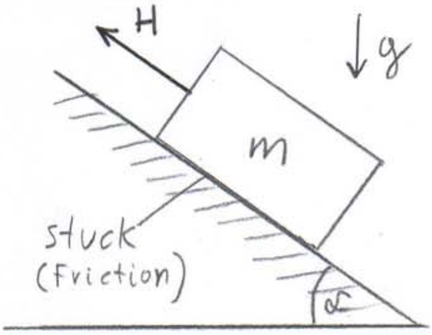
EXERCISE 3.1



given

$m = 20\,kg$

$H = 60\,N$

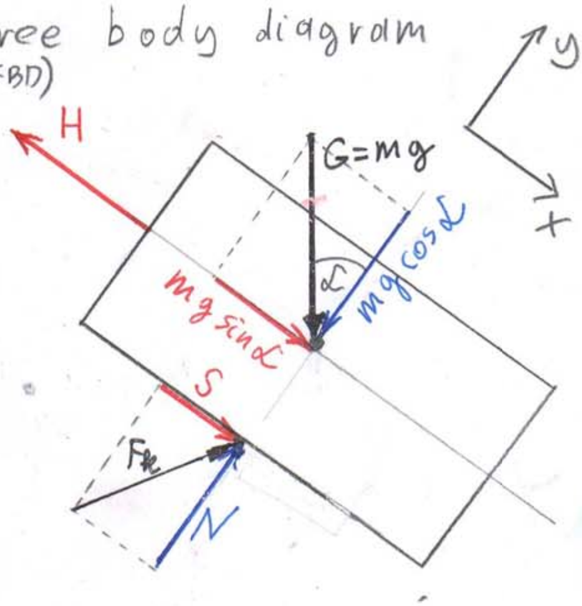
$g = 9.81\,\frac{m}{s^2}$

$\alpha = 35^\circ$

task

- normal Force (N)
- Friction Force (S)
- Constraint force (F_R)

- free body diagram (FBD)



- equilibrium equations (EE)

$$\left.\begin{array}{l} \Sigma F_x = 0 \Rightarrow 1)\; S - H + mg\sin\alpha = 0 \\ \Sigma F_y = 0 \Rightarrow 2)\; N - mg\cos\alpha = 0 \end{array}\right\}$$

- solution of the equilibrium equations:

$$1) \Rightarrow S = H - mg\sin\alpha = 60 - 20\cdot 9.81\sin 35^\circ = -52.5\,N\ (\nwarrow)$$

The friction force is opposed to the assumed direction

$$2) \Rightarrow N = mg\cos\alpha = 20\cdot 9.81\cdot\cos 35^\circ = 160.7\,N\ (\nearrow)$$

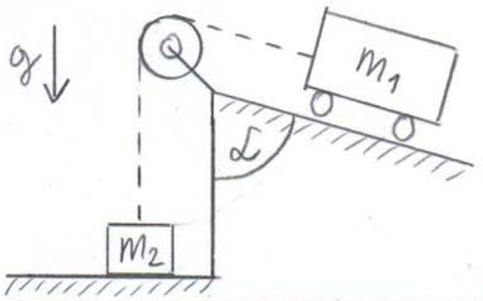
- results in vector form:

$$\vec{N} = N\vec{j} = (160.7\,\vec{j})\,N$$

$$\vec{S} = S\vec{i} = (-52.5\,\vec{i})\,N$$

$$\vec{F_R} = \vec{S} + \vec{N} = (-52.5\,\vec{i} + 160.7\,\vec{j})\,N$$

EXERCISE 3.2



given:

$m_1 = 200\ kg$

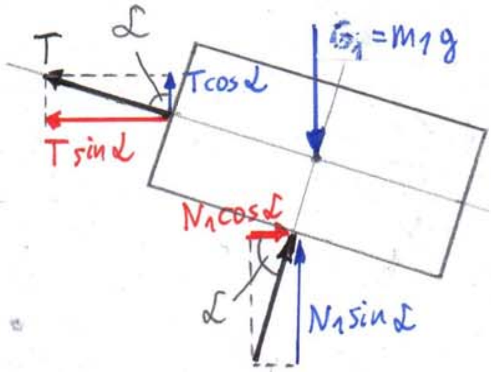
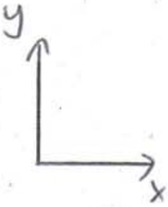
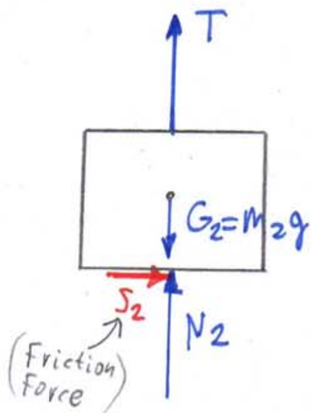
$m_2 = 300\ kg$

$g = 9.81\ \frac{m}{s^2}$

$\alpha = 70^\circ$

task:

- normal forces
- tension forces

	body ①	body ②
1. FBD	 	
2. EE	$\Sigma F_x = 0 \Rightarrow 1)\ N_1 \cos\alpha - T \sin\alpha = 0$ $\Sigma F_y = 0 \Rightarrow 2)\ -m_1 g + N_1 \sin\alpha + T\cos\alpha = 0$	$\Sigma F_x = 0 \Rightarrow 3)\ S_2 = 0$ $\Sigma F_y = 0 \Rightarrow 4)\ N_2 + T - m_2 g = 0$

3. EE sol.

$3) \Rightarrow S_2 = 0$

$1) \Rightarrow N_1 = T\frac{\sin\alpha}{\cos\alpha} = T\tan\alpha \longrightarrow N_1 = 671.9\tan 70^\circ = 1846N$

$1) \to 2) \Rightarrow -m_1 g + T\overbrace{\tan\alpha \sin\alpha}^{2.58} + T\overbrace{\cos\alpha}^{0.34} = 0$

$-1962 + 2.58T + 0.34T = 0$

$2.92T = 1962$

$T = 671.9N$

4.

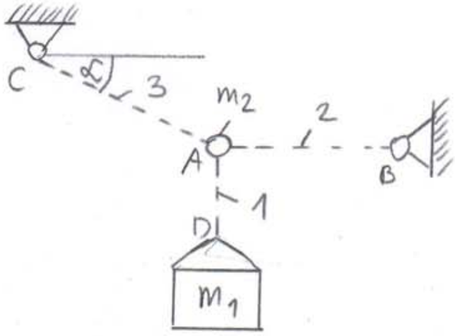
$\vec{T_1} = -T\sin\alpha\,\vec{i} + T\cos\alpha\,\vec{j} = (-631.4\vec{i} + 229.8\vec{j})N$

$\vec{N_1} = N_1\cos\alpha\,\vec{i} + N_1\sin\alpha\,\vec{j} = (631.4\vec{i} + 1734\vec{j})N$

$\vec{T_2} = T\vec{j} = (671.9\vec{j})N$

$\vec{N_2} = N_2\vec{j} = (2271.1\vec{j})N$

EXERCISE 3.3



given:

$m_1 = 75\,kg$

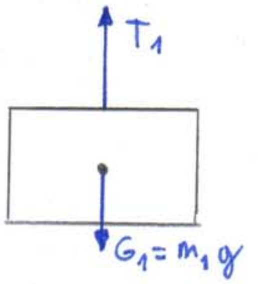
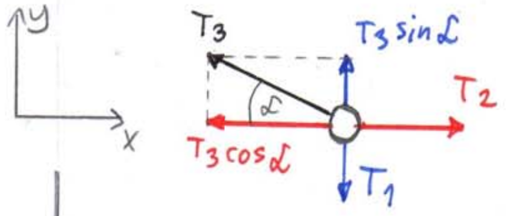
$m_2 \approx 0\,kg$

$g = 9.81\,\frac{m}{s^2}$

$\alpha = 15°$

task:

- tension forces

	body ①	body ②
1. FBD		
2. EE	$\Sigma F_x = 0 \Rightarrow 1)\ 0 = 0$ $\Sigma F_y = 0 \Rightarrow 2)\ T_1 - m_1 g = 0$	$\Sigma F_x = 0 \Rightarrow 3)\ T_2 - T_3 \cos\alpha = 0$ $\Sigma F_y = 0 \Rightarrow 4)\ T_3 \sin\alpha - T_1 = 0$

3. EE SOL.

$2) \Rightarrow T_1 = m_1 g = 75 \cdot 9.81 = 735.8\,N$

$2) \to 4) \Rightarrow T_3 = \frac{T_1}{\sin\alpha} = \frac{735.8}{\sin 15°} = 2842.9\,N$

$4) \to 3) \Rightarrow T_2 = T_3 \cos\alpha = 2842.9 \cdot \cos 15° = 2746\,N$

4.

Body ①:

$\vec{T}_1^{①} = T_1 \vec{j} = (735.8\,\vec{j})\,N$

Body ②:

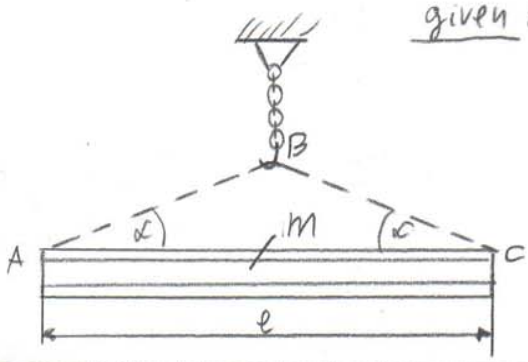
$\vec{T}_1^{②} = -T_1 \vec{j} = (-735.8\,\vec{j})\,N$

$\vec{T}_2^{②} = T_2 \vec{i} = (2746\,\vec{i})\,N$

$\vec{T}_3^{②} = -T_3 \cos\alpha\,\vec{i} + T_3 \sin\alpha\,\vec{j} =$

$= (-2746\,\vec{i} + 735.8\,\vec{j})\,N$

EXERCISE 3.4



given: $m = 350\,kg$

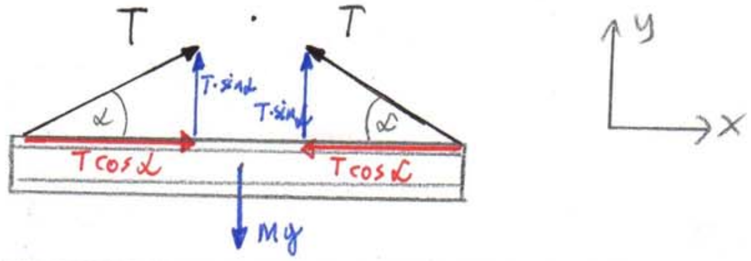
$l = 6\,m$

$g = 9.81\,\frac{m}{s^2}$

task:

Determine the shortest cable ABC that can be used to lift the beam if the maximum force the cable can sustain is 7500N

1. Free body diagram (FBD):



2. equations of equilibrium (EE)

$\Sigma F_x = 0 \Rightarrow 1)\ T\cos\alpha - T\cos\alpha = 0$

$\Sigma F_y = 0 \Rightarrow 2)\ T\sin\alpha + T\sin\alpha - mg = 0$

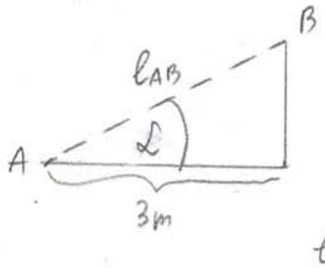
3. solution of the equations of equilibrium

max. force: 7500 N $\Rightarrow T = 7500N$

$2) \Rightarrow 2T\sin\alpha = mg$

$$\sin\alpha = \frac{mg}{2T} = \frac{350 \cdot 9.81}{2 \cdot 7500} = 0.2289$$

$$\alpha = \sin^{-1}(0.2289) = 13.23^\circ$$



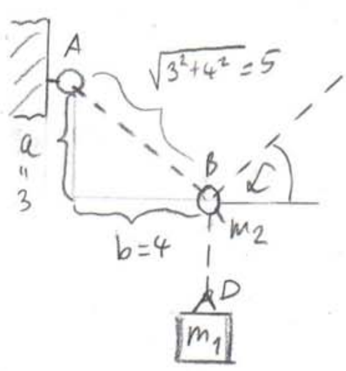
$$\cos\alpha = \frac{3}{l_{AB}}$$

$$\Downarrow$$

$$l_{AB} = \frac{3}{\cos\alpha} = \frac{3}{\cos(13.23)} = 3.082$$

$$l_{ABC} = 2 \cdot l_{AB} = 2 \cdot 3.082 = 6.164$$

EXERCISE 3.5



given:

$a = 3m$

$b = 4m$

$\mathcal{L} = 45°$

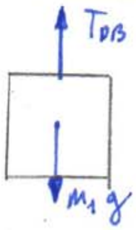
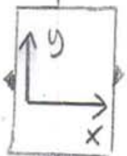
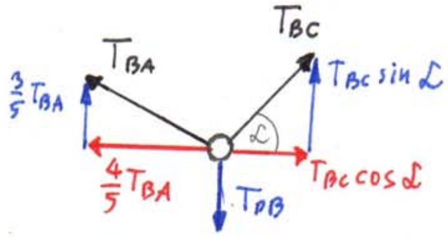
$m_1 = 60kg$

$m_2 \approx 0kg$

$g = 9.81 \frac{m}{s^2}$

task:

Determine the tension in cables BA, BC and DB

	body ①	body ②
1. FBD		 
2. EE	$\Sigma F_x = 0 \Rightarrow 1)\ 0 = 0$ $\Sigma F_y = 0 \Rightarrow 2)\ T_{DB} - m_1 g = 0$	$\Sigma F_x = 0 \Rightarrow 3)\ T_{BC}\cos\mathcal{L} - \frac{4}{5} T_{BA} = 0$ $\Sigma F_y = 0 \Rightarrow 4)\ \frac{3}{5} T_{BA} + T_{BC}\sin\mathcal{L} - T_{DB} = 0$

3. EE SOL

$2) \Rightarrow T_{DB} = m_1 g = 60 \cdot 9.81 = 588.6N$

$3) \Rightarrow T_{BA} = T_{BC}\cos\mathcal{L} \cdot \frac{5}{4} = 0.8839 T_{BC}$ ⟶ $T_{BA} = 0.8839 \cdot 475.8 = 420.6N$

$3) \to 4) \Rightarrow \frac{3}{5} \cdot \underbrace{0.8839 T_{BC}}_{T_{BA}} + T_{BC} \underbrace{\sin\mathcal{L}}_{0.7071} - \underbrace{588.6}_{T_{DB}} = 0$

$1.237 T_{BC} = 588.6$

$T_{BC} = 475.8N$

4.

Body ①:

$\vec{T}_{DB}^{①} = T_{DB}\vec{j} = (588.6\vec{j})N$

Body ②:

$\vec{T}_{DB}^{②} = -T_{DB}\vec{j} = (-588.6\vec{j})N$

$\vec{T}_{BA}^{②} = -\frac{4}{5}T_{BA}\vec{i} + \frac{3}{5}T_{BA}\vec{j} =$

$= -\frac{4}{5} \cdot 420.6\vec{i} + \frac{3}{5}420.6\vec{j} =$

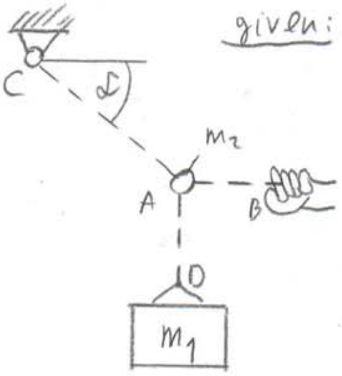
$= (-336.48\vec{i} + 252.36\vec{j})N$

$\vec{T}_{BC}^{②} = T_{BC}\cos\mathcal{L}\,\vec{i} + T_{BC}\sin\mathcal{L}\,\vec{j} =$

$= 475.8\cos 45°\vec{i} + 475.8\sin 45°\vec{j} =$

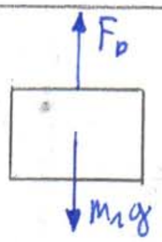
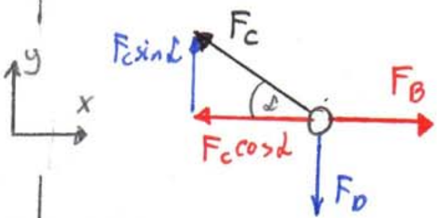
$= (336.48\vec{i} + 336.48\vec{j})N$

EXERCISE 3.6



given: $m_1 = 200$ kg
$m_2 \approx 0$ kg
$g = 9.81 \frac{m}{s^2}$

task: Each rope can withstand a maximum force of 10 kN before it breaks
If AB remains always horizontal, determine the smallest angle α to which the crate can be suspended before one of the ropes breaks

	body ①	body ②
1. FBD		
2. EE	$\Sigma F_x = 0 \Rightarrow 1)\ 0 = 0$ $\Sigma F_y = 0 \Rightarrow 2)\ F_D - m_1 g = 0$	$\Sigma F_x = 0 \Rightarrow 3)\ F_B - F_C \cos\alpha = 0$ $\Sigma F_y = 0 \Rightarrow 4)\ F_C \sin\alpha - F_D = 0$

3. EF SOL

F_C is always greater than F_B since $\cos\alpha \leq 1 \Rightarrow$ rope AC will reach the maximum tensile force of 10 kN $\Rightarrow F_C = 10$ kN

$2) \Rightarrow F_D = m_1 g = 200 \cdot 9.81 = 1962\ N$

$2) \to 4) \Rightarrow \sin\alpha = \frac{F_D}{F_C} = \frac{1962}{10\,000} = 0.1962$

$\alpha = \sin^{-1}(0.1962) = 11.31^\circ$

$3) \to F_B = F_C \cos\alpha = 10\,000 \cdot \cos(11.31) = 9805.8 N$

4.

body ①:

$\vec{F}_D^{①} = F_D \vec{j} = (1962 \vec{j})\ N$

body ②:

$\vec{F}_D^{②} = -F_D \vec{j} = (-1962 \vec{j})\ N$

$\vec{F}_B^{②} = F_B \vec{i} = (9805.8 \vec{i})\ N$

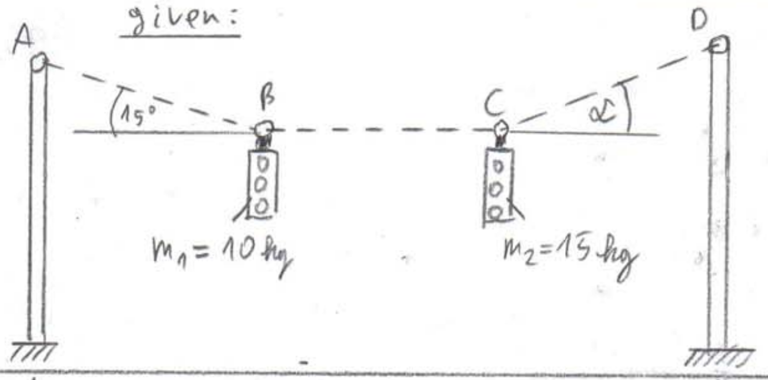
$\vec{F}_C^{②} = -F_C \cos\alpha\, \vec{i} + F_C \sin\alpha\, \vec{j} =$

$= -10\,000 \cos 11.31^\circ \vec{i} + 10\,000 \sin 11.31^\circ \vec{j} =$

$= (-9805.8 \vec{i} + 1962 \vec{j})\ N$

EXERCISE 3.7

given:

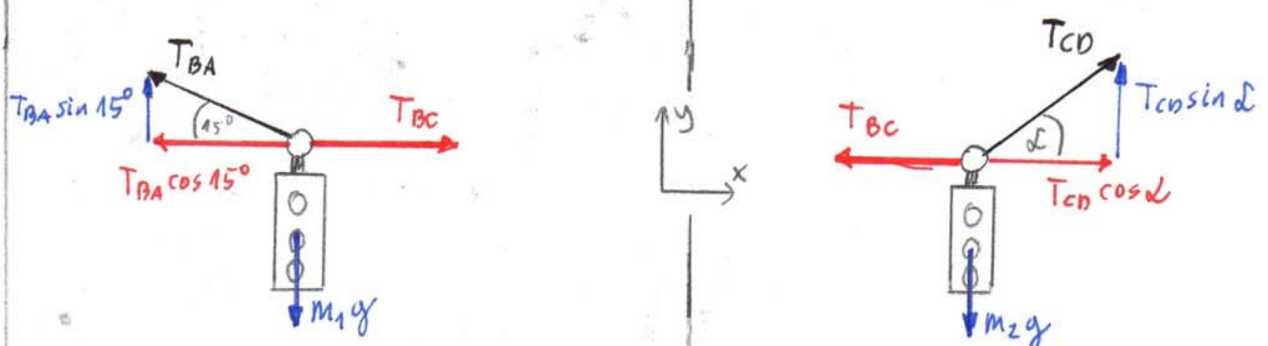


task:

- Determine the tension in cables AB, BC and CD, necessary to support the 10 kg and 15 kg traffic lights
- Also find the angle α

body ① | body ②

1. FBD



2. EE

Body ①:

$\Sigma F_x = 0 \Rightarrow 1)\ T_{BC} - T_{BA}\cos 15° = 0$

$\Sigma F_y = 0 \Rightarrow 2)\ T_{BA}\sin 15° - m_1 g = 0$

Body ②:

$\Sigma F_x = 0 \Rightarrow 3)\ T_{CD}\cos\alpha - T_{BC} = 0$

$\Sigma F_y = 0 \Rightarrow 4)\ T_{CD}\sin\alpha - m_2 g = 0$

3. EE SOL.

$2) \Rightarrow T_{BA} = \dfrac{m_1 g}{\sin 15°} = \dfrac{10 \cdot 9.81}{\sin 15°} = 379\,N$

$2) \to 1) \Rightarrow T_{BC} = T_{BA}\cos 15° = 379 \cdot \cos 15° = 366.1\,N$

$3) \Rightarrow T_{CD} = \dfrac{T_{BC}}{\cos\alpha} \longrightarrow T_{CD} = \dfrac{366.1}{\cos(21.9°)} = 394.6\,N$

$3) \to 4) \Rightarrow \dfrac{T_{BC}}{\cos\alpha}\sin\alpha - m_2 g = 0$

$T_{BC}\tan\alpha = m_2 g$

$\tan\alpha = \dfrac{m_2 g}{T_{BC}} = \dfrac{15 \cdot 9.81}{366.1} = 0.402 \Rightarrow \alpha = \tan^{-1}(0.402) = 21.9°$

4.

Body ①:

$\vec{T}_{BA}^{①} = -T_{BA}\cos 15°\,\vec{i} + T_{BA}\sin 15°\,\vec{j} = (-366.1\,\vec{i} + 98.1\,\vec{j})\,N$

$\vec{T}_{BC}^{①} = T_{BC}\,\vec{i} = (366.1\,\vec{i})\,N$

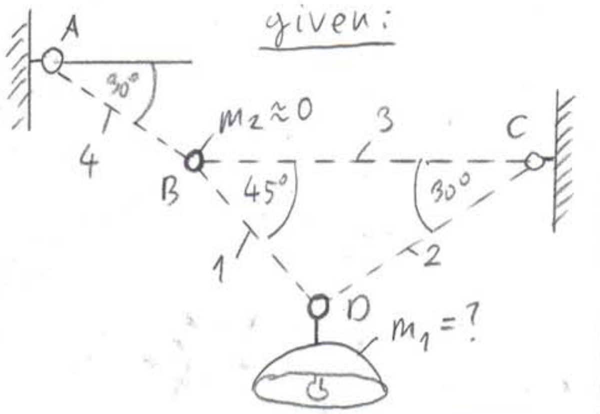
Body ②:

$\vec{T}_{CD}^{②} = T_{CD}\cos\alpha\,\vec{i} + T_{CD}\sin\alpha\,\vec{j} = (366.1\,\vec{i} + 147.2\,\vec{j})\,N$

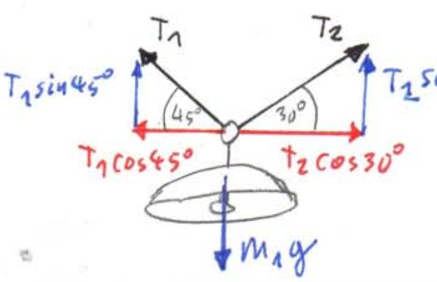
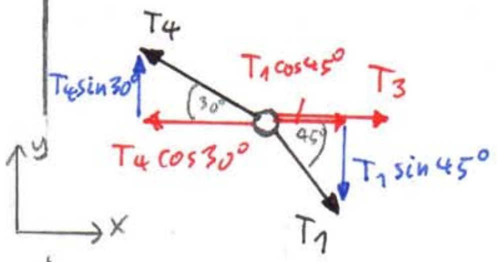
$\vec{T}_{BC}^{②} = -T_{BC}\,\vec{i} = (-366.1\,\vec{i})\,N$

EXERCISE 3.8

given:



task: If the tension in each of the four wires is not allowed to exceed 600 N, determine the maximum mass of the chandelier that can be supported.

	body ①	body ②
1 FBD		
2 EE	$\Sigma F_x = 0 \Rightarrow 1)\ T_2 \cos 30° - T_1 \cos 45° = 0$ $\Sigma F_y = 0 \Rightarrow 2)\ T_1 \sin 45° + T_2 \sin 30° - m_1 g = 0$	$\Sigma F_x = 0 \Rightarrow 3)\ T_3 + T_1 \cos 45° - T_4 \cos 30° = 0$ $\Sigma F_y = 0 \Rightarrow 4)\ T_4 \sin 30° - T_1 \sin 45° = 0$

3 EE SOL

$$T_{max} = T_4 = 600\text{N}$$

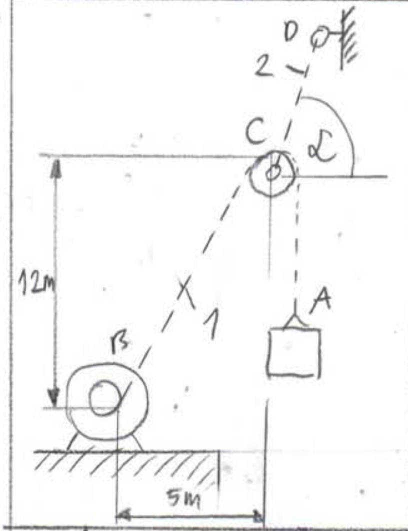
$$4) \Rightarrow T_1 = \frac{T_4 \sin 30°}{\sin 45°} = \frac{600 \cdot \sin 30°}{\sin 45°} = 424.3\text{N}$$

$$3) \Rightarrow T_3 = T_4 \cos 30° - T_1 \cos 45° = 600 \cos 30° - 424.3 \cos 45° = 219.6\text{N}$$

$$1) \Rightarrow T_2 = \frac{T_1 \cos 45°}{\cos 30°} = \frac{424.3 \cos 45°}{\cos 30°} = 346.4\text{N}$$

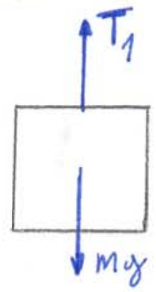
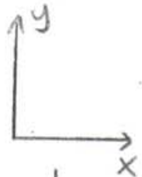
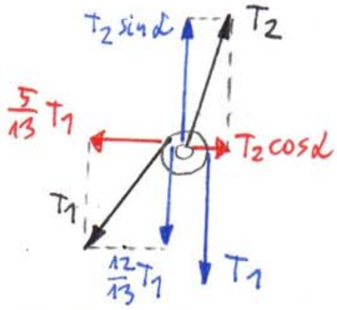
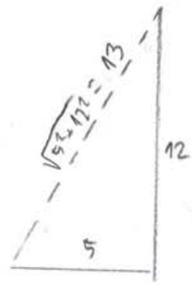
$$2) \Rightarrow m_1 = \frac{T_1 \sin 45° + T_2 \sin 30°}{g} = \frac{424.3 \sin 45° + 346.4 \sin 30°}{9.81} = \underline{\underline{48.2\ \text{kg}}}$$

EXERCISE 3.9



task:

The cords BCA and CD can each support a maximum load of 500N Determine the maximum weight of the crate that can be hoisted at a constant velocity and the angle α for equilibrium Neglect the size of the smooth pulley at C.

	body ①	body ②
1 FBD	 	 
2 EE	$\Sigma F_x = 0 \Rightarrow 1)\ 0 = 0$ $\Sigma F_y = 0 \Rightarrow 2)\ T_1 - mg = 0$	$\Sigma F_x = 0 \Rightarrow 3)\ T_2 \cos\alpha - \frac{5}{13}T_1 = 0$ $\Sigma F_y = 0 \Rightarrow 4)\ T_2 \sin\alpha - T_1 - \frac{12}{13}T_1 = 0$

3 EE sol

$T_{max} = T_2 = 500N$

$2) \Rightarrow T_1 = mg$

$3) \Rightarrow T_1 = \frac{13 T_2 \cos\alpha}{5} \longrightarrow 3)\ 5mg = 13 T_2 \cos\alpha$

$4) \Rightarrow T_2 \sin\alpha - T_1\left(1 + \frac{12}{13}\right) = 0$

$T_2 \sin\alpha = \frac{25}{13} T_1$

$T_1 = \frac{13 T_2 \sin\alpha}{25}$

$4)\ 25mg = 13 T_2 \sin\alpha$

$4):3):\quad 5 = \frac{\sin\alpha}{\cos\alpha}$

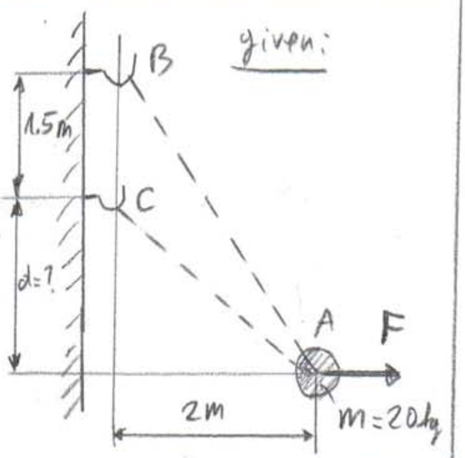
$5 = \tan\alpha$

$\alpha = 78.69^\circ$

$3) \rightarrow m = \frac{13 T_2 \cos\alpha}{5g} = \frac{13 \cdot 500 \cdot \cos 78.69^\circ}{5 \cdot 9.81} = 26\ kg$

$2) \Rightarrow T_1 = mg = 26 \cdot 9.81 = 225.63\ N$

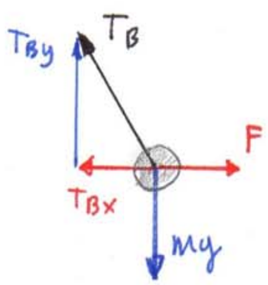
EXERCISE 3.10



task:

If a force F = 100 N is applied horizontally on the ball, determine the dimension d, so that the force in cable AC is zero.

1. Free body diagram



2. equations of equilibrium

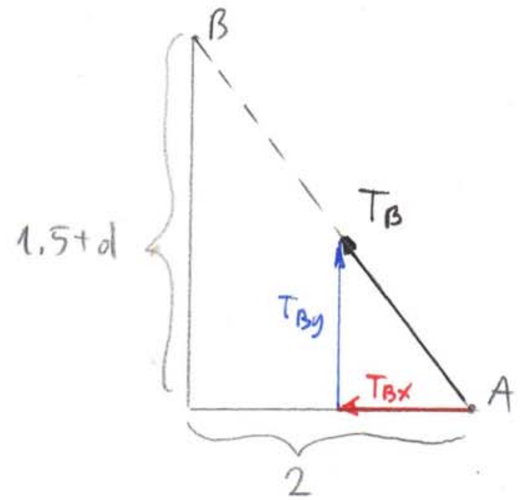
$\Sigma F_x = 0 \Rightarrow 1)\ -T_{Bx} + F = 0$

$\Sigma F_y = 0 \Rightarrow 2)\ T_{By} - mg = 0$

3. solution of the equation of equilibrium

$1) \Rightarrow T_{Bx} = F = 100N$

$2) \Rightarrow T_{By} = mg = 20 \cdot 9.81 = 196.2$



$$\frac{T_{By}}{T_{Bx}} = \frac{1.5+d}{2}$$

$$d = 2 \cdot \frac{T_{By}}{T_{Bx}} - 1.5 = 2 \cdot \frac{196.2}{100} - 1.5 = \underline{\underline{2.424m}}$$