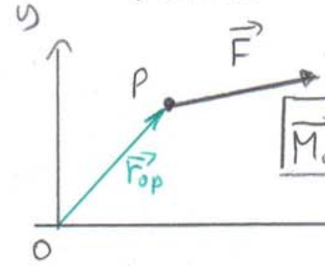
SUMMARY

• MOMENT

a) moment about a point

⇓

vector

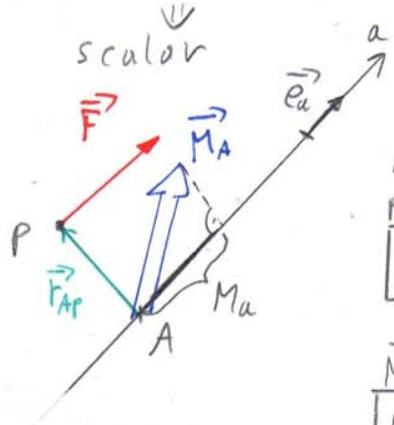


$$\boxed{\vec{M}_O = \vec{r}_{OP} \times \vec{F}}$$

b) moment about an axis

⇓

scalar



step 1:

Moment about an arbitrary point of the axis:

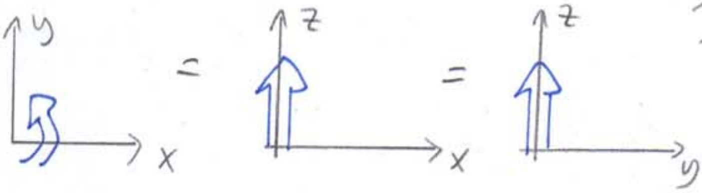
$$\boxed{\vec{M}_A = \vec{r}_{AP} \times \vec{F}}$$

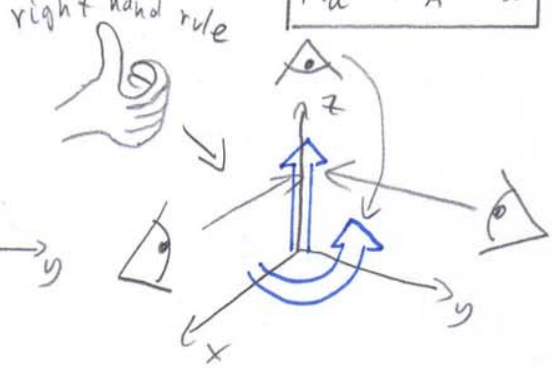
step 2:

Moment about the axis

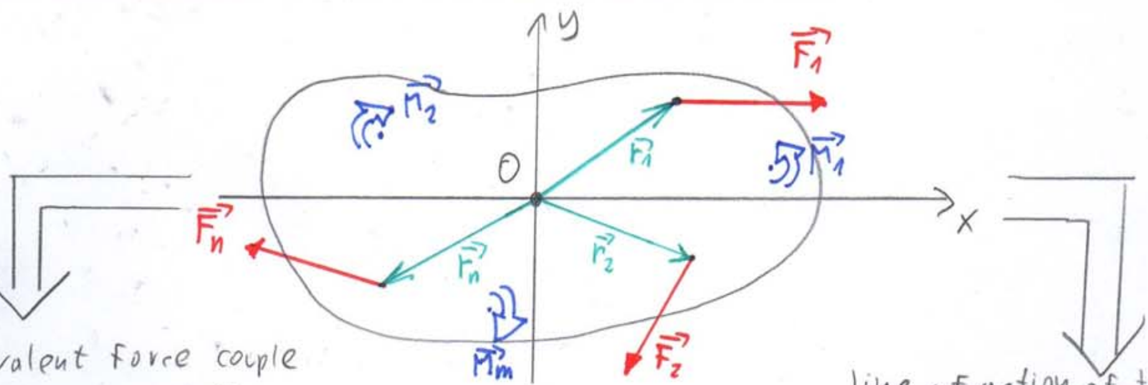
$$\boxed{M_a = \vec{M}_A \cdot \vec{e}_a}$$

notation of moment:

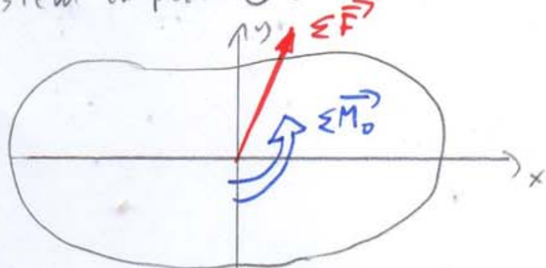




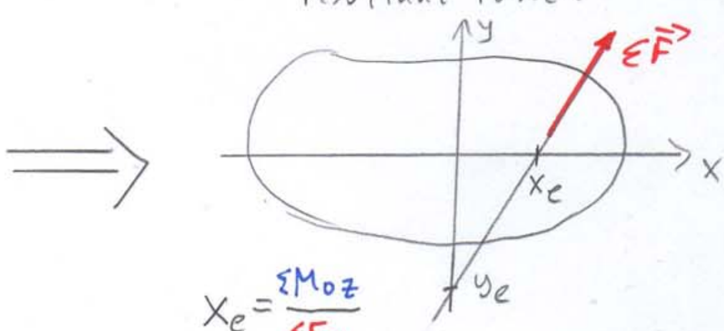
• EQUIVALENT FORCE COUPLE SYSTEM



equivalent force couple system in point O:



line of action of the resultant force:



$$\Sigma\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \left(\Sigma F_x \vec{i} + \Sigma F_y \vec{j}\right)$$

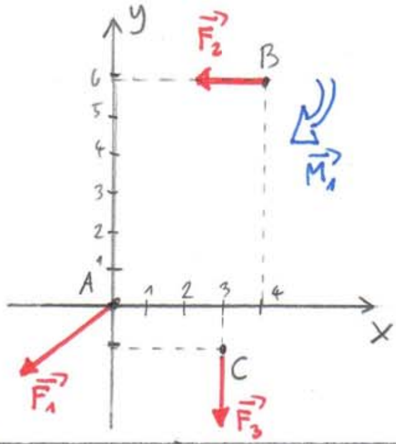
$$\Sigma\vec{M}_O = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{m} \vec{M}_j = \Sigma M_{Oz} \vec{k}$$

$$x_e = \frac{\Sigma M_{Oz}}{\Sigma F_y}$$

$$y_e = -\frac{\Sigma M_{Oz}}{\Sigma F_x}$$

EXERCISE 4.1

given:



$\vec{F_1} = (-8\vec{i} - 5\vec{j})\,N$

$F_2 = 12\,N$

$F_3 = 20\,N$

$M_1 = 12\,Nm$

task:

a) equivalent Force couple system in point O

b) equivalent Force couple system in point C

a) • Force vectors: $\vec{F_1} = (-8\vec{i} - 5\vec{j})\,N$

$\vec{F_2} = (-12\vec{i})\,N$

$\vec{F_3} = (-20\vec{j})\,N$

• position vectors: $\vec{r_1} = \vec{r_{AA}} = \vec{0}$

$\vec{r_2} = \vec{r_{AB}} = (4\vec{i} + 6\vec{j})\,m$

$\vec{r_3} = \vec{r_{AC}} = (3\vec{i} - \vec{j})\,m$

• resultant Force: $\Sigma\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} = (-20\vec{i} - 25\vec{j})\,N$

• resultant moment about point A: $\Sigma\vec{M_A} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2} + \vec{r_3}\times\vec{F_3} + \vec{M_1} = *$

• $\vec{r_1}\times\vec{F_1} = \vec{0}$

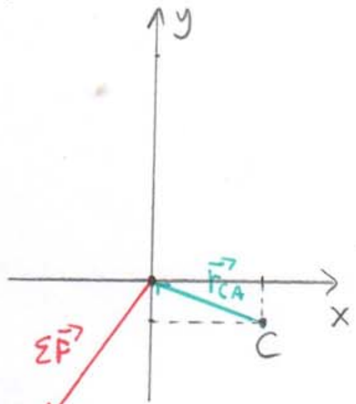
• $\vec{r_2}\times\vec{F_2} = (4\vec{i} + 6\vec{j})\times(-12\vec{i}) = \overbrace{4\cdot(-12)}^{-48}\overbrace{\vec{i}\times\vec{i}}^{\vec{0}} + \overbrace{6\cdot(-12)}^{-72}\overbrace{\vec{j}\times\vec{i}}^{-\vec{k}} = (72\vec{k})\,Nm$

• $\vec{r_3}\times\vec{F_3} = (3\vec{i} - \vec{j})\times(-20\vec{j}) = \underbrace{3\cdot(-20)}_{-60}\underbrace{\vec{i}\times\vec{j}}_{\vec{k}} + \underbrace{(-1)\cdot(-20)}_{20}\underbrace{\vec{j}\times\vec{j}}_{\vec{0}} = (-60\vec{k})\,Nm$

• $\vec{M_1} = (-12\vec{k})\,Nm$

$* = \vec{0} + 72\vec{k} - 60\vec{k} - 12\vec{k} = \vec{0}$

b) the original Force system can be replaced with it's equivalent Force couple system, so we can utilize this



$\Sigma\vec{F} = (-20\vec{i} - 25\vec{j})\,N$

$\Sigma\vec{M_C} = \vec{r_{CA}}\times\Sigma\vec{F} + \Sigma\vec{M_A} = *$

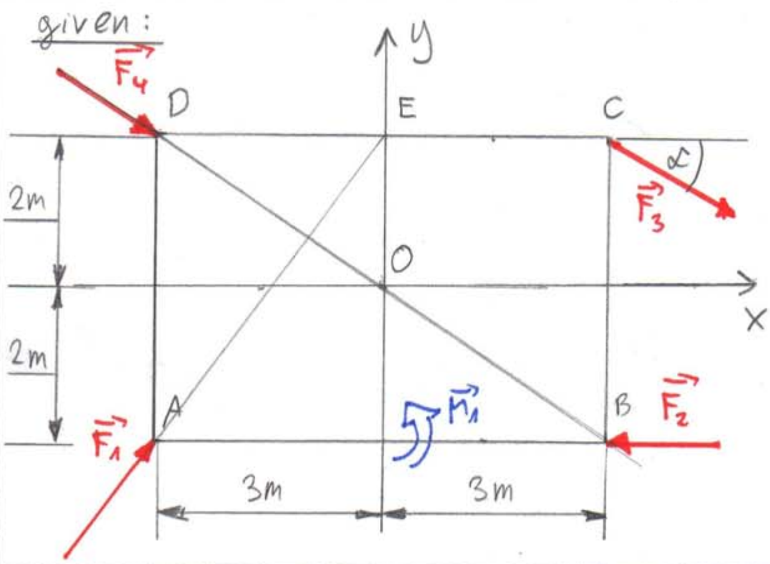
• $\vec{r_{CA}}\times\Sigma\vec{F} = (-3\vec{i} + \vec{j})\times(-20\vec{i} - 25\vec{j}) =$

$= (-3)\cdot(-20)\underbrace{\vec{i}\times\vec{i}}_{\vec{0}} + \underbrace{(-3)(-25)}_{75}\underbrace{\vec{i}\times\vec{j}}_{\vec{k}} + \underbrace{1\cdot(-20)}_{-20}\underbrace{\vec{j}\times\vec{i}}_{-\vec{k}} + 1\cdot(-25)\underbrace{\vec{j}\times\vec{j}}_{\vec{0}} =$

$= (95\vec{k})\,Nm$

$* = 95\vec{k} + \vec{0} = (95\vec{k})\,Nm$

EXERCISE 4.2



$F_1 = 15\,kN$

$F_2 = 12\,kN$

$F_3 = 5\,kN$

$F_4 = 2\sqrt{13}\,kN$

$M_1 = 45\,kNm$

$\alpha = 36.87^\circ$

task:

a) equivalent force couple system in point O

b) equivalent force couple system in point B

c) line of action of the resultant force

a)

- Force vectors:

$$\vec{F_1} = F_1\vec{e_1} = F_1\frac{\vec{r_{AE}}}{|\vec{r_{AE}}|} = 15\cdot\frac{3\vec{i}+4\vec{j}}{\sqrt{3^2+4^2}} = 15\left(\frac{3}{5}\vec{i}+\frac{4}{5}\vec{j}\right) = (9\vec{i}+12\vec{j})\,kN$$

$$\vec{F_2} = F_2\vec{e_2} = F_2(-\vec{i}) = (-12\vec{i})\,kN$$

$$\vec{F_3} = F_3\vec{e_3} = F_3\cos\alpha\,\vec{i} - F_3\sin\alpha\,\vec{j} = 5\cdot\cos 36.87\,\vec{i} - 5\sin 36.87\,\vec{j} = (4\vec{i}-3\vec{j})\,kN$$

$$\vec{F_4} = F_4\vec{e_4} = F_4\frac{\vec{r_{DB}}}{|\vec{r_{DB}}|} = 2\sqrt{13}\,\frac{6\vec{i}-4\vec{j}}{\underbrace{\sqrt{6^2+4^2}}_{\sqrt{52}=2\sqrt{13}}} = 2\sqrt{13}\left(\frac{6}{2\sqrt{13}}\vec{i}-\frac{4}{2\sqrt{13}}\vec{j}\right) = (6\vec{i}-4\vec{j})\,kN$$

- resultant force:

$$\Sigma\vec{F} = \vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4} = (9\vec{i}+12\vec{j})+(-12\vec{i})+(4\vec{i}-3\vec{j})+(6\vec{i}-4\vec{j}) = (7\vec{i}+5\vec{j})\,kN$$

- moments of the forces:

$$\vec{r_1}\times\vec{F_1} = (-3\vec{i}-2\vec{j})\times(9\vec{i}+12\vec{j}) = -3\cdot 9\,\overbrace{\vec{i}\times\vec{i}}^{\vec{0}} + \overbrace{(-3)\cdot 12}^{-36}\,\overbrace{\vec{i}\times\vec{j}}^{\vec{k}} + \overbrace{(-2)\cdot 9}^{-18}\,\overbrace{\vec{j}\times\vec{i}}^{-\vec{k}} + (-2)\cdot 12\,\overbrace{\vec{j}\times\vec{j}}^{\vec{0}} = (-18\vec{k})\,kNm$$

$$\vec{r_2}\times\vec{F_2} = (3\vec{i}-2\vec{j})\times(-12\vec{i}) = 3\cdot(-12)\underbrace{\vec{i}\times\vec{i}}_{\vec{0}} + \underbrace{(-2)\cdot(-12)}_{24}\underbrace{\vec{j}\times\vec{i}}_{-\vec{k}} = (-24\vec{k})\,kNm$$

$$\vec{r_3}\times\vec{F_3} = (3\vec{i}+2\vec{j})\times(4\vec{i}-3\vec{j}) = 3\cdot 4\underbrace{\vec{i}\times\vec{i}}_{\vec{0}} + \underbrace{3\cdot(-3)}_{-9}\underbrace{\vec{i}\times\vec{j}}_{\vec{k}} + \underbrace{2\cdot 4}_{8}\underbrace{\vec{j}\times\vec{i}}_{-\vec{k}} + 2\cdot(-3)\underbrace{\vec{j}\times\vec{j}}_{\vec{0}} = (-17\vec{k})\,kNm$$

$\vec{r_4}\times\vec{F_4} = \vec{0}$ (the line of action of $\vec{F_4}$ goes through point O)

- concentrated moments:

$$\vec{M_1} = M_1\vec{k} = (45\vec{k})\,kNm$$

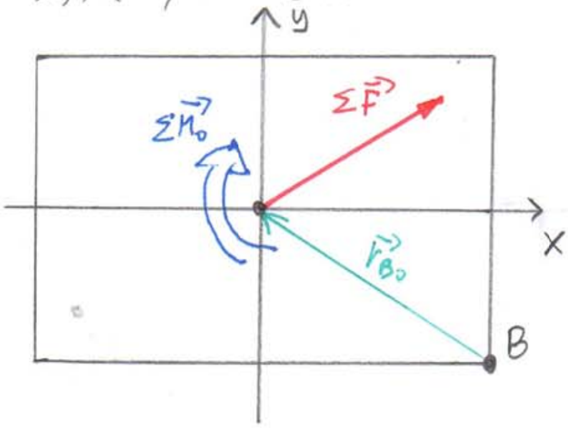
• resultant moment about point O:

$$\Sigma \vec{M}_O = \sum_{i=1}^{4} \vec{r}_i \times \vec{F}_i + \vec{M}_1 = -18\vec{k} - 24\vec{k} - 17\vec{k} + \vec{0} + 45\vec{k} = (-14\vec{k})\ kNm$$

the equivalent force couple system in point O:

$$\boxed{\begin{aligned} \Sigma \vec{F} &= (\overbrace{7}^{\Sigma F_x}\vec{i} + \overbrace{5}^{\Sigma F_y}\vec{j})\ kN \\ \Sigma \vec{M}_O &= (\underbrace{-14}_{\Sigma M_{Oz}}\vec{k})\ kNm \end{aligned}}$$

b) the original force system can be replaced by it's equivalent force couple system, so we can utilize this:



its the same everywhere

$$\Sigma \vec{F} = (7\vec{i} + 5\vec{j})\ kN$$

$$\Sigma \vec{M}_B = \vec{r}_{BO} \times \Sigma\vec{F} + \Sigma\vec{M}_O =$$

$$= (-3\vec{i} + 2\vec{j}) \times (7\vec{i} + 5\vec{j}) + (-14\vec{k}) =$$

$$= \overbrace{(-3)\cdot 5}^{-15} \cdot \overbrace{\vec{i}\times\vec{j}}^{\vec{k}} + \overbrace{2\cdot 7}^{14} \cdot \overbrace{\vec{j}\times\vec{i}}^{-\vec{k}} + (-14\vec{k}) =$$

$$= -15\vec{k} - 14\vec{k} - 14\vec{k} = (-43\vec{k})\ kNm$$

the equivalent force couple system in point B:

$$\boxed{\begin{aligned} \Sigma \vec{F} &= (7\vec{i} + 5\vec{j})\ kN \\ \Sigma \vec{M}_B &= (-43\vec{k})\ kNm \end{aligned}}$$

c) we would like to find those "E" points about which the resultant moment is zero:

this is the goal

$$\Sigma \vec{M}_E = \vec{r}_{EO} \times \Sigma\vec{F} + \Sigma\vec{M}_O \overset{!}{=} 0$$

$$(x_{EO}\vec{i} + y_{EO}\vec{j}) \times (\Sigma F_x \vec{i} + \Sigma F_y \vec{j}) = -\Sigma M_{Oz}\vec{k}$$

$$x_{EO}\,\Sigma F_y\,\vec{k} - y_{EO}\,\Sigma F_x\,\vec{k} = -\Sigma M_{Oz}\vec{k} \quad / \cdot \vec{k}$$

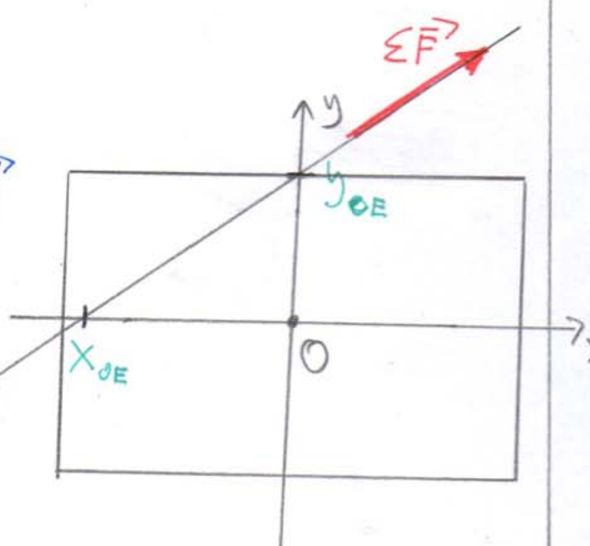
$$x_{EO}\,\Sigma F_y - y_{EO}\,\Sigma F_x = -\Sigma M_{Oz}$$

$x_{EO} = -x_{OE}$
$y_{EO} = -y_{OE}$

$$-x_{OE}\,\Sigma F_y + y_{OE}\,\Sigma F_x = -\Sigma M_{Oz}$$

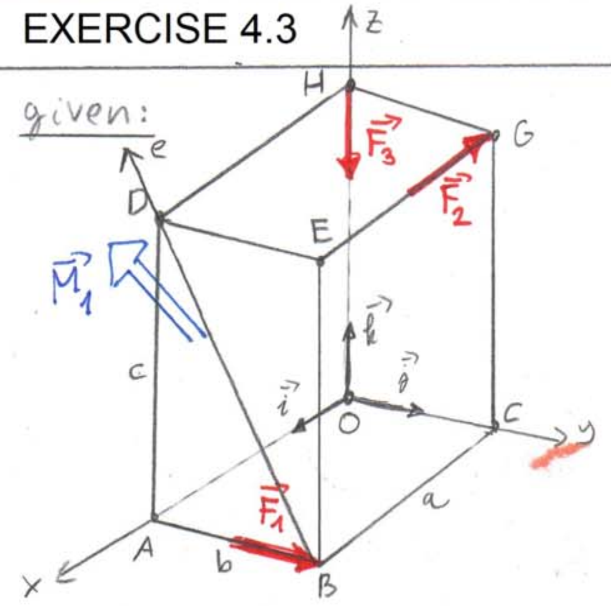
if $x_{OE} = 0 \Rightarrow$ $\boxed{y_{OE} = -\dfrac{\Sigma M_{Oz}}{\Sigma F_x}} = -\dfrac{-14}{7} = 2$

if $y_{OE} = 0 \Rightarrow$ $\boxed{x_{OE} = \dfrac{\Sigma M_{Oz}}{\Sigma F_y}} = \dfrac{-14}{5} = -2{,}8$



EXERCISE 4.3

given:



$a = 2\,m$

$b = 1\,m$

$c = 3\,m$

$F_1 = 10\,kN$

$F_2 = 5\,kN$

$F_3 = 2\,kN$

$\vec{M}_1 = (10\vec{i} + 10\vec{k})\,kNm$

a) equivalent force couple system in point O

b) moments about axis x, y and z

c) moment about axis e

d) what force and moment at point O should be used to balance the body?

a) • force vectors: $\vec{F}_1 = F_1\vec{j} = (10\vec{j})\,kN$

$\vec{F}_2 = -F_2\vec{i} = (-5\vec{i})\,kN$

$\vec{F}_3 = -F_3\vec{k} = (-2\vec{k})\,kN$

• resultant force: $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (-5\vec{i} + 10\vec{j} - 2\vec{k})\,kN$

• position vectors of the forces: $\vec{r}_1 = \vec{r}_B = a\vec{i} + b\vec{j} = (2\vec{i} + \vec{j})\,m$

$\vec{r}_2 = \vec{r}_G = b\vec{j} + c\vec{k} = (\vec{j} + 3\vec{k})\,m$

$\vec{r}_3 = \vec{r}_H = c\vec{k} = (3\vec{k})\,m$

• resultant moment about point O: $\Sigma\vec{M}_0 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 + \vec{M}_1 = *$

• $\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \overset{+}{\vec{i}} & \overset{-}{\vec{j}} & \overset{+}{\vec{k}} \\ 2 & 1 & 0 \\ 0 & 10 & 0 \end{vmatrix} = \vec{i}\underbrace{(1\cdot 0 - 0\cdot 10)}_{0} - \vec{j}\underbrace{(2\cdot 0 - 0\cdot 0)}_{0} + \vec{k}\underbrace{(2\cdot 10 - 1\cdot 0)}_{20} = (20\vec{k})\,kNm$

• $\vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \overset{+}{\vec{i}} & \overset{-}{\vec{j}} & \overset{+}{\vec{k}} \\ 0 & 1 & 3 \\ -5 & 0 & 0 \end{vmatrix} = \vec{i}\overbrace{(1\cdot 0 - 3\cdot 0)}^{0} - \vec{j}\overbrace{(0\cdot 0 - 3\cdot(-5))}^{15} + \vec{k}\overbrace{(0\cdot 0 - 1\cdot(-5))}^{5} =$

$= (-15\vec{j} + 5\vec{k})\,kNm$

• $\vec{r}_3 \times \vec{F}_3 = \begin{vmatrix} \overset{+}{\vec{i}} & \overset{-}{\vec{j}} & \overset{+}{\vec{k}} \\ 0 & 0 & 3 \\ 0 & 0 & -2 \end{vmatrix} = \vec{i}\underbrace{(0\cdot(-2) - 3\cdot 0)}_{0} - \vec{j}\underbrace{(0\cdot(-2) - 3\cdot 0)}_{0} + \vec{k}\underbrace{(0\cdot 0 - 0\cdot 0)}_{0} = \vec{0}$

$* = (20\vec{k}) + (-15\vec{j} + 5\vec{k}) + \vec{0} + (10\vec{i} + 10\vec{k}) = (10\vec{i} - 15\vec{j} + 35\vec{k})\,kNm$

- the equivalent force couple system in point O:

$$\Sigma\vec{F} = (-5\vec{i} + 10\vec{j} - 2\vec{k})\ \text{kN}$$

$$\Sigma\vec{M}_O = (10\vec{i} - 15\vec{j} + 35\vec{k})\ \text{kNm}$$

b) moments about x, y, and z axes:

$$M_x = \Sigma\vec{M}_O \cdot \vec{i} = (10\vec{i} - 15\vec{j} + 33\vec{k}) \cdot \vec{i} = 10\ \text{kNm}$$

$$M_y = \Sigma\vec{M}_O \cdot \vec{j} = (10\vec{i} - 15\vec{j} + 35\vec{k}) \cdot \vec{j} = -15\ \text{kNm}$$

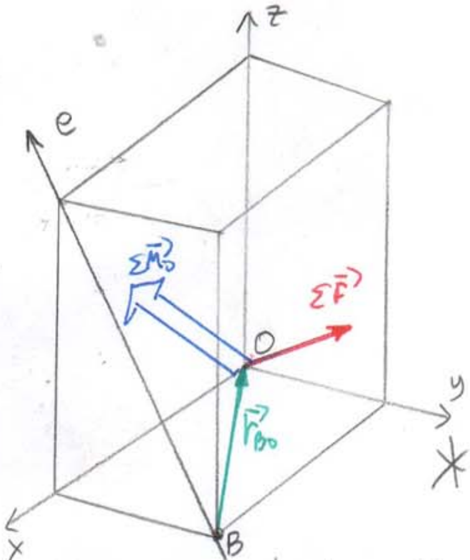
$$M_z = \Sigma\vec{M}_O \cdot \vec{k} = (10\vec{i} - 15\vec{j} + 35\vec{k}) \cdot \vec{k} = 35\ \text{kNm}$$

c) moment about axis e

- unit direction vector of axis e:

$$\vec{e} = \frac{\vec{r}_{BO}}{|\vec{r}_{BO}|} = \frac{-1\vec{j} + 3\vec{k}}{\sqrt{1^2+3^2}} = -\frac{1}{\sqrt{10}}\vec{j} + \frac{3}{\sqrt{10}}\vec{k}$$

- moment about an arbitrary point of axis e (e.g. point B):



$$\Sigma\vec{M}_B = \vec{r}_{BO} \times \Sigma\vec{F} + \Sigma\vec{M}_O = *$$

$$\vec{r}_{BO} \times \Sigma\vec{F} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 0 \\ -5 & 10 & -2 \end{vmatrix} \end{array} =$$

$$= \vec{i}\left(-1\cdot(-2) - 0\cdot 10\right) - \vec{j}\left(-2\cdot(-2) - 0\cdot(-5)\right) + \vec{k}\left(-2\cdot 10 - (-1)\cdot(-5)\right) =$$

$$= (2\vec{i} - 4\vec{j} - 25\vec{k})\ \text{kNm}$$

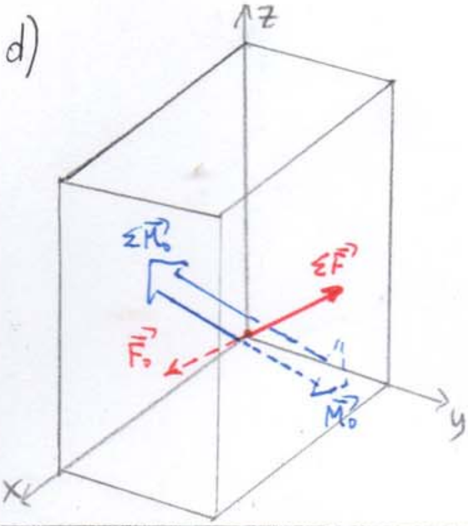
$$* = (2\vec{i} - 4\vec{j} - 25\vec{k}) + (10\vec{i} - 15\vec{j} + 33\vec{k}) = (12\vec{i} - 19\vec{j} + 10\vec{k})\ \text{kNm}$$

- moment about axis e:

$$M_e = \Sigma\vec{M}_B \cdot \vec{e} = (12\vec{i} - 19\vec{j} + 10\vec{k}) \cdot \left(0\vec{i} - \frac{1}{\sqrt{10}}\vec{j} + \frac{3}{\sqrt{10}}\vec{k}\right) =$$

$$= 12\cdot 0 + (-19)\cdot\frac{1}{\sqrt{10}} + 10\cdot\frac{3}{\sqrt{10}} = 3.48\ \text{kNm}$$

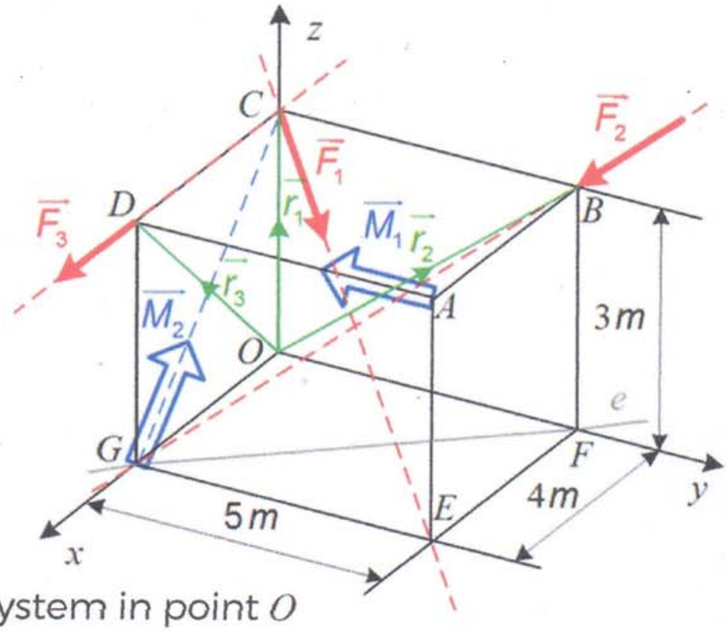
d)



$$\vec{F}_O = -\Sigma\vec{F} = (5\vec{i} - 10\vec{j} + 2\vec{k})\ \text{kN}$$

$$\vec{M}_O = -\Sigma\vec{M}_O = (-10\vec{i} + 15\vec{j} - 35\vec{k})\ \text{kNm}$$

EXERCISE 4.4



given:

$F_1 = 50\sqrt{2}$ N

$\vec{F}_2 = \left(20\vec{i} - 25\vec{j} - 15\vec{k}\right)$ N

$F_3 = 50$ N

$M_1 = 30$ Nm

$M_2 = 40$ Nm

tasks:

a) Equivalent force couple system in point O

b) Moments about axes x, y and z

c) Moment about axes e

solution:

a) Equivalent force couple system in point O

- force vectors:

$$\vec{F}_1 = F_1 \cdot \vec{e}_1 = F_1 \cdot \frac{\vec{r}_{CE}}{\left|\vec{r}_{CE}\right|} = 50\sqrt{2} \cdot \frac{4\vec{i} + 5\vec{j} - 3\vec{k}}{\underbrace{\sqrt{4^2 + 5^2 + 3^2}}_{\sqrt{50} = 5\sqrt{2}}} = 50\sqrt{2} \cdot \frac{4\vec{i} + 5\vec{j} - 3\vec{k}}{5\sqrt{2}} = \left(40\vec{i} + 50\vec{j} - 30\vec{k}\right) \text{N}$$

$$\vec{F}_2 = \left(20\vec{i} - 25\vec{j} - 15\vec{k}\right) \text{N}$$

$$\vec{F}_3 = F_3\vec{i} = \left(50\vec{i}\right) \text{N}$$

- resultant force:

$$\Sigma\vec{F} = \sum_{i=1}^{3}\vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 40\vec{i} + 50\vec{j} - 30\vec{k} + 20\vec{i} - 25\vec{j} - 15\vec{k} + 50\vec{i} =$$

$$= (40 + 20 + 50)\vec{i} + (50 - 25)\vec{j} + (-30 - 15)\vec{k} = \underline{\underline{(110\vec{i} + 25\vec{j} - 45\vec{k})\ \text{N}}}$$

- position vectors of the forces:

$$\vec{r}_1 = \vec{r}_{OG} = \left(3\vec{k}\right) \text{m}$$

$$\vec{r}_2 = \vec{r}_{OB} = \left(5\vec{j} + 3\vec{k}\right) \text{m}$$

$$\vec{r}_3 = \vec{r}_{OD} = \left(4\vec{i} + 3\vec{k}\right) \text{m}$$

- moments of the forces:

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 3 \\ 40 & 50 & -30 \end{vmatrix} = \vec{i}\left(0\cdot(-30) - 3\cdot 50\right) - \vec{j}\left(0\cdot(-30) - 3\cdot 40\right) + \\ +\vec{k}\left(0\cdot 50 - 0\cdot 40\right) = \left(-150\vec{i} + 120\vec{j}\right)\ \mathrm{Nm}$$

$$\vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5 & 3 \\ 20 & -25 & -15 \end{vmatrix} = \vec{i}\left(5\cdot(-15) - 3\cdot(-25)\right) - \vec{j}\left(0\cdot(-15) - 3\cdot 20\right) + \\ +\vec{k}\left(0\cdot(-25) - 5\cdot 20\right) = \left(60\vec{j} - 100\vec{k}\right)\ \mathrm{Nm}$$

$$\vec{r}_3 \times \vec{F}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 0 & 3 \\ 50 & 0 & 0 \end{vmatrix} = \vec{i}\left(0\cdot 0 - 3\cdot 0\right) - \vec{j}\left(4\cdot 0 - 0\cdot 50\right) + \\ +\vec{k}\left(4\cdot 0 - 0\cdot 50\right) = \left(150\vec{j}\right)\ \mathrm{Nm}$$

- concentrated moment vectors:

$$\vec{M}_1 = M_1\left(-\vec{j}\right) = \left(-30\vec{j}\right)\mathrm{Nm}$$

$$\vec{M}_2 = M_2 \cdot \vec{f}_2 = M_2 \cdot \frac{\vec{r}_{GC}}{\left|\vec{r}_{GC}\right|} = 40\cdot\frac{-4\vec{i} + 3\vec{k}}{\underbrace{\sqrt{4^2+3^2}}_{\sqrt{25}=5}} = 40\cdot\frac{-4\vec{i} + 3\vec{k}}{5} = \left(-32\vec{i} + 24\vec{k}\right)\mathrm{Nm}$$

- resultant moment about point O:

$$\vec{M}_O = \sum_{i=1}^{3} \vec{r}_i \times \vec{F}_i + \sum_{j=1}^{2} \vec{M}_j = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 + \vec{M}_1 + \vec{M}_2 =$$

$$= -150\vec{i} + 120\vec{j} + 60\vec{j} - 100\vec{k} + 150\vec{j} - 30\vec{j} - 32\vec{i} + 24\vec{k} =$$

$$= (-150-32)\vec{i} + (120+60+150-30)\vec{j} + (-100+24)\vec{k} =$$

$$= \underline{\underline{(-182\vec{i} + 300\vec{j} - 76\vec{k})\ \mathrm{Nm}}}$$

b) Moments about axes x, y and z

$$M_x = \vec{M}_O \cdot \vec{i} = (-182\vec{i} + 300\vec{j} - 76\vec{k})\cdot\vec{i} = \underline{\underline{-182\ \mathrm{Nm}}}$$

$$M_y = \vec{M}_O \cdot \vec{j} = (-182\vec{i} + 300\vec{j} - 76\vec{k})\cdot\vec{j} = \underline{\underline{300\ \mathrm{Nm}}}$$

$$M_z = \vec{M}_O \cdot \vec{k} = (-182\vec{i} + 300\vec{j} - 76\vec{k})\cdot\vec{k} = \underline{\underline{-76\ \mathrm{Nm}}}$$

c) Moment about axes e

- unit direction vector of axis e:
$$\vec{e} = \frac{\vec{r}_{GF}}{|\vec{r}_{GF}|} = \frac{-4\vec{i}+5\vec{j}}{\sqrt{4^2+5^2}} = \left(-\frac{4}{\sqrt{41}}\vec{i}+\frac{5}{\sqrt{41}}\vec{j}\right)$$

- Moment about an arbitrary ponint of axis e (e.g. point G)
$$\vec{M}_G = \vec{r}_{GO} \times \vec{F} + \vec{M}_O$$

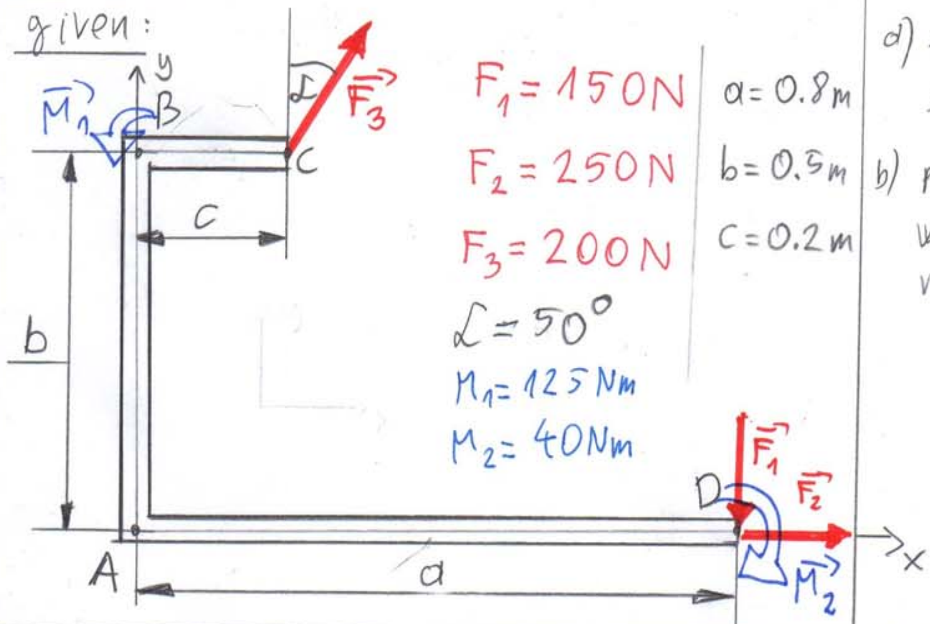
moment of $\vec{F}$:
$$\vec{r}_{GO} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 0 & 0 \\ 110 & 25 & -45 \end{vmatrix} = \vec{i}(0\cdot -45 - 0\cdot 25) - \vec{j}(-4\cdot(-45) - 0\cdot 110) + \\ +\vec{k}(-4\cdot 25 - 0\cdot 110) = (-180\vec{j} - 100\vec{k})\ \text{Nm}$$

Moment about point G:
$$\vec{M}_G = \vec{r}_{GO} \times \vec{F} + \vec{M}_O = -180\vec{j} - 100\vec{k} - 182\vec{i} + 300\vec{j} - 76\vec{k} = (-182\vec{i} + 120\vec{j} - 176\vec{k})\ \text{Nm}$$

Moment about axes´e
$$M_e = \vec{M}_G \cdot \vec{e} = (-182\vec{i} + 120\vec{j} - 176\vec{k}) \cdot \left(-\frac{4}{\sqrt{41}}\vec{i} + \frac{5}{\sqrt{41}}\vec{j} + 0\vec{k}\right) =$$
$$= \left(\frac{182\cdot 4}{\sqrt{41}} + \frac{120\cdot 5}{\sqrt{41}} - 176\cdot 0\right) = \underline{\underline{207.4\ \text{Nm}}}$$

EXERCISE 4.5

given:



$F_1 = 150\,N$ $\quad a = 0.8\,m$

$F_2 = 250\,N$ $\quad b = 0.5\,m$

$F_3 = 200\,N$ $\quad c = 0.2\,m$

$\alpha = 50^\circ$

$M_1 = 125\,Nm$

$M_2 = 40\,Nm$

a) equivalent force couple system in point O

b) point on section AB, about which the resultant moment is zero

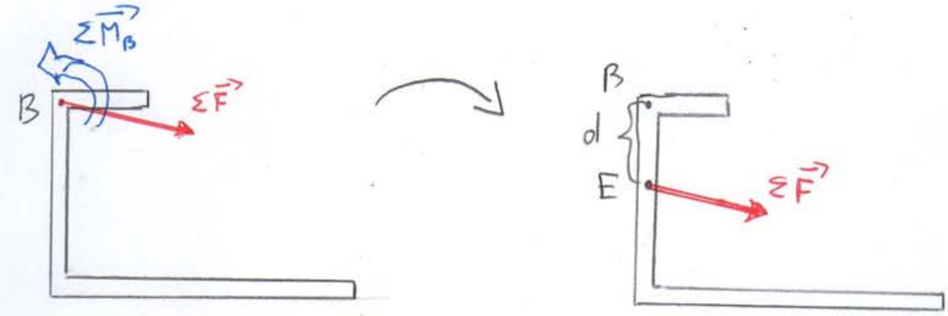
a) $\left.\begin{aligned} \Sigma F_x &= F_2 + F_3 \sin\alpha = 403.2\,N \\ \Sigma F_y &= -F_1 + F_3 \cos\alpha = -21.4\,N \end{aligned}\right\}$ $\Sigma \vec{F} = \Sigma F_x \vec{i} + \Sigma F_y \vec{j} = (403.2\,\vec{i} - 21.4\,\vec{j})\,N$

$$\Sigma M_{Bz} = c \cdot F_3 \cos\alpha + \underbrace{0 \cdot F_3 \sin\alpha}_{0} + b \cdot F_2 - a \cdot F_1 + M_1 - M_2 =$$

$$= 0.2 \cdot 200 \cdot \cos 50^\circ + 0.5 \cdot 250 - 0.8 \cdot 150 + 125 - 40 = 115.7\ Nm$$

$$\Sigma \vec{M_B} = \Sigma M_{Bz}\,\vec{k} = (115.7\,\vec{k})\,Nm$$

b)



$$\Sigma M_{Ez} = -d \cdot \Sigma F_x + \Sigma M_{Bz} = 0$$

this is the goal

$$\Downarrow$$

$$d = \frac{\Sigma M_{Bz}}{\Sigma F_x} = \frac{115.7}{403.2} = 0.287\,m$$

$$\vec{r_E} = (b-d)\,\vec{j} = (0.5 - 0.287)\,\vec{j} = (0.213\,\vec{j})\,m$$