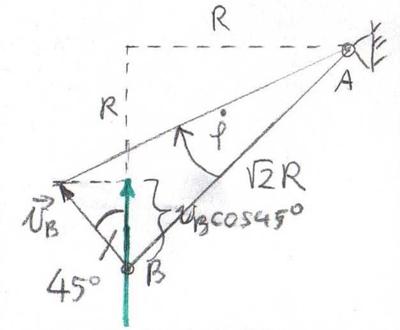
④ $\underbrace{Q_k + Q_g}_{v_B}$ meghatározása

$$v_d = \sqrt{2}R\dot{\varphi}\cos 45° - \dot{y}_g(t) =$$
$$= \sqrt{2}R\frac{\dot{y}_c}{2R}\cdot\frac{\sqrt{2}}{2} - \dot{y}_g(t) = \frac{1}{2}\dot{y}_c - \dot{y}_g(t)$$
$$D = \frac{1}{2}k v_d^2 = \frac{1}{2}k\left(\frac{1}{2}\dot{y}_c - \dot{y}_g(t)\right)^2$$
$$Q_k + Q_g = -\frac{\partial D}{\partial \dot{y}_c} = -k\left(\frac{1}{2}\dot{y}_c - \dot{y}_g(t)\right)\cdot\frac{1}{2} =$$
$$= \underbrace{-\overset{k_{red}}{\frac{1}{4}k}\,\dot{y}_c}_{Q_k} + \underbrace{\frac{1}{2}k\dot{y}_g(t)}_{Q_g}$$
$$k_{red} = \frac{1}{4}k = \frac{1}{4}\cdot 1000 = \underline{\underline{250\frac{Ns}{m}}}$$



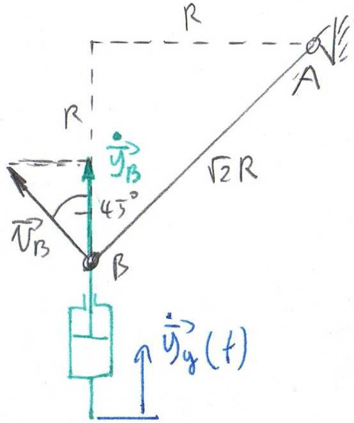
$\beta = 30° + 45° = 75°$





$$Q_g = \frac{1}{2}k\omega\cdot y_{g0}\cos(\omega t) =$$
$$= \frac{1}{2}\cdot 1000\cdot 50\cdot 0{,}015\cos(50t)$$
$$= 375\cos(50t)\,N$$

④ $Q_k + Q_y$ meghatározása (másik módszer)



$$v_B = \sqrt{2} R \dot{\varphi} = \sqrt{2} R \frac{\dot{y}_c}{2R} = \frac{\sqrt{2}}{2}\dot{y}_c$$

$$\vec{v}_B = v_B\left(-\sin 45° \vec{i} + \cos 45° \vec{j}\right) =$$

$$= \frac{\sqrt{2}}{2}\dot{y}_c\left(-\frac{\sqrt{2}}{2}\vec{i} + \frac{\sqrt{2}}{2}\vec{j}\right) =$$

$$= \underbrace{-\frac{1}{2}\dot{y}_c\vec{i}}_{\dot{\vec{x}}_B} + \underbrace{\frac{1}{2}\dot{y}_c\vec{j}}_{\dot{\vec{y}}_B}$$

$$\vec{v}_d = \left(\dot{\vec{y}}_B - \dot{\vec{y}}_y(t)\right)$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} + \vec{v}_{AB} = \vec{0} + \left(-\dot{\varphi}\vec{k}\right)\times\left(-R\vec{i} - R\vec{j}\right) =$$

$$= \dot{\varphi}R\underbrace{\left(\vec{k}\times\vec{i}\right)}_{\vec{j}} + \dot{\varphi}R\underbrace{\left(\vec{k}\times\vec{j}\right)}_{-\vec{i}} =$$

$$= -\dot{\varphi}R\vec{i} + \dot{\varphi}R\vec{j} = -\frac{\dot{y}_c}{2R}R\vec{i} + \frac{\dot{y}_c}{2R}R\vec{j} =$$

$$= \underbrace{-\frac{1}{2}\dot{y}_c\vec{i}}_{\dot{\vec{x}}_B} + \underbrace{\frac{1}{2}\dot{y}_c\vec{j}}_{\dot{\vec{y}}_B}$$

$$\vec{v}_d = \left(\dot{\vec{y}}_B - \dot{\vec{y}}_y(t)\right)$$

$$Q_k = \vec{F}_k \cdot \vec{\beta}_B$$

$$\vec{F}_k = -k\vec{v}_d = -k\left(\dot{\vec{y}}_B - \dot{\vec{y}}_y(t)\right) =$$

$$= -k\left(\frac{1}{2}\dot{y}_c\vec{j} - \dot{y}_y(t)\vec{j}\right) =$$

$$= -k\cdot\left(\frac{1}{2}\dot{y}_c - \dot{y}_y(t)\right)\vec{j}$$

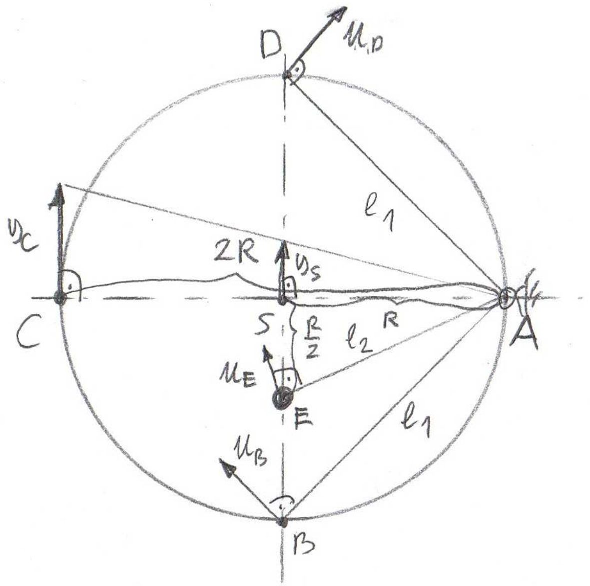
$$\vec{\beta}_B = \frac{\partial \vec{v}_B}{\partial \dot{y}_c} = \frac{\partial\left(-\frac{1}{2}\dot{y}_c\vec{i} + \frac{1}{2}\dot{y}_c\vec{j}\right)}{\partial \dot{y}_c} =$$

$$= -\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j}$$

$$Q_k = \vec{F}_k\cdot\vec{\beta}_B = -k\left(\frac{1}{2}\dot{y}_c - \dot{y}_y(t)\right)\vec{j}\cdot\left(-\frac{1}{2}\vec{i} + \frac{1}{2}\vec{j}\right) = \underbrace{-\frac{1}{2}k\dot{y}_c}_{Q_k} + \underbrace{\frac{1}{2}k\dot{y}_y(t)}_{Q_y}$$

Megjegyzés: f nélkül arányosság alapján:

A forgásponttól távolodva az elmozdulás (és annak deriváltjai) lineárisan nőnek



ált koord: y_C

$\Downarrow$

$$y_S = \frac{\cancel{R}}{2\cancel{R}} y_C = \frac{1}{2} y_C$$

$$u_D = \frac{l_1}{2R} y_C = \frac{\sqrt{2}R}{2R} y_C = \frac{\sqrt{2}}{2} y_C \Rightarrow V = \frac{1}{2}\frac{f_1^2}{c_1} = \frac{1}{2}\frac{(u_D\cos\beta)^2}{c_1} = \frac{1}{2}\frac{\left(\frac{\sqrt{2}}{2} y_c\cos\beta\right)^2}{c_1} =$$

$$= \frac{1}{2}\left(\frac{\frac{1}{2}\cos^2\beta}{c_1}\right) y_c^2$$

$$v_B = \dot{u}_B = \frac{l_1}{2R}\dot{y}_c = \frac{\sqrt{2}R}{2R}\dot{y}_c = \frac{\sqrt{2}}{2}\dot{y}_c \Rightarrow D = \frac{1}{2}k\left(\dot{u}_B\cos 45^\circ - \dot{y}_g(t)\right)^2 =$$

$$= \frac{1}{2}k\left(\frac{\sqrt{2}}{2}\dot{y}_c\frac{\sqrt{2}}{2} - \dot{y}_g(t)\right)^2 = \frac{1}{2}k\left(\frac{1}{2}\dot{y}_c - \dot{y}_g(t)\right)^2$$

$$v_2 = v_E = \dot{u}_E = \frac{l_2}{2R}\dot{y}_c = \frac{\sqrt{R^2+\frac{R^2}{4}}}{2R}\dot{y}_c = \sqrt{\frac{R^2+\frac{R^2}{4}}{4R^2}}\,\dot{y}_c = \sqrt{\frac{1}{4}+\frac{1}{16}}\,\dot{y}_c = \sqrt{\frac{5}{16}}\,\dot{y}_c$$

$$\Rightarrow E_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_2\left(\sqrt{\frac{5}{16}}\,\dot{y}_c\right)^2 = \frac{1}{2}\left(\frac{5}{16}m_2\right)\dot{y}_c^2$$