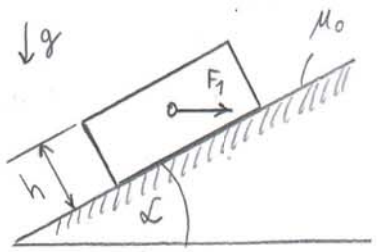
EXERCISE 12.1



given:

$m = 3\,kg$

$\alpha = 30°$

$g = 9.81\,\frac{m}{s^2}$

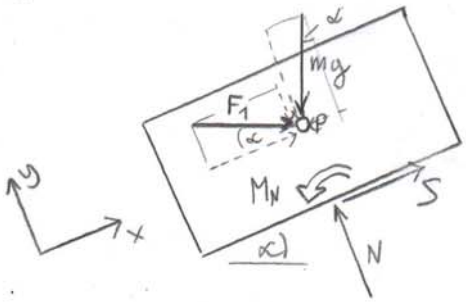
$h = 0.1m$

$\mu_0 = 0.4$

task

$F_1 = ?$ so that the body does not move

① Free body diagram



② equation of equilibrium

$\Sigma F_x: 1)\ F_1 \cos\alpha - mg \sin\alpha + S = 0$

$\Sigma F_y: 2)\ -F_1 \sin\alpha - mg \cos\alpha + N = 0$

$\Sigma M_{P_z}: 3)\ M_N + S\,\frac{h}{2} = 0$

③ static Friction and normal force in function of F_1

$1) \Rightarrow S = mg \sin\alpha - F_1 \cos\alpha = 14.7 - 0.866 F_1$

$2) \Rightarrow N = mg \cos\alpha + F_1 \sin\alpha = 25.5 + 0.5 F_1$

④ condition of sticking (when the body does not move):

$$|S| \leq \mu_0 N$$

case 1:

$S \leq \mu N$

$14.7 - 0.866 F_1 \leq 0.4 (25.5 + 0.5 F_1)$

$4.54 \leq 1.066 F_1$

$$F_1 \geq 4.26\,N$$

case 2:

$-S \leq \mu N$

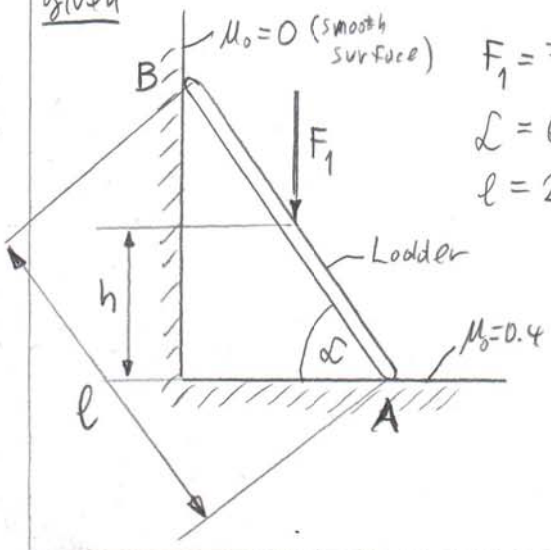
$-14.7 + 0.866 F_1 \leq 0.4 (25.5 + 0.5 F_1)$

$0.666 F_1 \leq 24.86$

$$F_1 \leq 37.3\,N$$

EXERCISE 12.1

given



$F_1 = 700N$

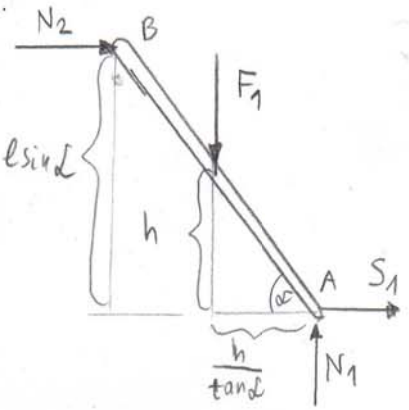
$\alpha = 60°$

$\ell = 2m$

task:

How high can you climb the ladder so that it does not slip?

① Free body diagram



② equations of equilibrium

ΣF_x: 1) $S_1 + N_2 = 0$

ΣF_y: 2) $N_1 - F_1 = 0$

ΣM_{Az}: 3) $\frac{h}{\tan\alpha} F_1 - N_2 \ell \sin\alpha = 0$

③ normal and static friction forces as function of h

2) $\Rightarrow N_1 = F_1 = 700N$

3) $\Rightarrow N_2 = \frac{\frac{h}{\tan\alpha} F_1}{\ell \sin\alpha} = \frac{F_1}{\ell \sin\alpha \tan\alpha} h = 233.3\,h$

1) $\Rightarrow S_1 = -N_2 = -233.3\,h$

④ condition of sticking

$|S_1| \leq \mu_0 N_1 \Rightarrow -S_1 \leq \mu_0 N_1$

$233.3\,h \leq 0.4 \cdot 700$

$\boxed{h \leq 1.2m}$

$\rightarrow S_1 = -233.3 \cdot 1.2 = -280N$

$N_2 = 233.3 \cdot 1.2 = 280N$

$\vec{A} = S_1 \vec{i} + N_1 \vec{j} = (-280\vec{i} + 700\vec{j})N$

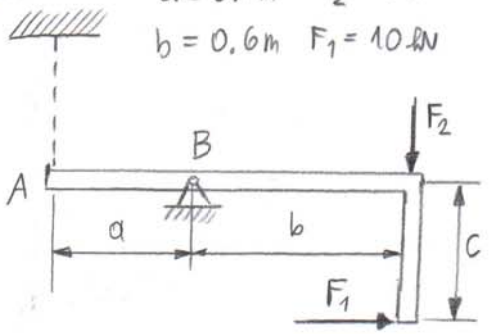
$\vec{B} = N_2 \vec{i} = (280\vec{i})N$

EXERCISE 12.3

given:

$a = 0.2m \quad F_2 = 4\,kN$

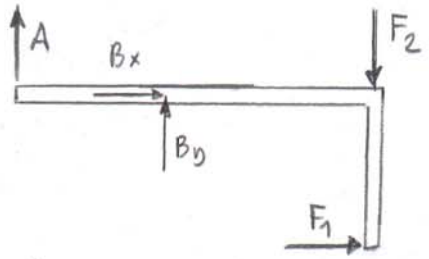
$b = 0.6m \quad F_1 = 10\,kN$



task:

- What should be the length of drum c to maintain equilibrium
- How much is the reaction force, if $c = 0.3m$?

① Free body diagram



② equations of equilibrium

ΣF_x: 1) $B_x + F_1 = 0$

ΣF_y: 2) $B_y + A - F_2 = 0$

ΣM_{B_z}: 3) $cF_1 - bF_2 - aA = 0$

③ reaction force A in function of c:

$$3) \Rightarrow A = \frac{cF_1 - bF_2}{a}$$

④ the condition for the rope to function:

$$\boxed{A \geq 0} \Rightarrow \frac{cF_1 - bF_2}{a} \geq 0$$

$$cF_1 - bF_2 \geq 0 \Rightarrow \boxed{c \geq b\frac{F_2}{F_1} = 0.24m}$$

- if $c = 0.3m \; (> 0.24m)$

$$A = \frac{cF_1 - bF_2}{a} = 3\,kN \; (>0 \; \checkmark)$$

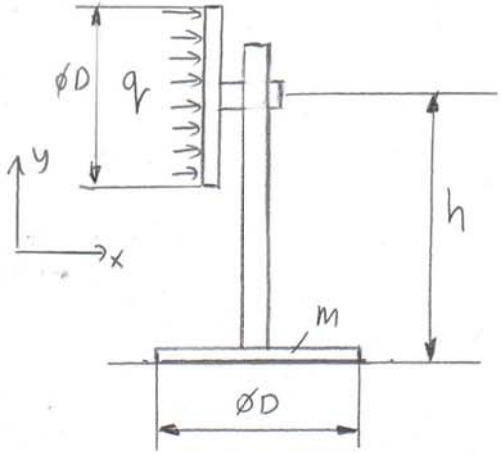
1) $\Rightarrow B_x = -F_1 = -10\,kN$

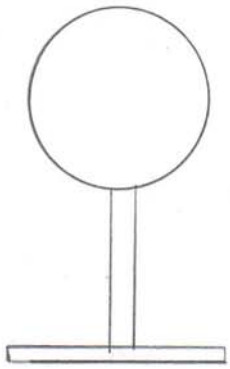
2) $\Rightarrow B_y = F_2 - A = 1\,kN$

$$\vec{A} = A\vec{j} = (3\vec{j})\,kN$$

$$\vec{B} = B_x\vec{i} + B_y\vec{j} = (-10\vec{i} + 1\vec{j})\,kN$$

EXERCISE 12.4





$D = 300\,mm$
$q = 500\,\frac{N}{m^2}$
$\mu = 0,2$
$g = 9.81\,\frac{m}{s^2}$
$h = 2m$

- what should be at least the mass of the base to prevent tilting with $n = 2$ safety factor
- check for slipping

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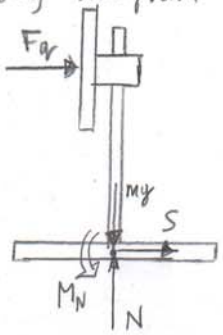
① Free body diagram



② equations of equilibrium

ΣF_x: 1) $F_q + S = 0$

ΣF_y: 2) $N - mg = 0$

ΣM_{Pz}: 3) $-hF_q + M_N = 0$

$F_q = q \cdot A = q \cdot \frac{D^2\pi}{4} = 35.34N$

③

$S = -F_q = -35.34N$

$M_N = F_q \cdot h = 70.68\,Nm$

$N = mg = 9.81m$

- condition to prevent tilting: $\boxed{|M_N| \leq N \cdot \frac{D}{2}}$

$\Downarrow M_N > 0$

$M_N \leq N \cdot \frac{D}{2}$

$70.68 \leq 9.81 \cdot m \cdot 0.15$

$m \geq \frac{70.68}{9.81 \cdot 0.15} = 48\,kg$

$m_{min} = 2 \cdot 48 = 96\,kg$

- check for slipping: $m_{min} \rightarrow N = m_{min} \cdot g = 941.8N$

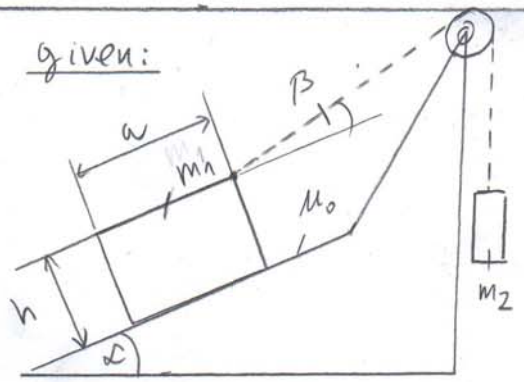
- condition for sticking: $|S| \leq \mu_0 \cdot N$

$35.34 \leq 0.2 \cdot 9418 = 188,36$ ✓

it won't slip

EXERCISE 12.5

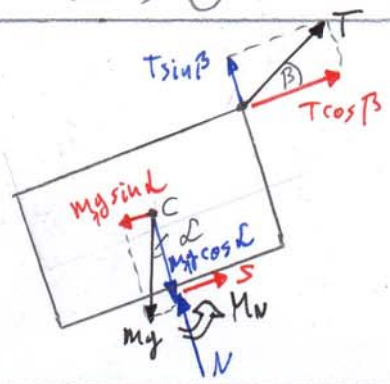
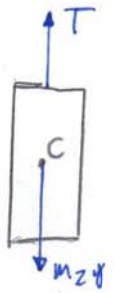
given:



$m_1 = 1\,kg$
$\mu_0 = 0.2$
$\alpha = 30°$
$\beta = 15°$
$g = 10\,\frac{m}{s^2}$
$h = 0.2\,m$

task:

- Find the mass of box2 to mass of box 2 to maintain equilibrium
- a_{min} to prevent tilting in case of maximum m_2 mass

	body ①	body ②
FBD		
EE	ΣF_x: 1) $T\cos\beta + S - m_1 g \sin\alpha = 0$ ΣF_y: 2) $T\sin\beta + N - m_1 g \cos\alpha = 0$ ΣM_{Cz}: 3) $\frac{h}{2}S + M_N - \frac{h}{2}T\cos\beta + \frac{a}{2}T\sin\beta = 0$	ΣF_x: 4) $0 = 0$ ΣF_y: 5) $T - m_2 g = 0$

SOL.

5) $\Rightarrow T = m_2 g$

- static friction and normal forces in function of m_2

1) $\Rightarrow S = m_1 g \sin\alpha - m_2 g \cos\beta = 5 - 9.66 m_2$

2) $\Rightarrow N = m_1 g \cos\alpha - m_2 g \sin\beta = 8.66 - 2.59 m_2$

- condition of sticking:

$$|S| \le \mu_0 N$$

case 1:

$S \le \mu_0 N$

$5 - 9.66 m_2 \le 0.2 \cdot (8.66 - 2.59 m_2)$

$0.2 \cdot 2.59 m_2 - 9.66 m_2 \le 0.2 \cdot 8.66 - 5$

$-9.142 m_2 \le -3.268$

$m_2 \ge \frac{-3.268}{-9.142}$

$m_2 \ge 0.3575\,kg$

case 2:

$-S \le \mu_0 N$

$-5 + 9.66 m_2 \le 0.2 (8.66 - 2.59 m_2)$

$0.2 \cdot 2.59 m_2 + 9.66 m_2 \le 0.2 \cdot 8.66 + 5$

$10.178 m_2 \le 6.732$

$m_2 \le \frac{6.732}{10.178}$

$m_2 \le 0.6615\,kg$

- M_N moment in function of a :

$$3) \Rightarrow M_N = \frac{h}{2} T\cos\beta - \frac{h}{2} S - \frac{a}{2} m_2 g \sin\beta =$$

$$= \frac{h}{2} m_2 g \cos\beta - \frac{h}{2}(m_1 g \sin\alpha - m_2 g \cos\beta) - \frac{a}{2} m_2 g \sin\beta =$$

$$= 0.778 - 0.856\,a$$

- condition to prevent tilting :

$$\boxed{|M_N| \leq N \frac{a}{2}}$$

$$\Downarrow \quad M_N > 0$$

$$M_N \leq N \frac{a}{2}$$

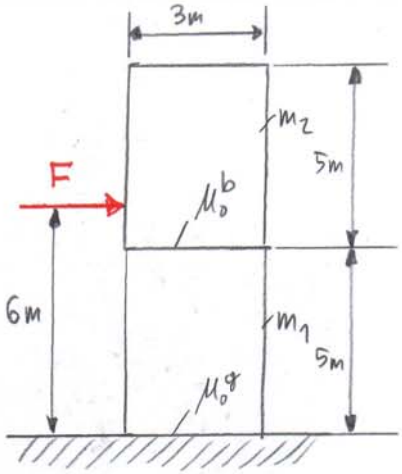
$$0.778 - 0.856\,a \leq \overbrace{(8.66 - 2.59 \cdot 0.6615)}^{6.947} \frac{a}{2}$$

$$0.778 - 0.856\,a \leq 3.473\,a$$

$$0.778 \leq 4.329\,a$$

$$0.18\,m \leq a$$

EXERCISE 12.6



$m_1 = m_2 = 100\,kg$

$\mu_0^g = 0.3$

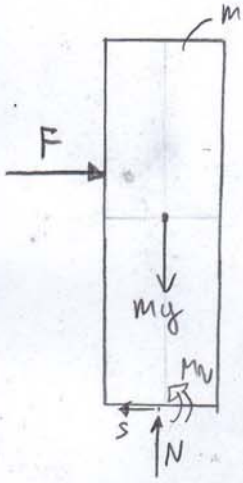
$\mu_0^b = 0.65$

$g = 10\,\frac{m}{s^2}$

task:

- Find the Force F where motion occurs
- what kind of motion occurs?

A) whole stack slides:



ΣF_x: 1) $F - S = 0$

ΣF_y: 2) $N - mg = 0$

1) $\Rightarrow S = F$

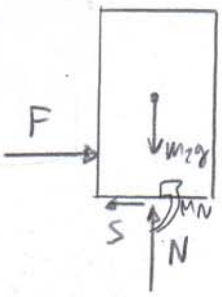
2) $\Rightarrow N = mg = 2000\,N$

- condition of sticking

$S \leq \mu_0^g N$,

$F \leq 0.3 \cdot 2000 = \boxed{600\,N}$

B) top box slides only:



ΣF_x: 1) $F - S = 0$

ΣF_y: 2) $N - m_2 g = 0$

1) $\Rightarrow F = S$

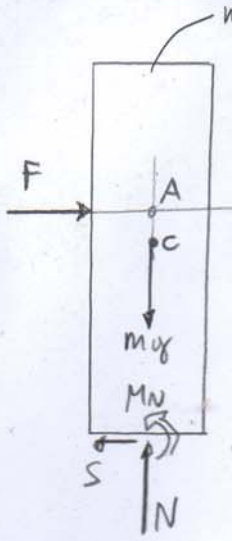
2) $\Rightarrow N = m_2 g = 1000\,N$

- condition of sticking:

$S \leq \mu_0^b N$

$F \leq 0.65 \cdot 1000 = \boxed{650\,N}$

C) whole stack tips:



ΣF_x: 1) $F - S = 0$

ΣF_y: 2) $N - mg = 0$

ΣM_{A_z}: 3) $-6S + M_N = 0$

1) $\Rightarrow S = F$

2) $\Rightarrow N = mg = 2000\,N$

3) $\Rightarrow M_N = 6S = 6F$

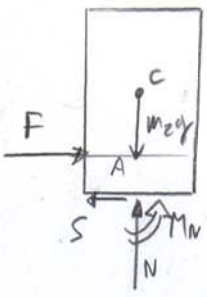
- condition to prevent tipping

$M_N \leq 1.5\,N$

$6F \leq 1.5 \cdot 2000$

$F \leq \frac{1.5 \cdot 2000}{6} = \boxed{500\,N}$

D) top box tips only:



ΣF_x: 1) $F - S = 0$

ΣF_y: 2) $N - m_2 g = 0$

ΣM_{A_z}: 3) $-1 \cdot S + M_N = 0$

1) $\Rightarrow S = F$

2) $\Rightarrow N = m_2 g = 1000\,N$

3) $\Rightarrow M_N = 1 \cdot S = F$

- condition to prevent tipping:

$M_N \leq 1.5\,N$

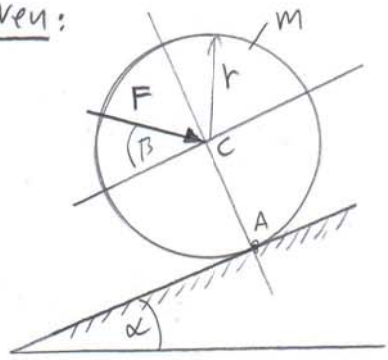
$F \leq 1.5 \cdot 1000$

$F \leq \boxed{1500\,N}$

Result: the whole stack tips when the Force F reaches 500 N

EXERCISE 12.7

given:



$m = 6\,kg$

$f = 3\,mm$ ↓ arm of the rolling resistance

$\alpha = 14°$

$\beta = 30°$

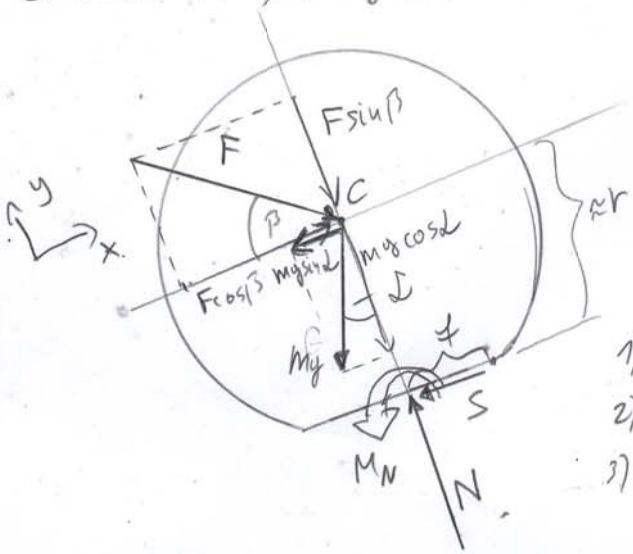
$r = 200\,mm$

$g = 10\,m/s^2$

task:

- $F_{max} = ?$ to prevent rolling
- $\mu_0^{min} = ?$ to prevent slipping

① Free body diagram



② equations of equilibrium:

$$\Sigma F_x: 1)\ F\cos\beta - mg\sin\alpha - S = 0$$

$$\Sigma F_y: 2)\ -F\sin\beta - mg\cos\alpha + N = 0$$

$$\Sigma M_{Cz}: 3)\ -rS + M_N = 0$$

$$1) \Rightarrow S = F\cos\beta - mg\sin\alpha = 0.866F - 14.52$$

$$2) \Rightarrow N = F\sin\beta + mg\cos\alpha = 0.5F + 58.22$$

$$3) \Rightarrow M_N = rS = rF\cos\beta - rmg\sin\alpha =$$
$$= 173.2F - 2903.06$$

- condition to prevent rolling:

$$\boxed{M_N \leq N f}$$

$$173.2F - 2903.06 \leq (0.5F + 58.22)\cdot 3$$

$$173.2F - 2903.06 \leq 1.5F + 174.66$$

$$171.7F \leq 3077.72$$

$$F \leq 17.92\,N$$

- $F_{max} = 17.92\,N$

- condition to prevent slipping:

$$\boxed{S \leq \mu_0 N}$$

$$S = 0.866 \cdot 17.92 - 14.52 = 1\,N$$

$$N = 0.5 \cdot 17.92 + 58.22 = 62.18$$

$$1 \leq \mu_0 \cdot 62.18$$

$$\frac{1}{62.18} \leq \mu_0$$

$$0.015 \leq \mu_0$$

$$\mu_0^{min} = 0.015$$