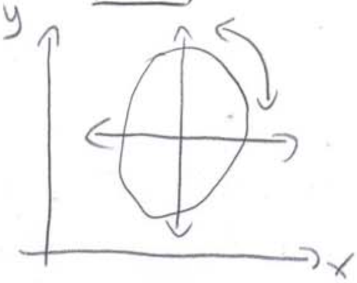
SUMMARY

in planar case:

- number of degrees of freedom (DOF): $\boxed{n=3}$

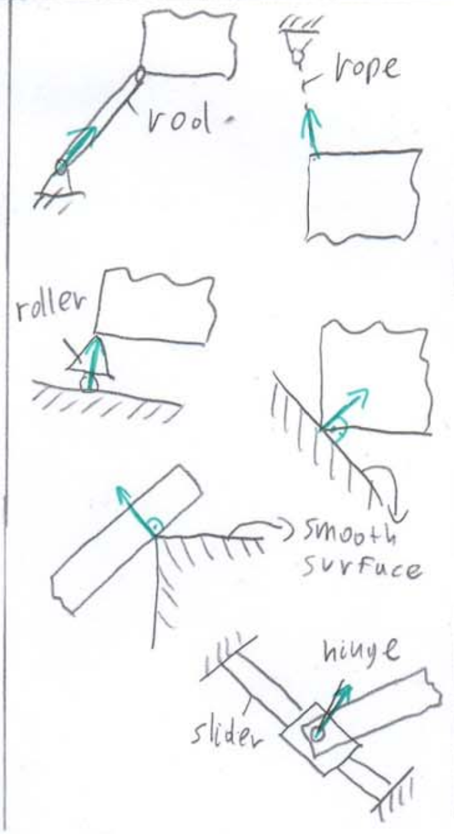
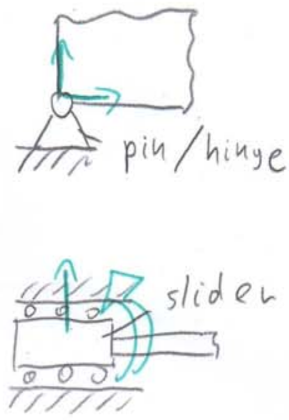
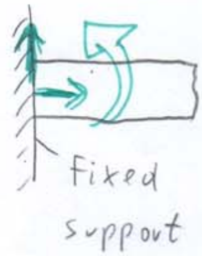


- number of equations of equilibrium: $\boxed{3}$

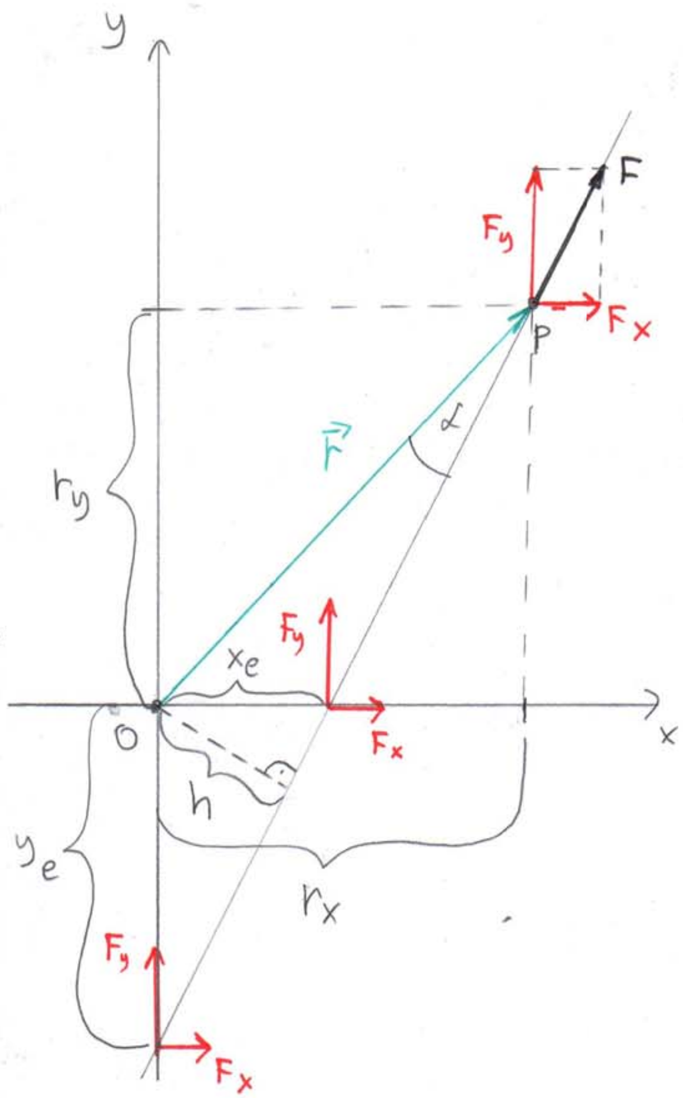
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_{A_z} = 0$$

arbitrary point A

- classification of supports:

number of restrained DOF = number of unknown reaction forces/moment	$n_c = 1$	$n_c = 2$	$n_c = 3$
examples:			

if $\boxed{n = n_c}$ $\Rightarrow$ structure is statically determinate methods to calculate M_{oz}:



① $M_{oz} = \overbrace{(\vec{r} \times \vec{F})}^{\vec{M_o}} \cdot \vec{k}$

where: $|\vec{r} \times \vec{F}| = |\vec{F}| \underbrace{|\vec{r}| \sin\alpha}_{\text{Force arm} \rightarrow h}$

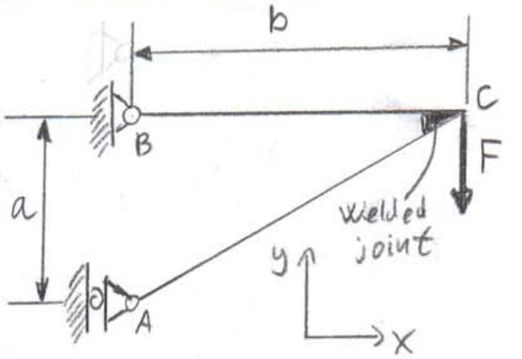
② $M_{oz} = h \cdot F$

③ $M_{oz} = r_x \cdot F_y - r_y F_x$

④ $M_{oz} = x_e \cdot F_y$

$M_{oz} = y_e \cdot F_x$

EXERCISE 5.1



given:

$a = 1.25\,m$

$b = 2.75\,m$

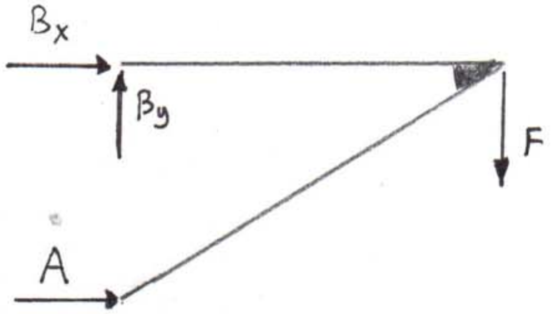
$F = 350\,N$

task:

reaction forces

Because of the welded joint, it is one rigid body

1. Free body diagram:



2. equations of equilibrium (EE)

$\Sigma F_x = 0: \quad 1)\ A + B_x = 0$

$\Sigma F_y = 0: \quad 2)\ B_y - F = 0$

$\Sigma M_{Bz} = 0: \quad 3)\ A \cdot a - F \cdot b = 0$

3. solution of the EE:

$2) \Rightarrow B_y = F = 350\,N\ (\uparrow)$

$3) \Rightarrow A = F\,\frac{b}{a} = 350 \cdot \frac{2.75}{1.25} = 770\,N\ (\rightarrow)$

$1) \Rightarrow B_x = -A = -770\,N\ (\leftarrow)$

2. equations of equilibrium (EE)

$\Sigma F_x = 0: \quad 1)\ A + B_x = 0$

$\Sigma F_y = 0: \quad 2)\ B_y - F = 0$

$\Sigma M_{Az} = 0: \quad 3)\ -B_x\, a - F \cdot b = 0$

3. solution of the EE

$2) \Rightarrow B_y = F = 350\,N\ (\uparrow)$

$3) \Rightarrow B_x = -F\,\frac{b}{a} = -350 \cdot \frac{2.75}{1.25} = -770\,N\ (\leftarrow)$

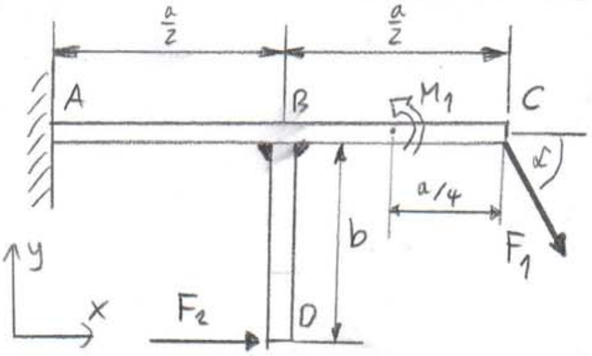
$1) \Rightarrow A = -B_x = -(-770) = 770\,N\ (\rightarrow)$

4. results in vector form:

$\vec{A} = A\,\vec{i} = (770\,\vec{i})\,N$

$\vec{B} = B_x\,\vec{i} + B_y\,\vec{j} = (-770\,\vec{i} + 350\,\vec{j})\,N$

EXERCISE 5.2



given:

$a = 2m$ $\quad M_1 = 200Nm$

$b = 0.8m$ $\quad F_2 = 150N$

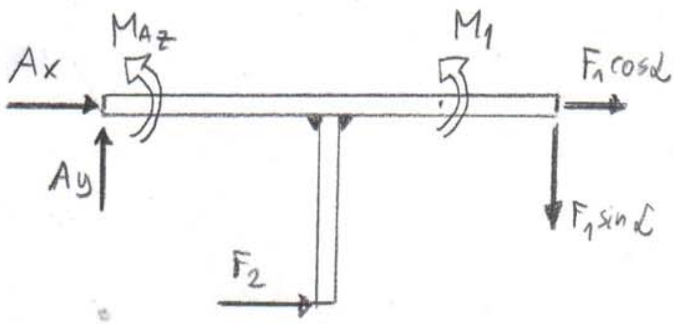
$F_1 = 400N$

$\alpha = 60°$

task:

- reaction forces
- moment of F_1 about point D

1. Free body diagram (FBD)



2. equations of equilibrium (EE)

$$\Sigma F_x = 0 \Rightarrow 1)\ A_x + F_1\cos\alpha + F_2 = 0$$

$$\Sigma F_y = 0 \Rightarrow 2)\ A_y - F_1\sin\alpha = 0$$

$$\Sigma M_{Az} = 0 \Rightarrow 3)\ M_{Az} + M_1 + F_2 \cdot b - F_1\sin\alpha\, a = 0$$

3. solution of the EE:

$$1) \Rightarrow A_x = -F_2 - F_1\cos\alpha = -150 - 400\cos 60° = -350N\ (\leftarrow)$$

$$2) \Rightarrow A_y = F_1\sin\alpha = 400\sin 60° = 346N\ (\uparrow)$$

$$3) \Rightarrow M_{Az} = F_1\sin\alpha\, a - F_2 \cdot b - M_1 = 400\sin 60° \cdot 2 - 150 \cdot 0.8 - 200 = 372.8\,Nm\ (\circlearrowleft)$$

4. results in vector form:

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = (-350\vec{i} + 346\vec{j})N$$

$$\vec{M_A} = M_{Az}\vec{k} = (372.8\vec{k})\,Nm$$

moment of F_1 about point D:

- method 1:

$$\vec{F_1} = F_1\cos\alpha\,\vec{i} - F_1\sin\alpha\,\vec{j} = (200\vec{i} - 346.4\vec{j})N$$

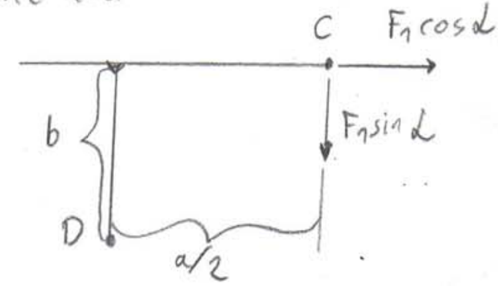
$$\vec{r}_{DC} = \frac{a}{2}\vec{i} + b\vec{j} = (1\vec{i} + 0.8\vec{j})\,m$$

$$\vec{M_D} = \vec{r}_{DC} \times \vec{F_1} = \begin{vmatrix} \overset{+}{\vec{i}} & \overset{-}{\vec{j}} & \overset{+}{\vec{k}} \\ 1 & 0.8 & 0 \\ 200 & -346.4 & 0 \end{vmatrix} =$$

$$= \vec{i}\,\overbrace{(0.8 \cdot 0 - 0 \cdot (-346.4))}^{0} - \vec{j}\,\overbrace{(1 \cdot 0 - 0 \cdot 200)}^{0} +$$

$$+ \vec{k}\,\underbrace{(1 \cdot (-346.4) - 0.8 \cdot 200)}_{-506.4} = (-506.4\vec{k})\,Nm$$

- method 2:



$$M_{Dz} = -F_1\cos\alpha\, b - F_1\sin\alpha\,\frac{a}{2} =$$

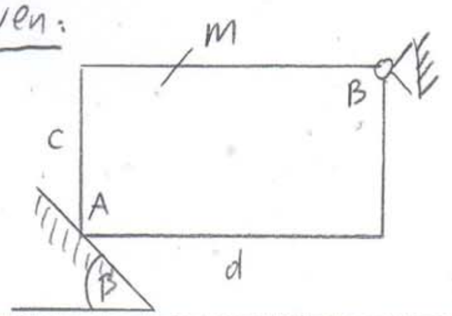
$$= -400\cos 60° \cdot 0.8 - 400\sin 60° \cdot \frac{2}{2} =$$

$$= -506.4\,Nm$$

$$\vec{M_D} = M_{Dz}\vec{k} = (-506.4\vec{k})\,Nm$$

EXERCISE 5.3

given:



$c = 300\,mm$

$d = 700\,mm$

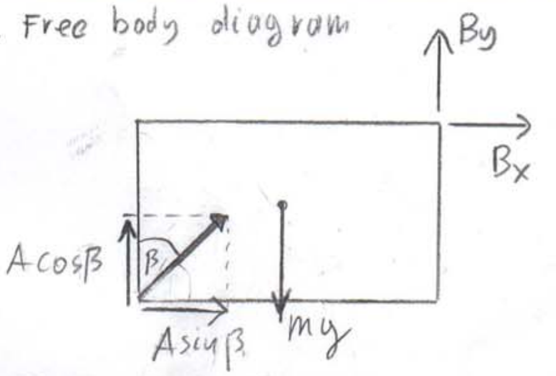
$m = 24\,kg$

$g = 9.81\,\frac{m}{s^2}$

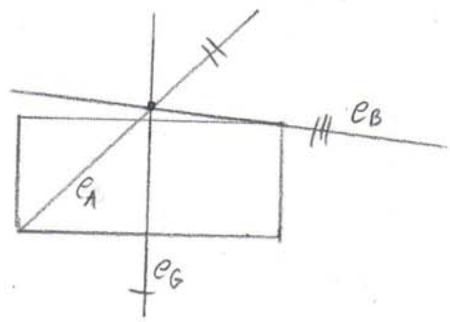
$\beta = 45^\circ$

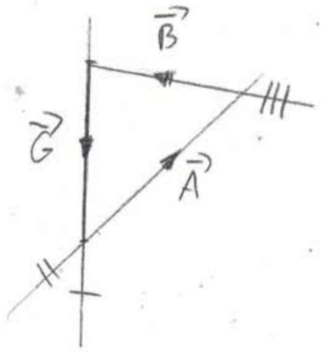
task: reaction forces

1. Free body diagram



checking with construction





2. equations of equilibrium (EE)

$\Sigma F_x = 0$: 1) $A \sin\beta + B_x = 0$

$\Sigma F_y = 0$: 2) $A \cos\beta - mg + B_y = 0$

$\Sigma M_{Bz} = 0$: 3) $c \cdot A \sin\beta - d A \cos\beta + \frac{d}{2} mg = 0$

3. solution of the EE:

$3) \Rightarrow A(c \sin\beta - d\cos\beta) = -\frac{d}{2} mg$

$$A = -\frac{d\,m\,g}{2(c\sin\beta - d\cos\beta)} = -\frac{0{,}7 \cdot 24 \cdot 9.81}{2(0{,}3\sin 45^\circ - 0{,}7\cos 45^\circ)} = 291{,}3\,N\ (\nearrow)$$

$1) \Rightarrow B_x = -A\sin\beta = -291.3 \sin 45^\circ = -206\,N\ (\leftarrow)$

$2) \Rightarrow B_y = mg - A\cos\beta = 24 \cdot 9.81 - 291.3\cos 45^\circ = 29.5\,N\ (\uparrow)$

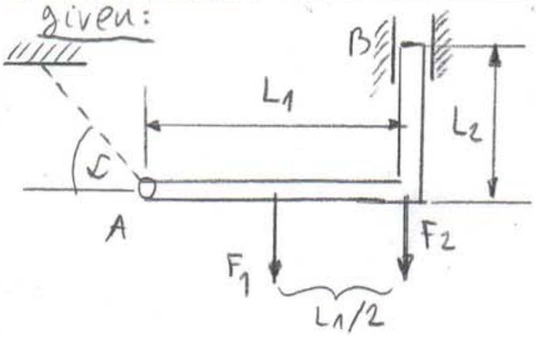
4. results in vector form:

$\vec{A} = A\sin\beta\,\vec{i} + A\cos\beta\,\vec{j} = (206\,\vec{i} + 206\,\vec{j})\,N$

$\vec{B} = B_x\vec{i} + B_y\vec{j} = (-206\,\vec{i} + 29.5\,\vec{j})\,N$

EXERCISE 5.4

given:



$L_1 = 0.6\,m$

$L_2 = 0.4\,m$

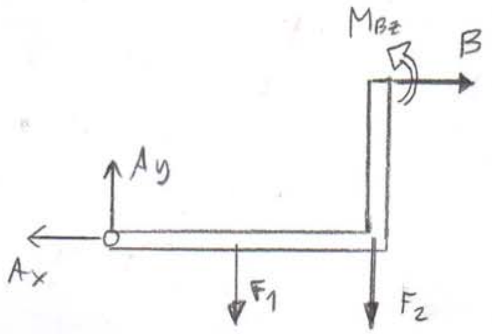
$F_1 = 270N$

$F_2 = 180N$

$\alpha = 30°$

task: reaction forces

1. free body diagram:



2. equations of equilibrium:

$\Sigma F_x = 0 : 1)\ B - A_x = 0$

$\Sigma F_y = 0 : 2)\ A_y - F_1 - F_2 = 0$

$\Sigma M_{Az} = 0 : 3)\ -\frac{L_1}{2} F_1 - L_1 F_2 - B L_2 + M_{Bz} = 0$

$+\ \frac{A_y}{A_x} = \tan\alpha = 0.577$

3. solution:

$2) \rightarrow A_y = F_1 + F_2 = 270 + 180 = 450N$

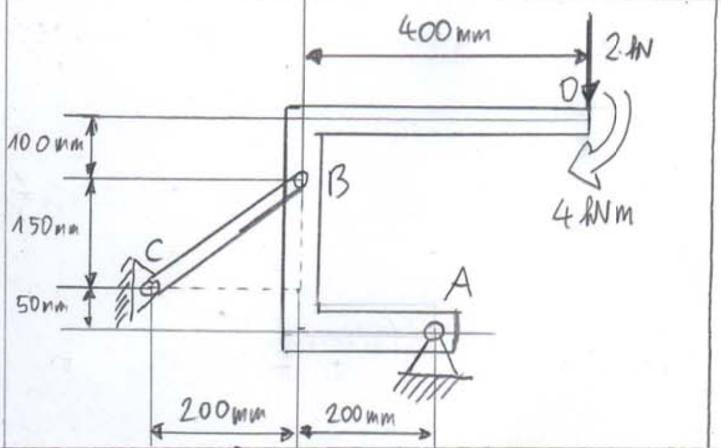
$A_x = \frac{A_y}{0.577} = 779.4N$

$1) \rightarrow B = A_x = 779.4N$

$3) \rightarrow M_{Bz} = BL_2 + F_1\frac{L_1}{2} + F_2 L_1 =$

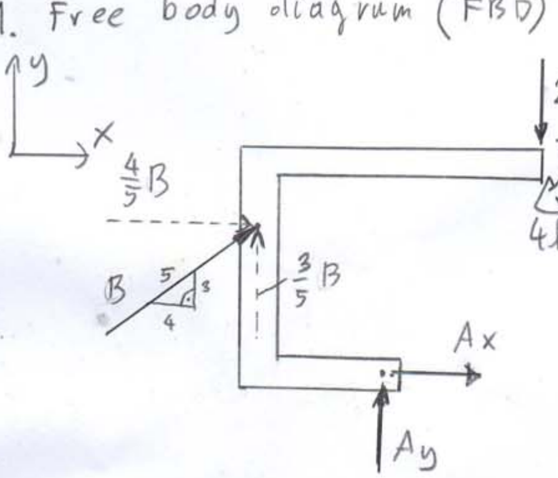
$= 311.8 + 81 + 108 = 500.8\,Nm$

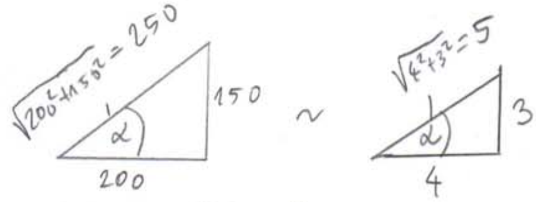
EXERCISE 5.5



task: reaction forces

1. Free body diagram (FBD)





$\cos\alpha = \frac{200}{250} = \frac{4}{5}$

$\sin\alpha = \frac{150}{250} = \frac{3}{5}$

2. equations of equilibrium (EE):

$\Sigma M_{Az} = 0 : 1)\ -0.2\cdot\frac{3}{5}B - 0.2\cdot\frac{4}{5}B - 0.2\cdot 2 - 4 = 0$

$\Sigma F_x = 0 : 2)\ \frac{4}{5}B + A_x = 0$

$\Sigma F_y = 0 : 3)\ \frac{3}{5}B + A_y - 2 = 0$

3. solution of the EE:

$1) \Rightarrow\ -\frac{3}{25}B - \frac{4}{25}B = 4.4$

$-\frac{7}{25}B = 4.4$

$B = -\frac{4.4\cdot 25}{7} = -15.71\ kN(\swarrow)$

$2) \Rightarrow A_x = -\frac{4}{5}B = -\frac{4}{5}\cdot(-15.71) = 12.57\ kN(\rightarrow)$

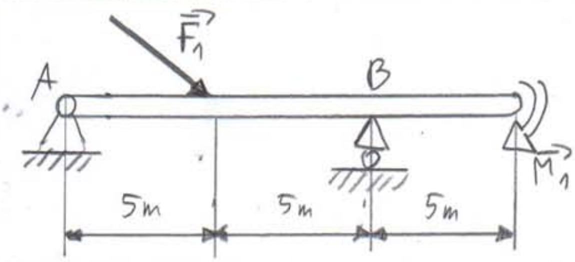
$3) \Rightarrow A_y = 2 - \frac{3}{5}B = 2 - \frac{3}{5}(-15.71) = 11.43\ kN(\uparrow)$

4. results in vector form:

$\vec{C} = \vec{B} = \frac{4}{5}B\vec{i} + \frac{3}{5}B\vec{j} = \frac{4}{5}\cdot(-15.71)\vec{i} + \frac{3}{5}(-15.71)\vec{j} = (-12.57\vec{i} - 9.43\vec{j})\ kN$

$\vec{A} = A_x\vec{i} + A_y\vec{j} = (12.57\vec{i} + 11.43\vec{j})\ kN$

EXERCISE 5.6



given:

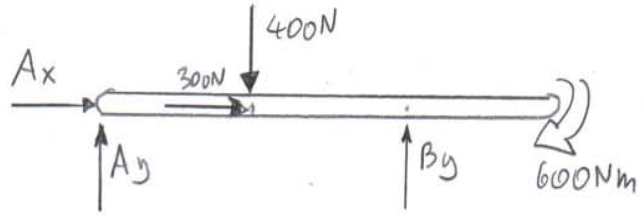
$\vec{F_1} = (300\vec{i} - 400\vec{j})\,N$

$\vec{M_1} = (-600\vec{k})\,Nm$

task:

reaction Forces

1. Free body diagram (FBD)



2. equations of equilibrium (EE)

$\Sigma M_{Az} = 0 \Rightarrow 1) -5\cdot 400 + 10\,B_y - 600 = 0$

$\Sigma F_x = 0 \Rightarrow 2)\ A_x + 300 = 0$

$\Sigma F_y = 0 \Rightarrow 3)\ A_y - 400 + B_y = 0$

3. solution of the EE:

$1) \Rightarrow B_y = \frac{600 + 5\cdot 400}{10} = \frac{2600}{10} = 260\,N\ (\uparrow)$

$2) \Rightarrow A_x = -300\,N\ (\leftarrow)$

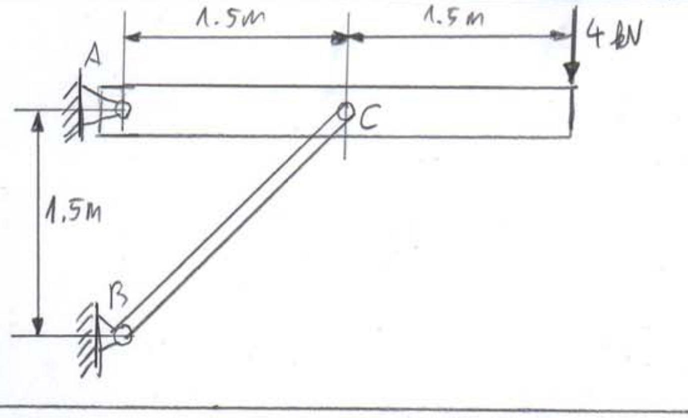
$3) \Rightarrow A_y = 400 - B_y = 400 - 260 = 140\,N\ (\uparrow)$

4. results in vector form:

$\vec{A} = A_x\vec{i} + A_y\vec{j} = (-300\vec{i} + 140\vec{j})\,N$

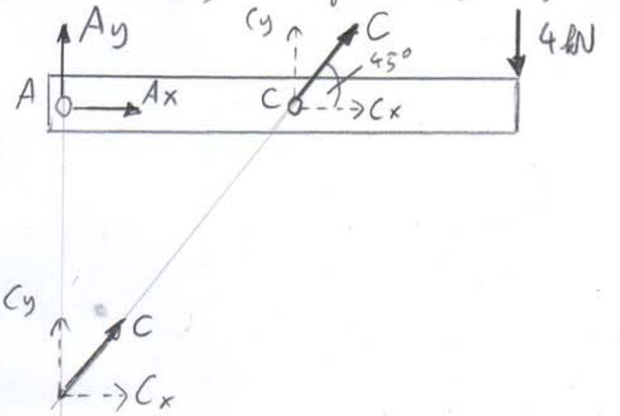
$\vec{B} = B_y\vec{j} = (260\vec{j})\,N$

EXERCISE 5.7



task: reaction forces

1. Free body diagram (FBD)



2. equations of equilibrium (EE)

$\Sigma M_{Az} = 0 \Rightarrow 1/A)\ 1.5\,C_y - 3\cdot 4 = 0$

$\Sigma M_{Az} = 0 \Rightarrow 1/B)\ 1.5\,C_x - 3\cdot 4 = 0$

$\Sigma F_x = 0 \Rightarrow 2)\ A_x + C_x = 0$

$\Sigma F_y = 0 \Rightarrow 3)\ A_y + C_y - 4 = 0$

3. solution:

$1/A) \Rightarrow C_y = \dfrac{3\cdot 4}{1.5} = 8\ \text{kN}\ (\uparrow)$

$1/B) \Rightarrow C_x = \dfrac{3\cdot 4}{1.5} = 8\ \text{kN}\ (\rightarrow)$

$2) \Rightarrow A_x = -C_x = -8\ \text{kN}\ (\leftarrow)$

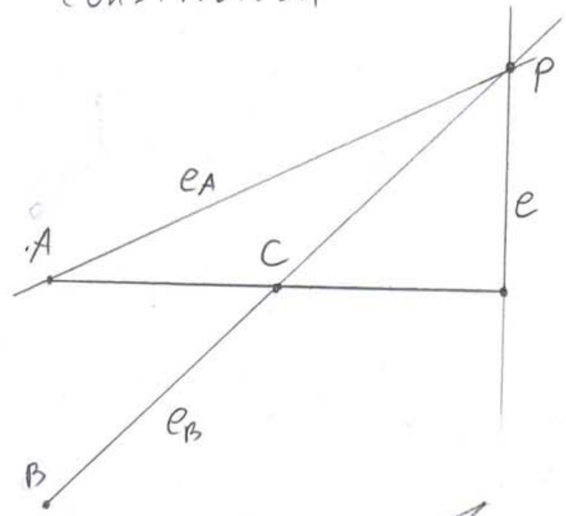
$3) \Rightarrow A_y = 4 - C_y = 4 - 8 = -4\ \text{kN}\ (\downarrow)$

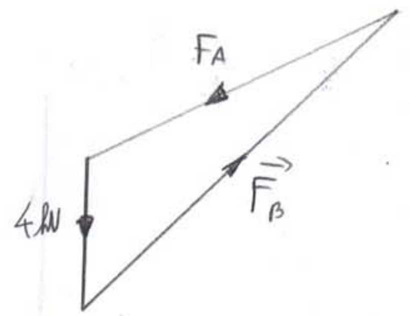
4. results in vector form

$\vec{A} = (-8\vec{i} - 4\vec{j})\ \text{kN}$

$\vec{B} = \vec{C} = (8\vec{i} + 8\vec{j})\ \text{kN}$

Construction





$\vec{A} + \vec{B} - 4\vec{j} = \vec{0}$

$\bullet\ \vec{A} = A\vec{e_A} = A\,\dfrac{3\vec{i} + 1.5\vec{j}}{\sqrt{3^2 + 1.5^2}} = A\cdot\dfrac{3}{\sqrt{11.25}}\vec{i} + A\,\dfrac{1.5}{\sqrt{11.25}}\vec{j}$

$\bullet\ \vec{B} = B\vec{e_B} = B\,\dfrac{3\vec{i} + 3\vec{j}}{\sqrt{3^2 + 3^2}} = B\cdot\dfrac{3}{\sqrt{18}}\vec{i} + B\cdot\dfrac{3}{\sqrt{18}}\vec{j}$

$A\,\dfrac{3}{\sqrt{11.25}}\vec{i} + A\,\dfrac{1.5}{\sqrt{11.25}}\vec{j} + B\cdot\dfrac{3}{\sqrt{18}}\vec{i} + B\cdot\dfrac{3}{\sqrt{18}}\vec{j} - 4\vec{j} = \vec{0}\ \ /\cdot\vec{i}/\vec{j}$

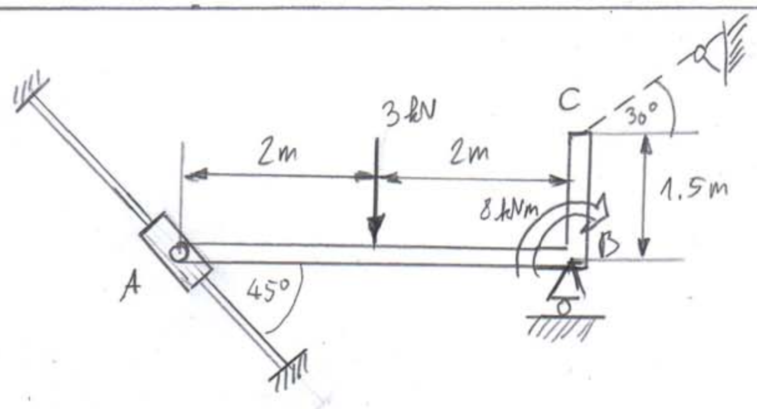
$$\left.\begin{aligned} 1)\ & \frac{3}{\sqrt{11.25}}A + \frac{3}{\sqrt{18}}B = 0 \\ 2)\ & \frac{1.5}{\sqrt{11.25}}A + \frac{3}{\sqrt{18}}B - 4 = 0 \end{aligned}\right\}$$

$1) - 2):\ \dfrac{1.5}{\sqrt{11.25}}A + 4 = 0 \qquad 1 - 2\cdot 2):\ -\dfrac{3}{\sqrt{18}}B + 8 = 0$

$\Rightarrow A = \dfrac{-4\sqrt{11.25}}{1.5}\ \text{kN} \qquad \Rightarrow B = \dfrac{8\sqrt{18}}{3}$

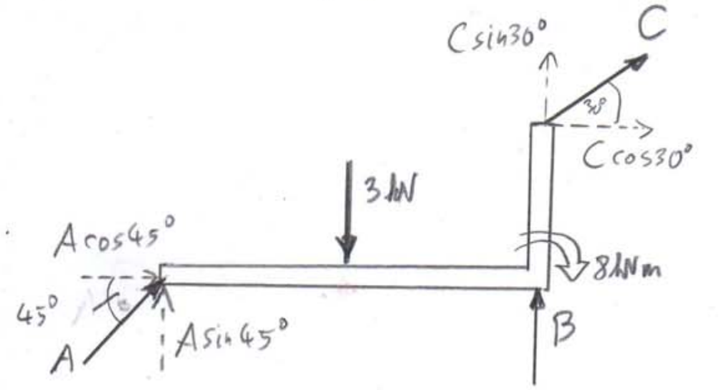
$\vec{A} = (-8\vec{i} - 4\vec{j})\ \text{kN} \qquad \vec{B} = (8\vec{i} + 8\vec{j})\ \text{kN}$

EXERCISE 5.8



task: reaction forces

1. Free body diagram (FBD):



2. equations of equilibrium (EE):

$$\sum M_{Cz} = 0: \quad 1)\; -4A\sin45° + 1.5A\cos45° + 2\cdot3 - 8 = 0$$

$$\sum F_x = 0: \quad 2)\; A\cos45° + C\cos30° = 0$$

$$\sum F_y = 0: \quad 3)\; A\sin45° - 3 + B + C\sin30° = 0$$

3. solution of the EE

$$1) \Rightarrow \quad A(-4\sin45° + 1.5\cos45°) = 2$$

$$A = \frac{2}{-4\sin45° + 1.5\cos45°} = -1.13\,\text{kN}\,(\swarrow)$$

$$2) \Rightarrow \quad C = -\frac{A\cos45°}{\cos30°} = -\frac{-1.13\cos45°}{\cos30°} = 0.92\,\text{kN}\,(\nearrow)$$

$$3) \Rightarrow \quad B = 3 - A\sin45° - C\sin30° = 3 - (-1.13)\sin45° - 0.92\sin30° = 3.34\,\text{kN}\,(\uparrow)$$

4. results in vector form:

$$\vec{A} = A\cos45°\,\vec{i} + A\sin45°\,\vec{j} = -1.13\cos45°\,\vec{i} + (-1.13)\sin45°\,\vec{j} = (-0.8\,\vec{i} + 0.8\,\vec{j})\,\text{kN}$$

$$\vec{B} = B\,\vec{j} = (3.34\,\vec{j})\,\text{kN}$$

$$\vec{C} = C\cos30°\,\vec{i} + C\sin30°\,\vec{j} = 0.92\cos30°\,\vec{i} + 0.92\sin30°\,\vec{j} = (0.8\,\vec{i} + 0.46\,\vec{j})\,\text{kN}$$