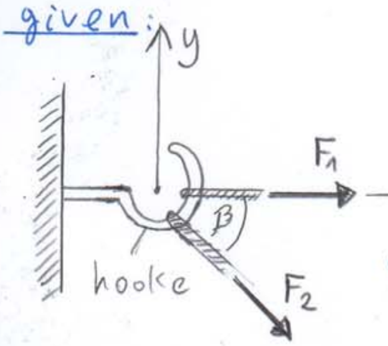
EXERCISE 2.1

given:



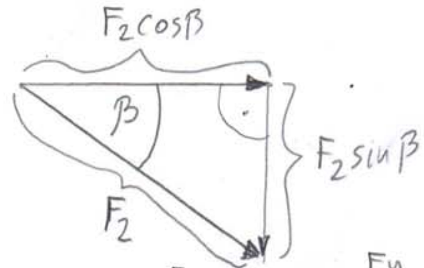
$F_1 = 300N$
$F_2 = 500N$
$\beta = 40°$

task:

a) $\vec{F_1}, \vec{F_2}$ Force vectors
b) resultant force ($\vec{F}$)
c) magnitude of the resultant force
d) direction of the resultant force
→ with unit vector
→ with angel

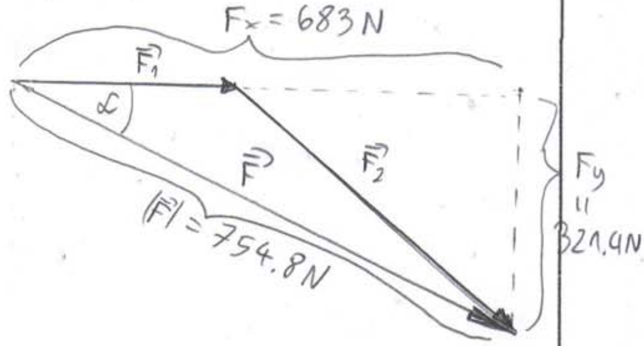
a) $\vec{F_1} = F_1 \vec{i} = (300\vec{i})N$

$\vec{F_2} = (F_2 \cos\beta)\vec{i} - (F_2 \sin\beta)\vec{j} =$
$= (500\cos 40°)\vec{i} - (500 \sin 40°)\vec{j} =$
$= (383\vec{i} - 321.4\vec{j})N$



b) $\vec{F} = \vec{F_1} + \vec{F_2} = (300\vec{i}) + (383\vec{i} - 321.4\vec{j}) = (\overbrace{683\vec{i}}^{F_x} - \overbrace{321.4\vec{j}}^{F_y})N$

c) $F = |\vec{F}| = \sqrt{683^2 + 321.4^2} = 754.8N$

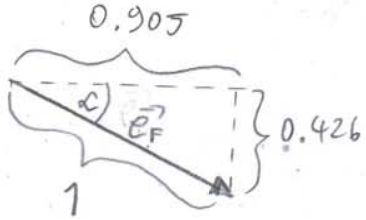


d) $\vec{e_F} = \frac{\vec{F}}{|\vec{F}|} = \frac{683\vec{i} - 321.4\vec{j}}{754.8} =$

$= 0.905\vec{i} - 0.426\vec{j}$

$\Updownarrow \quad \Updownarrow$

$\vec{e_F} = \cos\alpha\, \vec{i} + \sin\alpha\, \vec{j}$



$\Downarrow$

$\alpha = a\cos(0.905) = 25.2°$

or:

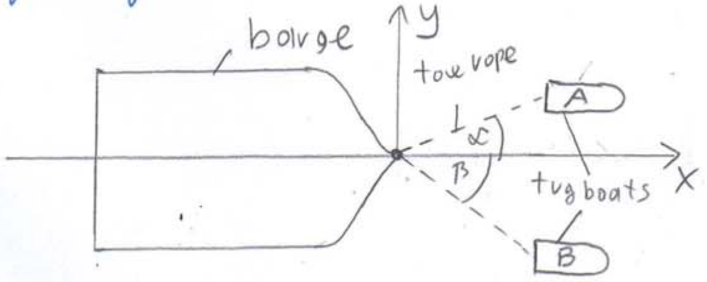
$\alpha = a\sin(0.426) = 25.2°$

or

$\alpha = atg \frac{F_y}{F_x} = atg \frac{321.4}{683} = 25.2°$

EXERCISE 2.2

Tugboats A and B tow a barge. The angles between the tow ropes and the direction of towing are α and β. Towing Force $\vec{F_V}$ along axis x is known (no acceleration occurs)



given:

$\alpha = 20°$

$\beta = 30°$

$\vec{F_V} = (50\vec{i})$ kN

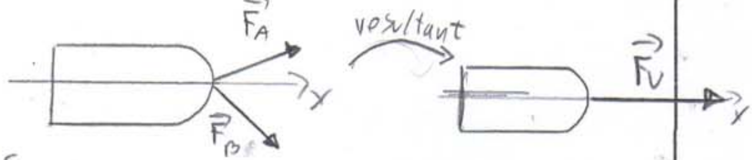
task:

towing forces arising in the tow ropes

F_A, F_B, $\vec{F_A}$, $\vec{F_B}$

- governing vector equation:

$$\vec{F_V} = \vec{F_A} + \vec{F_B}$$

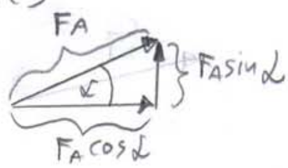


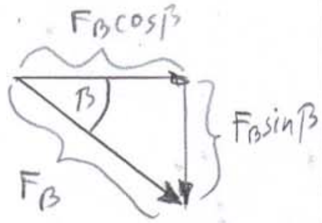
- Force vectors with x and y coordinates

$$\vec{F_V} = F_V \vec{i}$$

$$\vec{F_A} = F_A \cos\alpha \, \vec{i} + F_A \sin\alpha \, \vec{j}$$

$$\vec{F_B} = F_B \cos\beta \, \vec{i} - F_B \sin\beta \, \vec{j}$$





- governing vector equation with x and y coordinates:

$$F_V \vec{i} = F_A \cos\alpha \, \vec{i} + F_A \sin\alpha \, \vec{j} + F_B \cos\beta \, \vec{i} - F_B \sin\beta \, \vec{j} \quad / \cdot \vec{i} \, / \cdot \vec{j}$$

$/ \cdot \vec{i} \Rightarrow$ x dir. 1) $F_V = F_A \cos\alpha + F_B \cos\beta$

$/ \cdot \vec{j} \Rightarrow$ y dir. 2) $0 = F_A \sin\alpha - F_B \sin\beta$ $\Rightarrow F_B = F_A \dfrac{\sin\alpha}{\sin\beta} = F_A \dfrac{\sin 20°}{\sin 30°} =$

$$= 0.684 F_A$$

2) → 1): $F_V = F_A \cos\alpha + (0.684 F_A) \cos\beta$

$$50 = F_A \cdot 0.94 + 0.684 F_A \cdot 0.866$$

$$50 = 1.532 F_A$$

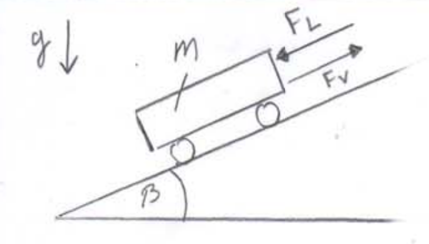
$$\underline{\underline{F_A = 32.64 \text{ kN}}} \Rightarrow \underline{\underline{F_B}} = 0.684 F_A = \underline{\underline{22.33 \text{ kN}}}$$

- Force vectors:

$$\vec{F_A} = 32.64 \cos 20° \, \vec{i} + 32.64 \sin 20° \, \vec{j} = (30.66 \vec{i} + 11.16 \vec{j}) \text{ kN}$$

$$\vec{F_B} = 22.33 \cos 30° \, \vec{i} - 22.33 \sin 30° \, \vec{j} = (19.34 \vec{i} - 11.16 \vec{j}) \text{ kN}$$

EXERCISE 2.3



given:

$F_L = 1200 N$ (air resistance)

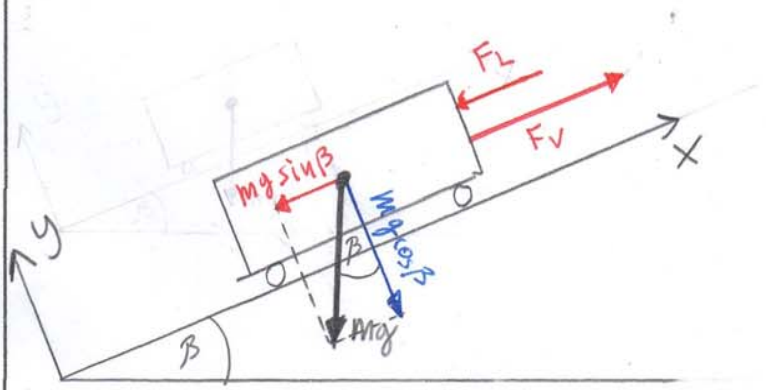
$m = 1.5 t$

$\beta = 10°$

$g = 9.81 \frac{m}{s^2}$

task:

- How much traction force (F_v) is needed to ensure steady motion
- resultant of the external forces



- Force vectors:

$$\vec{F_v} = F_v \vec{i}$$

$$\vec{F_L} = -F_L \vec{i} = (-1200 \vec{i}) N$$

$$\vec{F_g} = -mg \sin\beta \vec{i} - mg \cos\beta \vec{j} =$$

$$= -1500 \cdot 9.81 \sin 10° \vec{i} - 1500 \cdot 9.81 \cos 10° \vec{j} =$$

$$= (-2555.2 \vec{i} - 14491.4 \vec{j}) N$$

$$\vec{F}_{external} = \vec{F_v} + \vec{F_L} + \vec{F_g} = F_v \vec{i} - 1200 \vec{i} - 2555.2 \vec{i} - 14491.4 \vec{j} =$$

$$= \underbrace{(F_v - 1200 - 2555.2)}_{=0} \vec{i} - 14491.4 \vec{j}$$

$$\Downarrow$$

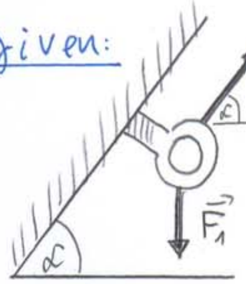
$$F_v = 1200 + 2555.2 = \underline{\underline{3755.2 N}}$$

$$\vec{F}_{external} = (-14491 \vec{j}) N$$

this is compensated by the normal force

EXERCISE 2.4

given:



$\alpha = 55°$

$F_1 = 400N$

task:

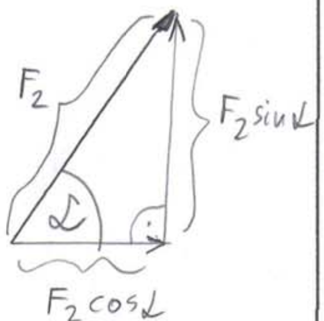
a) $F_2 = ?$ $\Rightarrow$ horizontal resultant force

b) force arising along the axis of the bolt

- governing vector equation:

$$\vec{F_1} + \vec{F_2} = \vec{F}$$

- the force vectors:



$$\vec{F_1} = -F_1 \vec{j} = -400\vec{j}$$

$$\vec{F_2} = F_2 \cos\alpha \, \vec{i} + F_2 \sin\alpha \, \vec{j} = 0.574 F_2 \vec{i} + 0.819 F_2 \vec{j}$$

$$\vec{F} = F\vec{i} \quad (\leftarrow \text{it must be horizontal})$$

- substituting into the governing equation:

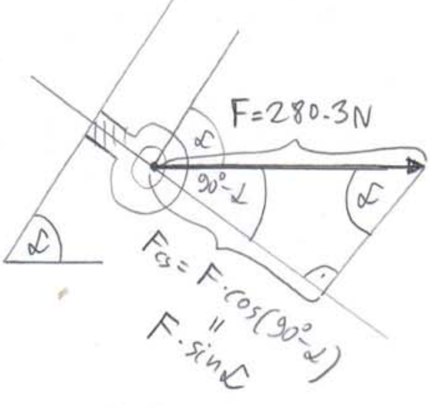
$$-400\vec{j} + 0.574 F_2 \vec{i} + 0.819 F_2 \vec{j} = F\vec{i} \quad / \cdot \vec{i} / \cdot \vec{j}$$

$$\left.\begin{aligned} /\cdot\vec{i} &\Rightarrow x \text{ dir}: \; 1) \; 0.574 F_2 = F \\ /\cdot\vec{j} &\Rightarrow y \text{ dir}: \; 2) \; -400 + 0.819 F_2 = 0 \end{aligned}\right\}$$

$$2) \rightarrow F_2 = \frac{400}{0.819} = 488.4N$$

$$1) \rightarrow F = 0.574 F_2 = 0.574 \cdot 488.4 = 280.3N$$

or

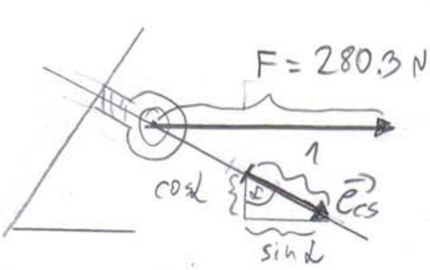


$$F_{cs} = F \cdot \cos(90° - \alpha) =$$

$$= 280.3 \cdot \cos 35° = 229.6N$$

or:

$$F_{cs} = F \cdot \sin\alpha = 229.6N$$



$$\vec{e_{cs}} = \sin\alpha \, \vec{i} - \cos\alpha \, \vec{j} = 0.819\vec{i} - 0.574\vec{j}$$

$$F_{cs} = \vec{F} \cdot \vec{e_{cs}} = (280.3\vec{i}) \cdot (0.819\vec{i} - 0.574\vec{j}) =$$

$$= 280.3 \cdot 0.819 \cdot \underbrace{\vec{i} \cdot \vec{i}}_{1} - 290.3 \cdot 0.574 \underbrace{\vec{i} \cdot \vec{j}}_{0} =$$

$$= 229.6N$$