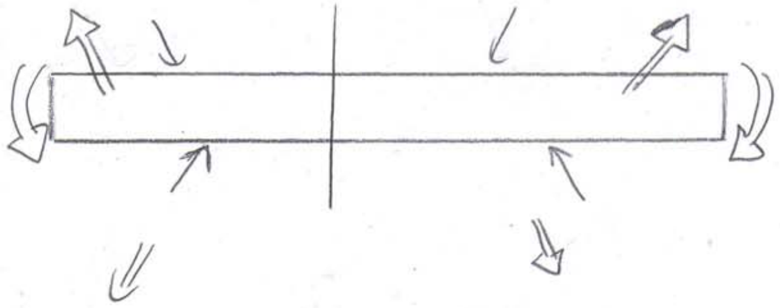
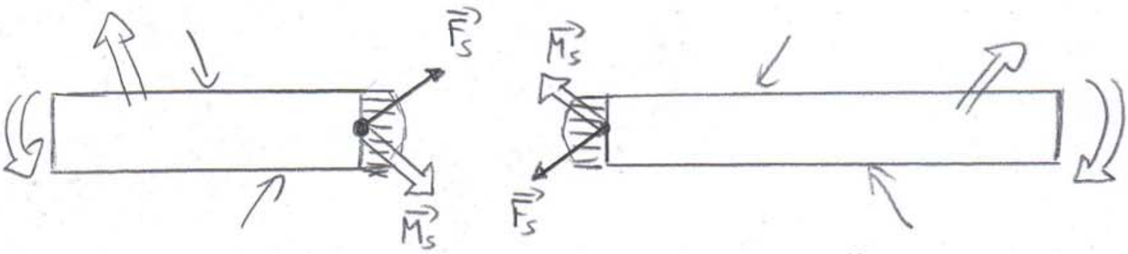
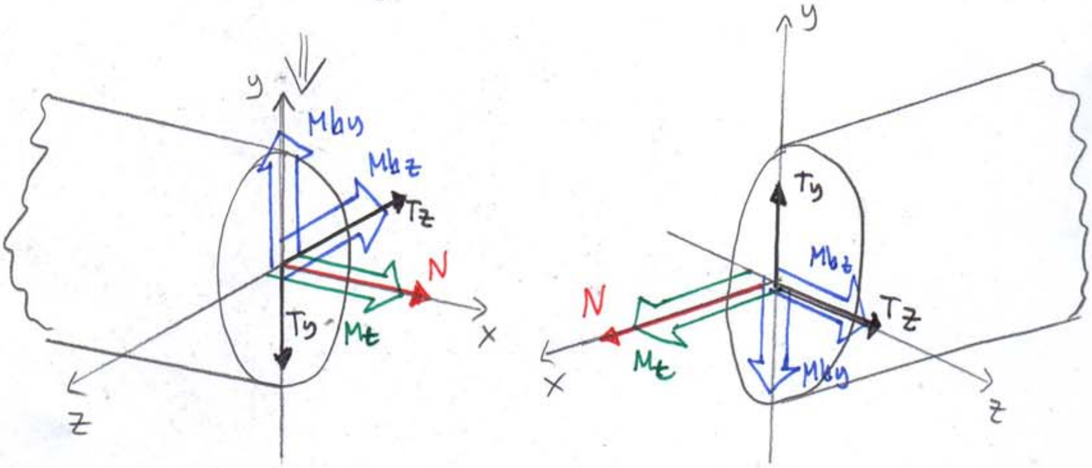


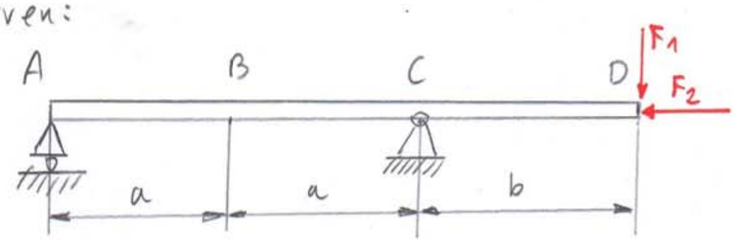




		⊕	⊖
axial/normal force	N		
shear force	T_y, T_z		
bending moment	Mb_y, Mb_z		
torsional moment	Mt		

EXERCISE 10.1

given:



$F_1 = 150N$

$F_2 = 250N$

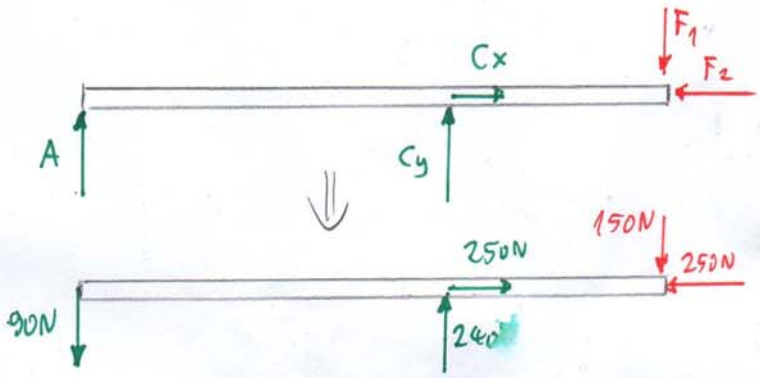
$a = 0.25m$

$b = 0.3m$

task:

internal forces at B, C^-, C^+, D^-, D^+ cross sections

• reaction forces:



ΣF_x: 1) $C_x - F_2 = 0$

ΣF_y: 2) $A + C_y - F_1 = 0$

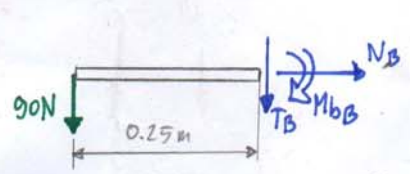
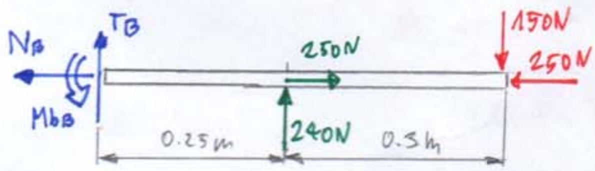
ΣM_{Cz}: 3) $-2a\,A - b \cdot F_1 = 0$

1) $\Rightarrow C_x = F_2 = 250N\ (\rightarrow)$

3) $\Rightarrow A = \dfrac{-bF_1}{2a} = \dfrac{-0.3 \cdot 150}{2 \cdot 0.25} = -90N\ (\downarrow)$

2) $\Rightarrow C_y = F_1 - A = 150 - (-90) = 240N\ (\uparrow)$

Ⓑ Let's divide the beam into two sections in cross section B

left-hand side part: ①	right-hand side part: ②
	
Method 1: keep ① and leave ②	Method 2: keep ② and leave ①

Method 1/A: from equilibrium of ①

ΣF_x: $N_B = 0 \quad \Longrightarrow N_B = 0$

ΣF_y: $-90 - T_B = 0 \quad \Longrightarrow T_B = -90N$

ΣM_{Bz}: $0.25 \cdot 90 - Mb_B = 0 \Longrightarrow M_{bB} = 22.5Nm$

Method 2/A: from equilibrium of ②

ΣF_x: $-N_B + 250 - 250 = 0 \Longrightarrow N_B = 0$

ΣF_y: $T_B + 240 - 150 = 0 \Longrightarrow T_B = -90N$

ΣM_{Bz}: $0.25 \cdot 240 - 0.55 \cdot 150 + M_{bB} = 0$

$\Longrightarrow M_{bB} = 0.55 \cdot 150 - 0.25 \cdot 240 = 22.5Nm$

Method 1/B: from equivalent force couple system of ②

$N_B \overset{\xrightarrow{+}}{=} 250 - 250 = 0$

$T_B \overset{\downarrow +}{=} 150 - 240 = -90N$

$M_{bB} \overset{\circlearrowright +}{=} 0.55 \cdot 150 - 0.25 \cdot 240 = 22.5Nm$

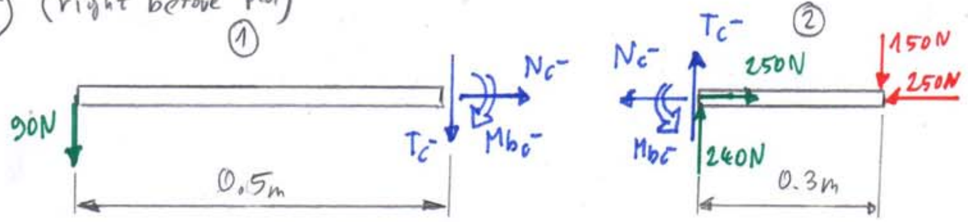
Method 2/B: from equivalent force couple system of ①

$N_B \overset{\xleftarrow{+}}{=} 0$

$T_B \overset{\uparrow +}{=} -90N$

$M_{bB} \overset{\circlearrowleft +}{=} 0.25 \cdot 90 = 22.5Nm$

$\bar{C}$ (right before pin)

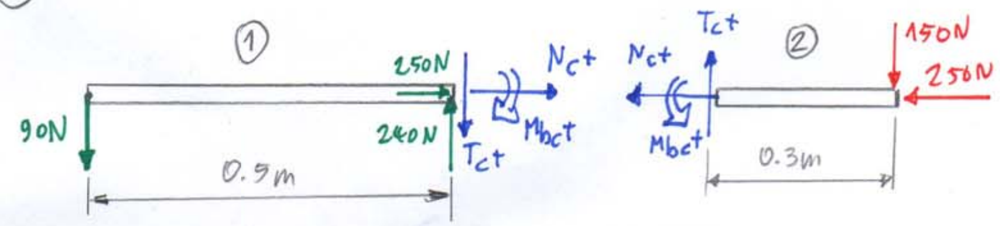


From equilibrium of ①: ΣF_x: $N_{c^-} = 0$

ΣF_y: $-90 - T_{c^-} = 0 \Rightarrow T_{c^-} = -90\,N$

$\Sigma M_{c^-_z}$: $0.5 \cdot 90 - M_{bc^-} = 0 \Rightarrow M_{bc^-} = 45\,Nm$

C^+ (right after pin)

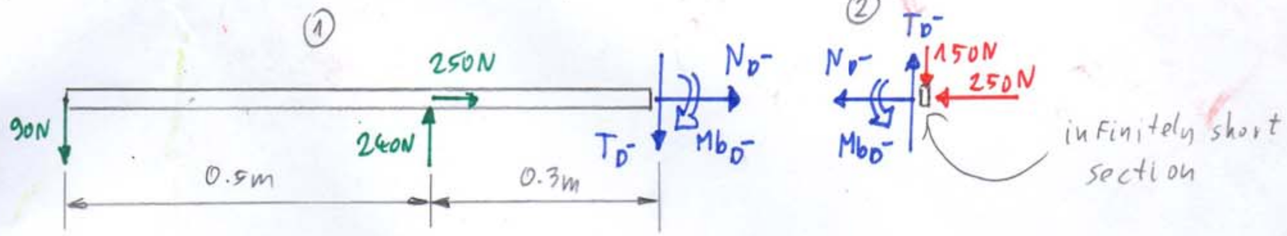


From equilibrium of ②: ΣF_x: $-N_{c^+} - 250 = 0 \Longrightarrow N_{c^+} = -250\,N$

ΣF_y: $T_{c^+} - 150 = 0 \Longrightarrow T_{c^+} = 150\,N$

$\Sigma M_{c^+_z}$: $-0.3 \cdot 150 + M_{bc^+} = 0 \Longrightarrow M_{bc^+} = 45\,Nm$

D^- (right before the end of the beam)

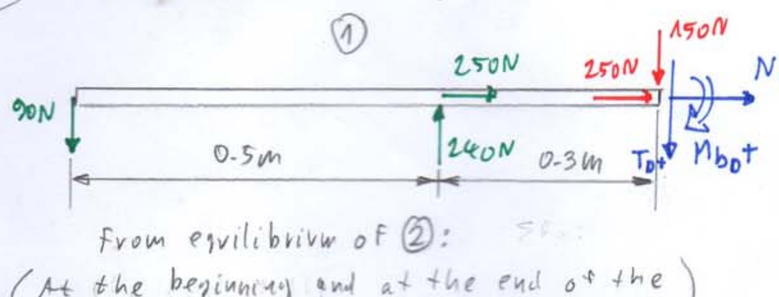


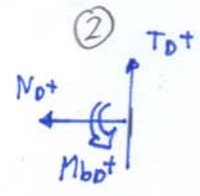
from equilibrium of ②: ΣF_x: $-N_{D^-} - 250 = 0 \Longrightarrow N_{D^-} = -250\,N$

ΣF_y: $T_{D^-} - 150 = 0 \Longrightarrow T_{D^-} = 150\,N$

$\Sigma M_{D^-_z}$: $M_{bD^-} = 0 \Longrightarrow M_{bD^-} = 0$

D^+ (at the end of the beam)





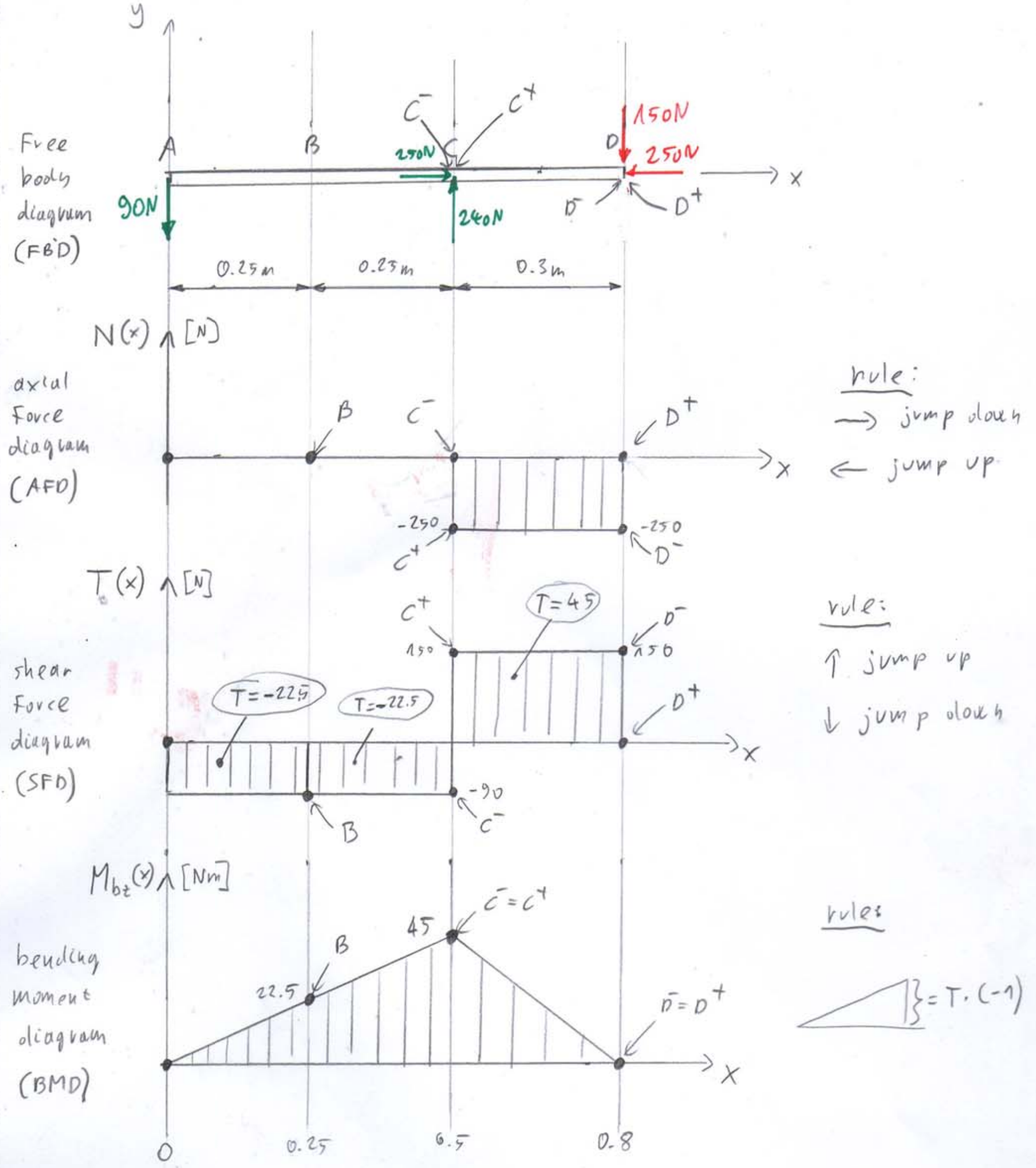
ΣF_x: $N_{D^+} = 0$

ΣF_y: $T_{D^+} = 0$

$\Sigma M_{D^+_z}$: $M_{bD^+} = 0$

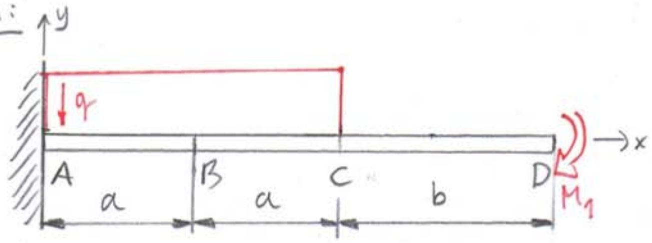
From equilibrium of ②:
(At the beginning and at the end of the beam everything is zero!)

• internal force diagrams:



EXERCISE 10.2

given:



$q = 200\,\frac{N}{m}$

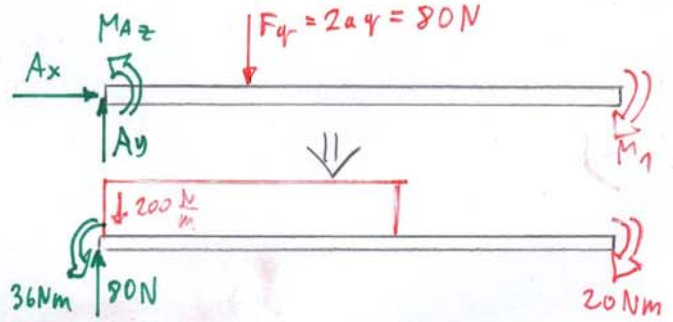
$a = 0.2\,m$

$b = 0.3\,m$

$M_1 = 20\,Nm$

task:
internal Forces at B, C, D^- cross sections

• reaction forces



ΣF_x: 1) $A_x = 0$

ΣF_y: 2) $A_y - F_q = 0$

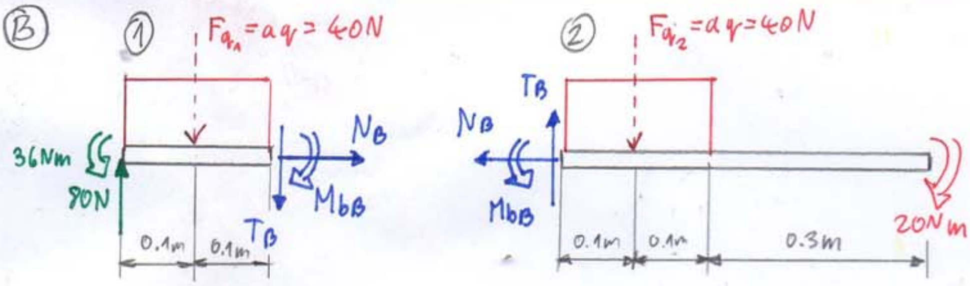
ΣM_{Az}: 3) $M_{Az} - a\,F_q - M_1 = 0$

1) $\Rightarrow A_x = 0$

2) $\Rightarrow A_y = F_q = 80N\,(\uparrow)$

3) $\Rightarrow M_{Az} = aF_q + M_1 = 36\,Nm\,(\circlearrowleft)$

(B)



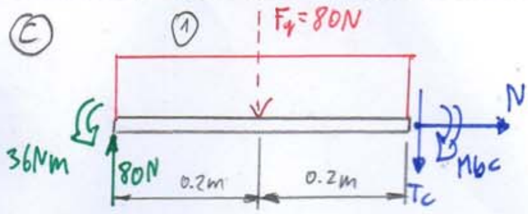
From equivalent force couple system of ②:

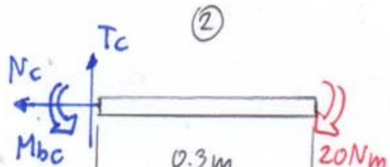
$$N_B \overset{\xrightarrow{+}}{=\!=} 0$$

$$T_B \overset{\downarrow +}{=\!=} 40N \quad \uparrow \Leftarrow \downarrow$$

$$M_{bB} \overset{\circlearrowright +}{=\!=} 0.1 \cdot 40 + 20 = 24Nm \quad \circlearrowleft \Leftarrow \circlearrowright$$

(C)





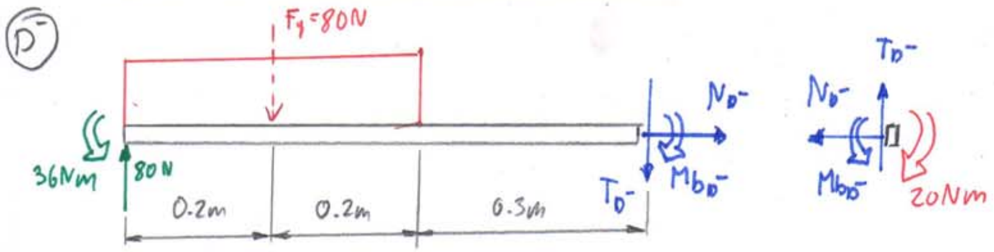
From equivalent force couple system of ②:

$$N_C \overset{\xrightarrow{+}}{=\!=} 0$$

$$T_C \overset{\downarrow +}{=\!=} 0$$

$$M_{bC} \overset{\circlearrowright +}{=\!=} 20\,Nm \quad \circlearrowleft \Leftarrow \circlearrowright$$

D⁻



From equivalent force couple system of ②:

$$N_{D^-} \overset{\xrightarrow{+}}{=} 0$$

$$T_{D^-} \overset{\downarrow +}{=} 0$$

$$M_{bD^-} \overset{\circlearrowright +}{=} 20\,Nm$$

• internal force diagrams: