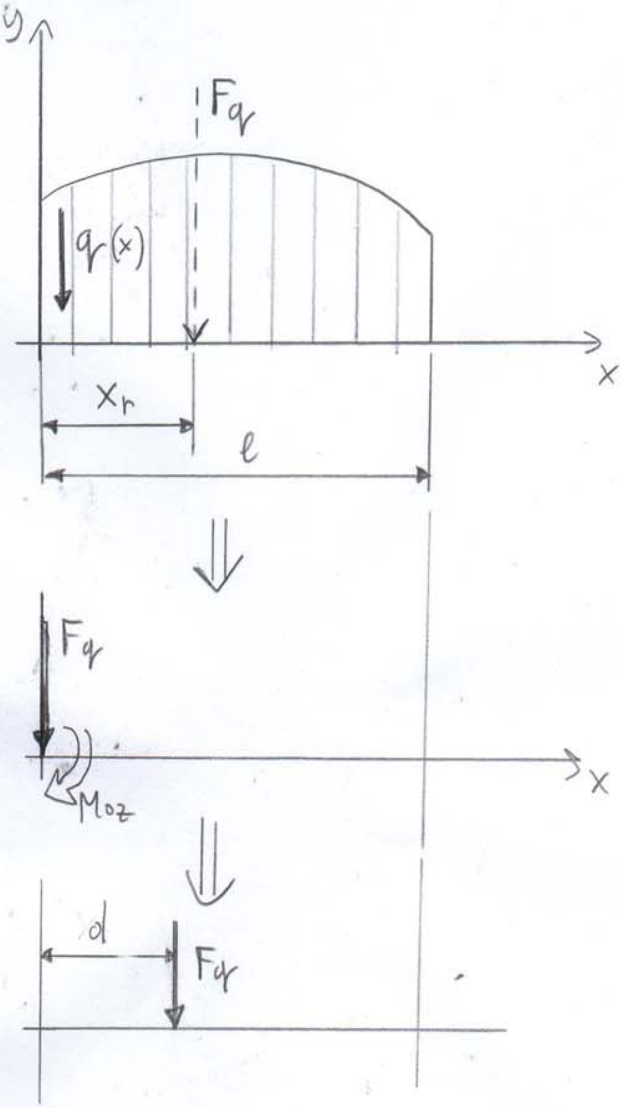
SUMMARY



$q(x)$: intensity of the distributed line load

$$q(x) = \frac{dF_q}{dx} \quad \left[\frac{N}{m}\right]$$

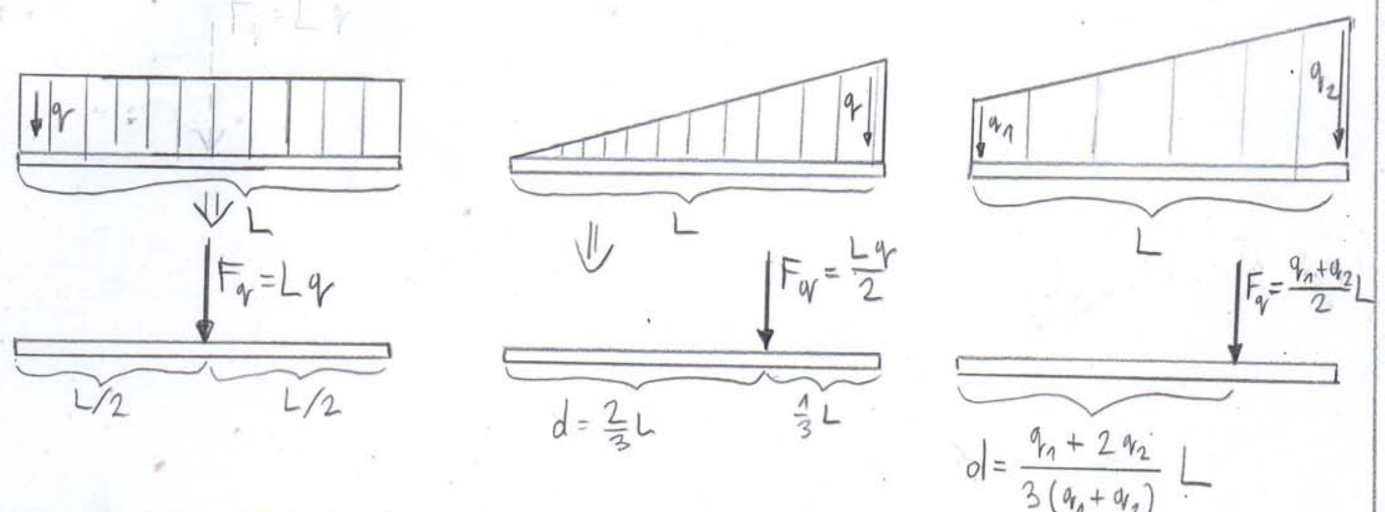
$$dF_q = q(x)\,dx$$

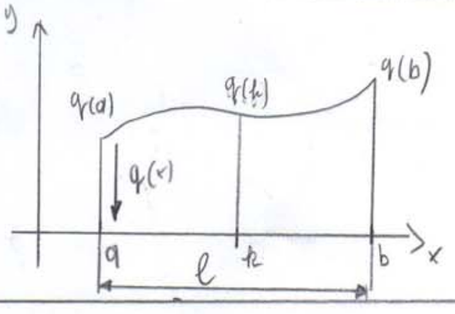
$$F_q = \int_0^{\ell} q(x)\,dx$$

$$M_{oz} = \int_0^{\ell} x\,q(x)\,dx$$

$$d = \frac{M_{oz}}{F_q}$$

line of action of the resultant force





Simpson rule:

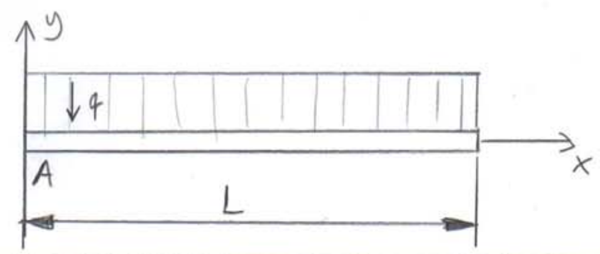
$$F_q = \int_a^b q(x)\,dx \approx \frac{\ell}{6}\left(q(a) + 4\,q(h) + q(b)\right)$$

$$M_{oz} = \int_a^b x\,q(x)\,dx \approx \frac{\ell}{6}\left(a\,q(a) + 4\cdot h\cdot q(h) + b\,q(b)\right)$$

$$d = \frac{M_{oz}}{F_q}$$

EXERCISE 6.1

given:



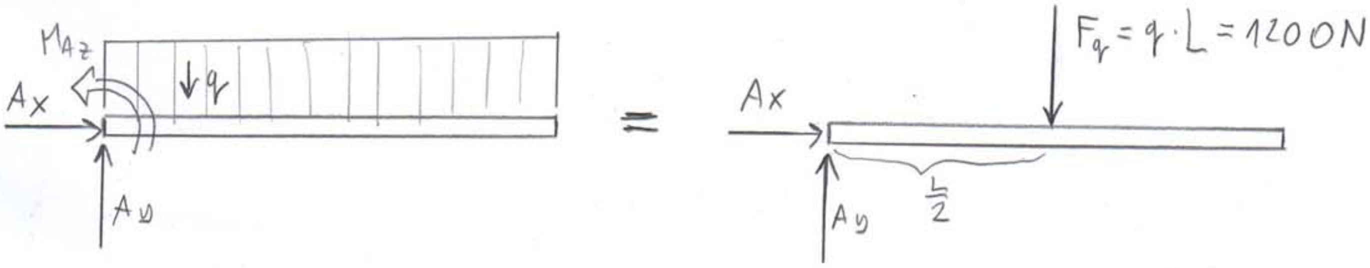
$L = 2m$

$q = 600 \frac{N}{m}$

task:

reaction Forces

1. Free body diagram



2. equations of equilibrium

$\sum F_x = 0 : 1)\ A_x = 0$

$\sum F_y = 0 : 2)\ A_y - F_q = 0$

$\sum M_{Az} = 0 : 3)\ M_{Az} - F_q \cdot \frac{L}{2} = 0$

3. solution of the equations of equilibrium

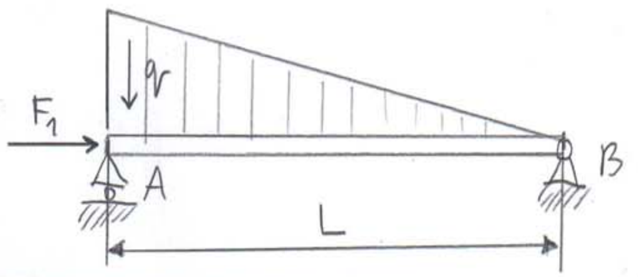
1) $\Rightarrow A_x = 0$

2) $\Rightarrow A_y = F_q = 1200\,N$

3) $\Rightarrow M_{Az} = F_q \frac{L}{2} = 1200 \cdot \frac{2}{2} = 1200\,Nm$

EXERCISE 6.2

given:



$q = 1{,}2 \ \frac{kN}{m}$

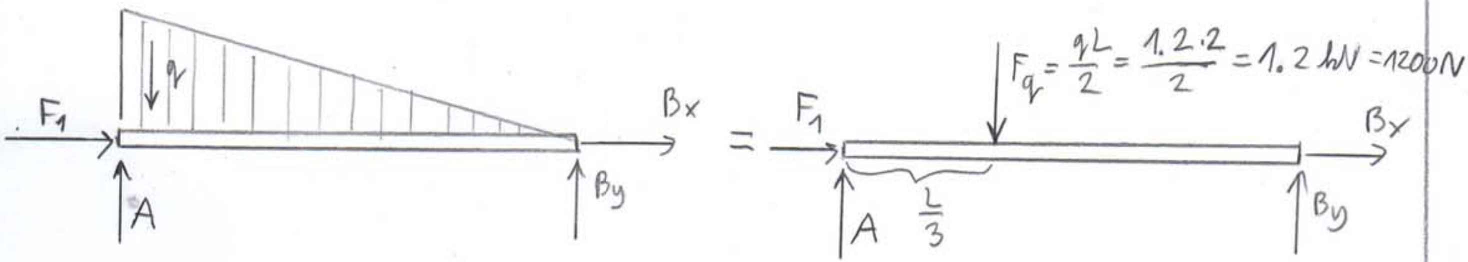
$L = 2\,m$

$F_1 = 600\,N$

task:

reaction forces

1. Free body diagram



2. equations of equilibrium

$\Sigma F_x = 0 : \quad 1)\ B_x + F_1 = 0$

$\Sigma F_y = 0 : \quad 2)\ A + B_y - F_q = 0$

$\Sigma M_{Az} = 0 : \quad 3)\ L\,B_y - \frac{L}{3} F_q = 0$

3. solution of the equations of equilibrium:

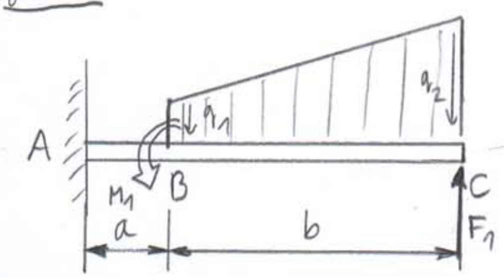
$1) \Rightarrow \ B_x = -F_1 = -600\,N \ (\leftarrow)$

$3) \Rightarrow \ B_y = \frac{\frac{L}{3} F_q}{L} = \frac{\frac{2}{3} \cdot 1200}{2} = 400\,N \ (\uparrow)$

$2) \Rightarrow \ A = F_q - B_y = 1200 - 400 = 800\,N \ (\uparrow)$

EXERCISE 6.3

given:



$a = 2m$

$b = 8m$

$F_1 = 70\,kN$

$q_1 = 6\,\frac{kN}{m}$

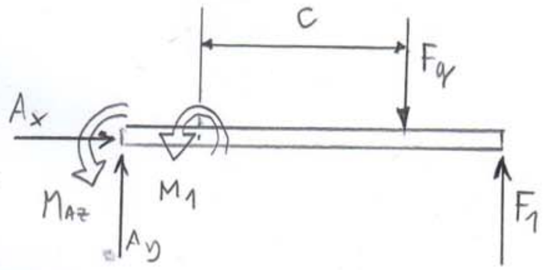
$q_2 = 10\,\frac{kN}{m}$

$M_1 = 80\,kNm$

task:

reaction forces

1. Free body diagram



$$F_q = \frac{q_1+q_2}{2}\,b = \frac{6+10}{2}\cdot 8 = 64\,kN$$

$$c = \frac{q_1+2q_2}{3(q_1+q_2)}\cdot b = \frac{6+2\cdot 10}{3(6+10)}\cdot 8 = 4.\dot{3}\,m$$

2. equations of equilibrium

$$\Sigma F_x = 0 \Rightarrow 1)\ A_x = 0$$

$$\Sigma F_y = 0 \Rightarrow 2)\ F_1 - F_q + A_y = 0$$

$$\Sigma M_{Az} = 0 \Rightarrow 3)\ M_{Az} + M_1 + F_1(a+b) - F_q(a+c) = 0$$

3. solution of the equation of equilibrium

$$1) \Rightarrow A_x = 0$$

$$2) \Rightarrow A_y = F_q - F_1 = 64 - 70 = -6\,kN\,(\downarrow)$$

$$3) \Rightarrow M_{Az} = F_q(a+c) - F_1(a+b) - M_1 = 64(2+4.\dot{3}) - 70(2+8) - 80 =$$

$$= -374.\dot{6}\,kNm\ (\circlearrowright)$$

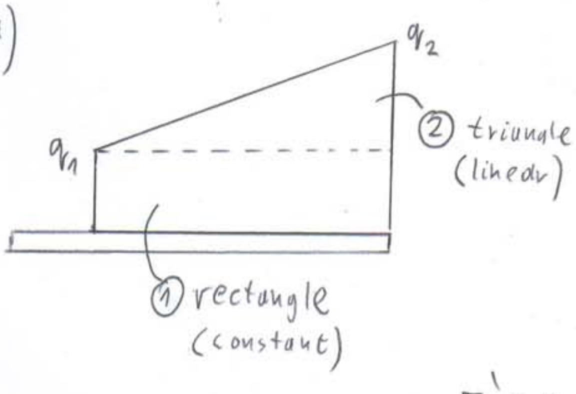
4. results in vector form:

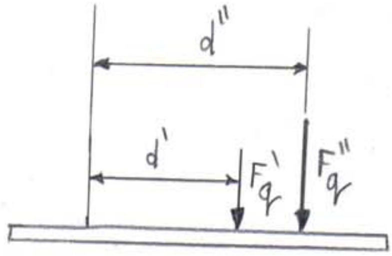
$$\vec{A} = A_x\vec{i} + A_y\vec{j} = (0\vec{i} - 6\vec{j})\,kN$$

$$\vec{M_A} = M_{Az}\,\vec{k} = (-374.\dot{6}\,\vec{k})\,kNm$$

• other ways to calculate F_q and c :

a)





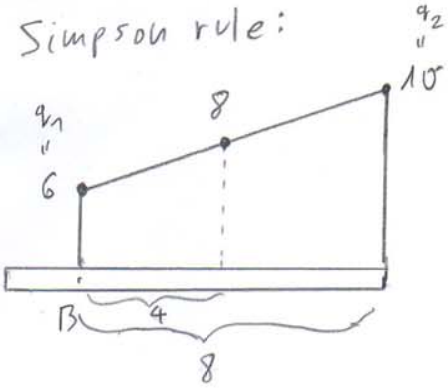
$$F_q' = q_1 \cdot b = 48\ kN \qquad d' = \frac{b}{2} = 4\ m$$

$$F_q'' = \frac{(q_2 - q_1)\cdot b}{2} = 16\ kN \qquad d'' = \frac{2}{3} b = 5.\dot{3}\ m$$

$$F_q = F_q' + F_q'' = 64\ kN$$

$$d = \frac{F_q' d' + F_q'' d''}{F_q' + F_q''} = 4.\dot{3}\ m$$

b) Simpson rule:

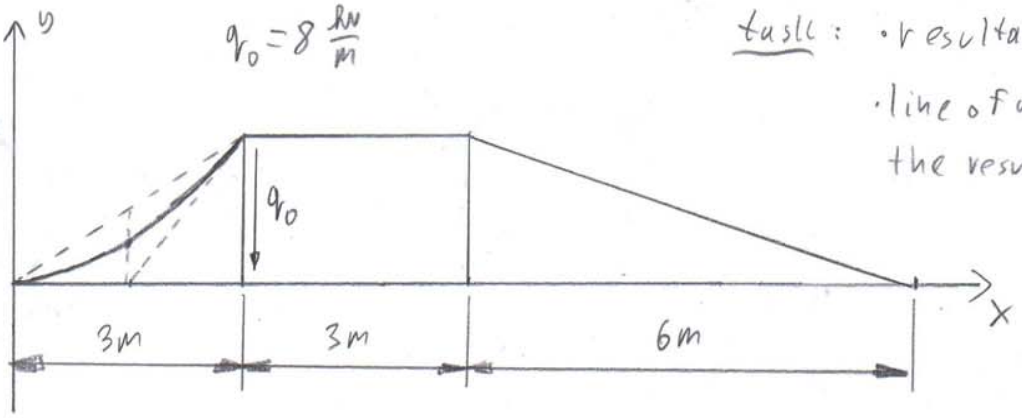


$$F_q = -\frac{8}{6}\cdot\left(6 + 4\cdot 8 + 10\right) = -64\ kN$$

$$M_{Bz} = -\frac{8}{6}\cdot\left(0\cdot 6 + 4\cdot 4\cdot 8 + 8\cdot 10\right) = -277.\dot{3}\ kNm$$

$$d = \frac{M_{Bz}}{F_q} = \frac{-277.\dot{3}}{-64} = 4.\dot{3}\ m$$

EXERCISE 6.4 - method 1



task: · resultant force (F_q)
· line of action of the resultant force (d)

$$F_{q_1} = -\frac{3}{6}\left(0 + 4\cdot 2 + 8\right) = -8\,kN(\downarrow)$$

$$F_{q_2} = -3\cdot 8 = -24\,kN(\downarrow)$$

$$F_{q_3} = -\frac{6\cdot 8}{2} = -24\,kN(\downarrow)$$

$$F_q = -8-24-24 = -56\,kN(\downarrow)$$

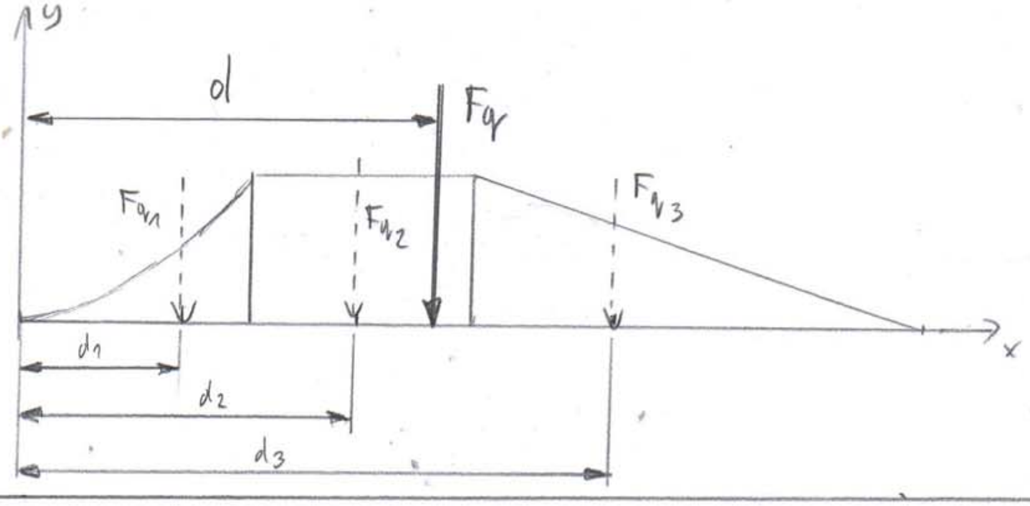
$$M_{oz_1} = -\frac{3}{6}\left(0\cdot 0 + 4\cdot 1{,}5\cdot 2 + 3\cdot 8\right) = -18\,kNm$$

$$d_1 = \frac{M_{oz1}}{F_{q1}} = \frac{-18}{-8} = 2.25\,m$$

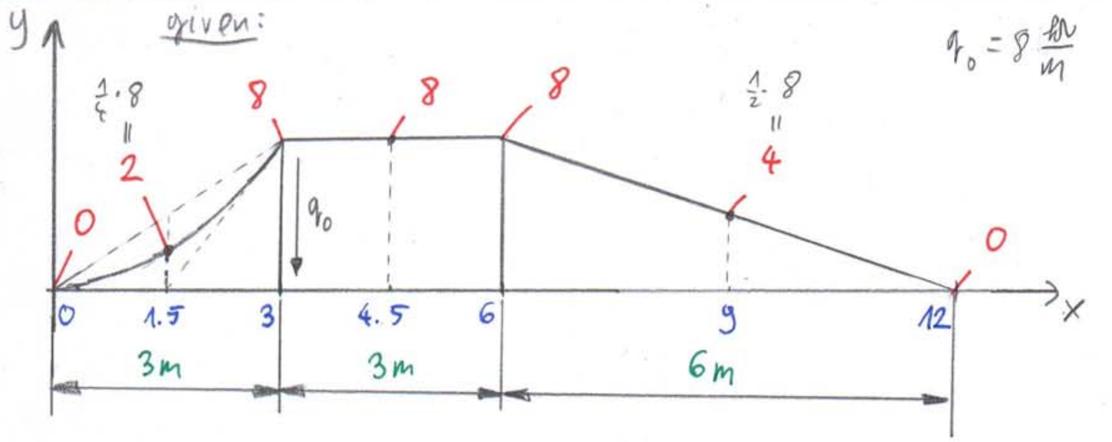
$$d_2 = 3 + \frac{3}{2} = 4.5\,m$$

$$d_3 = 3+3+\frac{1}{3}\cdot 6 = 8\,m$$

$$d = \frac{F_{q_1}d_1 + F_{q_2}d_2 + F_{q_3}d_3}{F_{q_1}+F_{q_2}+F_{q_3}} = \frac{-8\cdot 2.25 - 24\cdot 4.5 - 24\cdot 8}{-8-24-24} = 5.68\,m$$



EXERCISE 6.4 - method 2



task:

- resultant force (F_q)
- line of action of the resultant force (d)

- resultant force of each segments

$$F_{q_1} = \int_0^3 q_1(x)\,dx = -\frac{3}{6}\left(0+4\cdot 2+8\right) = -8\,kN\ (\downarrow)$$

$$F_{q_2} = \int_3^6 q_2(x)\,dx = -\frac{3}{6}\left(8+4\cdot 8+8\right) = -24\,kN\ (\downarrow)$$

$$F_{q_3} = \int_6^{12} q_3(x)\,dx = -\frac{6}{6}\left(8+4\cdot 4+0\right) = -24\,kN\ (\downarrow)$$

- resultant force:

$$F_q = F_{q_1} + F_{q_2} + F_{q_3} = -8-24-24 = -56\,kN\ (\downarrow)$$

- moment of each segment about origin:

$$M_{0z1} = \int_0^3 x\,q_1(x)\,dx = -\frac{3}{6}\left(0\cdot 0+4\cdot 1.5\cdot 2+3\cdot 8\right) = -18\,kNm\ (\circlearrowright)$$

$$M_{0z2} = \int_3^6 x\,q_2(x)\,dx = -\frac{3}{6}\left(3\cdot 8+4\cdot 4.5\cdot 8+6\cdot 8\right) = -108\,kNm\ (\circlearrowright)$$

$$M_{0z3} = \int_6^{12} x\,q_2(x)\,dx = -\frac{6}{6}\left(6\cdot 8+4\cdot 9\cdot 4+12\cdot 0\right) = -192\,kNm\ (\circlearrowright)$$

- moment about origin:

$$M_{0z} = M_{01} + M_{02} + M_{03} = -18-108-192 = -318\,kNm\ (\circlearrowright)$$

- line of action of the resultant force:

$$d = \frac{M_{0z}}{F_q} = \frac{-318}{-56} = 5{,}68\,m$$