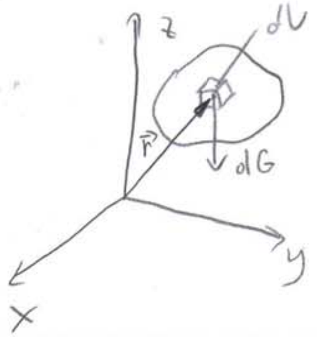
SUMMARY

- by definition:



$$\vec{r}_c = \frac{\int_{(V)} \vec{r}\, dG}{G} = \frac{\int_{(V)} \vec{r}\, \rho g\, dV}{\rho g V} = \frac{\int_{(V)} \vec{r}\, dV}{V}$$

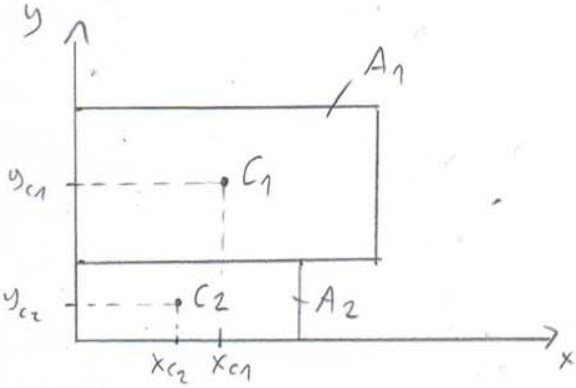
in plane: $$\vec{r}_c = \frac{\int_{(A)} \vec{r}\, dA}{A}$$

for lines: $$\vec{r}_c = \frac{\int_{(\ell)} \vec{r}\, ds}{\ell}$$

COG calculation of complex shapes using subshapes with known COG

method 1:

Complex shape is decomposed into subshapes of known COG



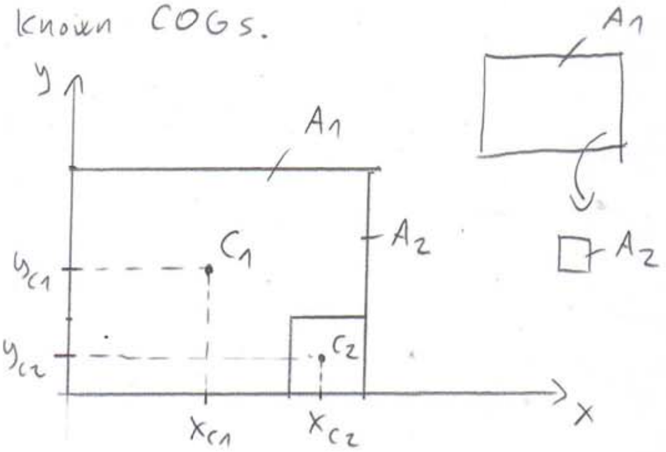
$$x_c = \frac{A_1 x_{c1} + A_2 x_{c2}}{A_1 + A_2}$$

$$y_c = \frac{A_1 y_{c1} + A_2 y_{c2}}{A_1 + A_2}$$

$$\vec{r}_c = x_c \vec{i} + y_c \vec{j}$$

method 2

Complex shape is complemented so that it consists of subshapes with known COGs.



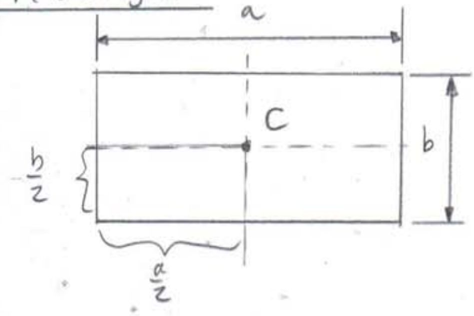
$$x_c = \frac{A_1 x_{c1} - A_2 x_{c2}}{A_1 - A_2}$$

$$y_c = \frac{A_1 y_{c1} - A_2 y_{c2}}{A_1 - A_2}$$

$$\vec{r}_c = x_c \vec{i} + y_c \vec{j}$$

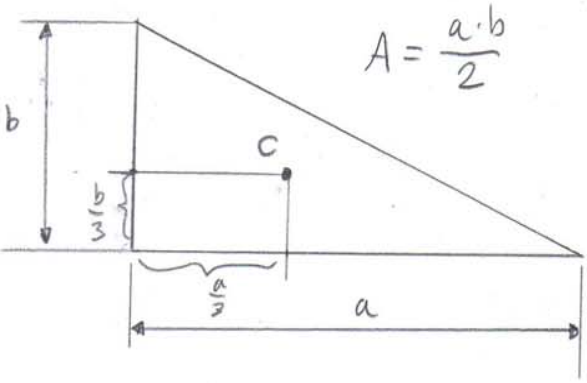
SUMMARY - COG of simple shapes

• rectangle :



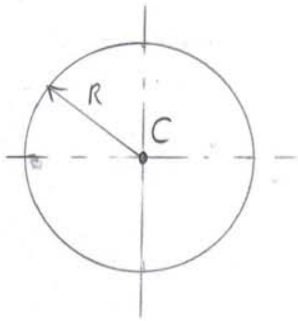
$A = a \cdot b$

• right triangle



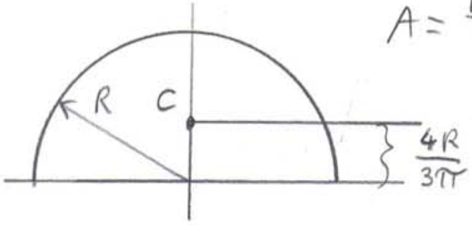
$A = \frac{a \cdot b}{2}$

• circle



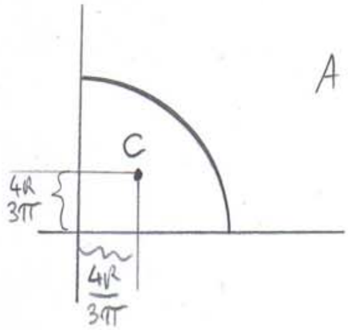
$A = R^2 \pi$

• half circle



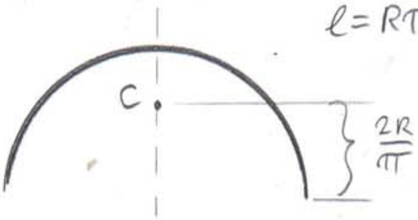
$A = \frac{R^2 \pi}{2}$

• quarter circle



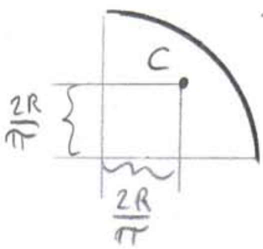
$A = \frac{R^2 \pi}{4}$

• half circle (line):



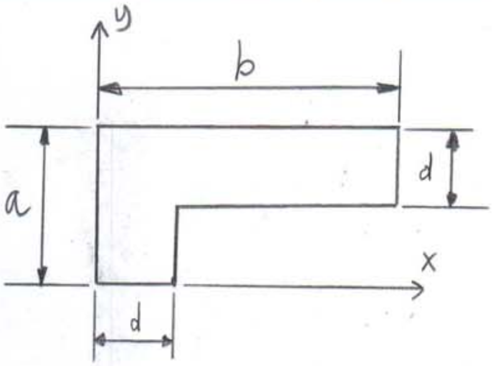
$\ell = R \pi$

• quarter circle (line):



$\ell = \frac{R \pi}{2}$

EXERCISE 7.1

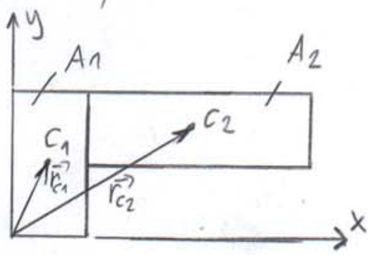


$a = 20\,mm$
$b = 40\,mm$
$d = 10\,mm$

plate of constant thickness

task:
where is the center of gravity?

method 1:
division, vectorial



$A_1 = a \cdot d = 200\,mm^2$

$\vec{r}_{C_1} = \frac{d}{2}\vec{i} + \frac{a}{2}\vec{j} =$
$= (5\vec{i} + 10\vec{j})\,mm$

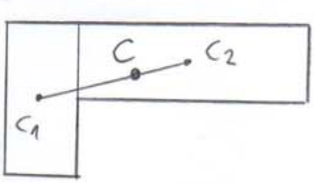
$A_2 = (b-d) \cdot d = 300\,mm^2$

$\vec{r}_{C_2} = \left(d + \frac{b-d}{2}\right)\vec{i} + \left(a - \frac{d}{2}\right)\vec{j} =$
$= (25\vec{i} + 15\vec{j})\,mm$

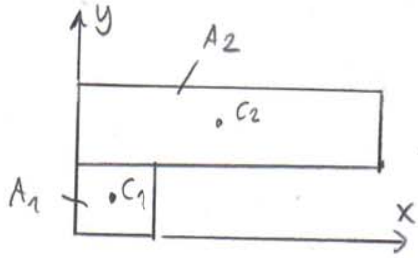
$\vec{r}_C = \frac{A_1 \vec{r}_{C_1} + A_2 \vec{r}_{C_2}}{A_1 + A_2} =$

$= \frac{1}{500}\left(200(5\vec{i} + 10\vec{j}) + 300(25\vec{i} + 13\vec{j})\right)$

$= \frac{1}{500}\left(1000\vec{i} + 2000\vec{j} + \right.$
$\left. + 7500\vec{i} + 4500\vec{j}\right) =$
$= (17\vec{i} + 13\vec{j})\,mm$



method 2
division, components



$A_1 = d(a-d) = 100\,mm^2$
$x_{C_1} = \frac{d}{2} = 5\,mm$
$y_{C_1} = \frac{d}{2} = 5\,mm$

$A_2 = d \cdot b = 400\,mm^2$
$x_{C_2} = \frac{b}{2} = 20\,mm$
$y_{C_2} = a - \frac{d}{2} = 15\,mm$

$X_C = \frac{A_1 x_{C_1} + A_2 x_{C_2}}{A_1 + A_2} =$

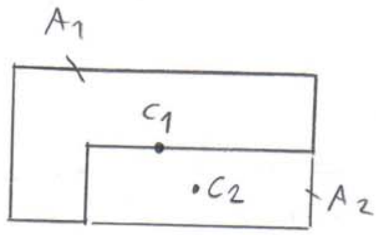
$= \frac{100 \cdot 5 + 400 \cdot 20}{500} = 17\,mm$

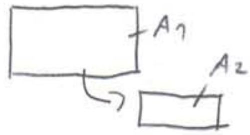
$y_C = \frac{A_1 y_{C_1} + A_2 y_{C_2}}{A_1 + A_2} =$

$= \frac{100 \cdot 5 + 400 \cdot 15}{500} = 13\,mm$

$\vec{r}_C = (17\vec{i} + 13\vec{j})\,mm$

method 3
complement, components





$A_1 = a \cdot b = 800\,mm^2$
$x_{C_1} = \frac{b}{20} = 20\,mm$
$y_{C_1} = \frac{a}{2} = 10\,mm$
$A_2 = (b-d)(a-d) = 300\,mm^2$
$x_{C_2} = d + \frac{b-d}{2} = 25\,mm$
$y_{C_2} = \frac{a-d}{2} = 5\,mm$

$X_C = \frac{A_1 x_{C_1} - A_2 x_{C_2}}{A_1 - A_2} =$

$= \frac{800 \cdot 20 - 300 \cdot 25}{800 - 300} = 17\,mm$

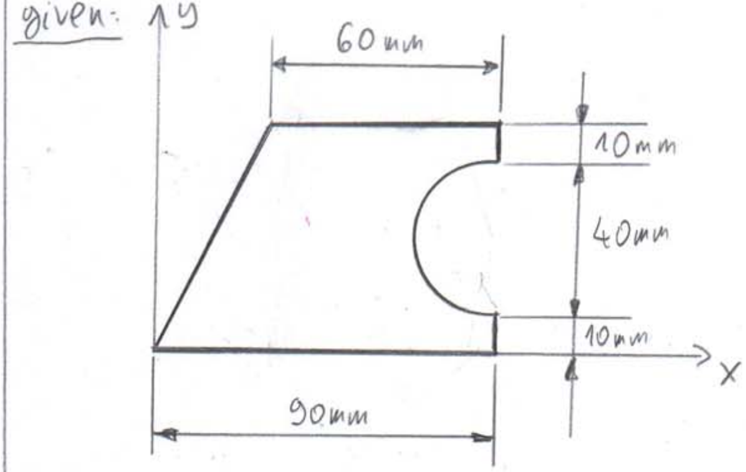
$y_C = \frac{A_1 y_{C_1} - A_2 y_{C_2}}{A_1 - A_2} =$

$= \frac{800 \cdot 10 - 300 \cdot 5}{800 - 300} = 13\,mm$

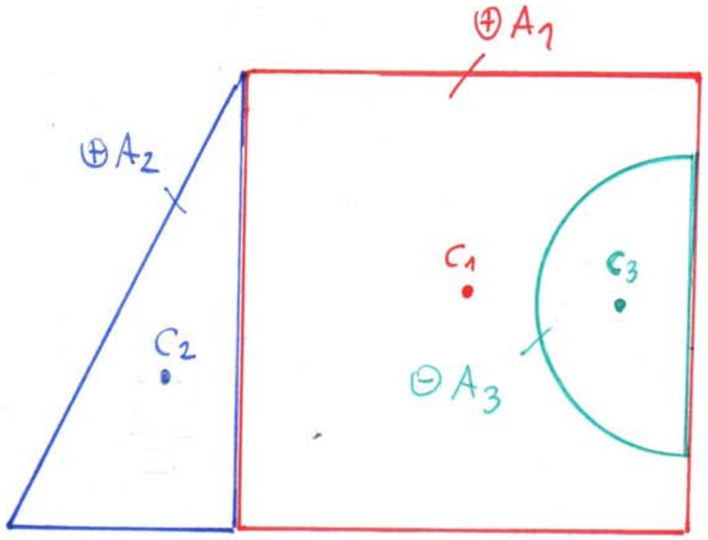
$\vec{r}_C = (17\vec{i} + 13\vec{j})\,mm$

EXERCISE 7.2

given:



task: center of gravity of the plate



$A_1 = 60 \cdot 60 = 3600\,mm^2$

$x_{c_1} = 30 + \frac{60}{2} = 60\,mm$

$y_{c_1} = \frac{60}{2} = 30\,mm$

$A_2 = \frac{30 \cdot 60}{2} = 900\,mm^2$

$x_{c_2} = \frac{2}{3} \cdot 30 = 20\,mm$

$y_{c_2} = \frac{1}{3} \cdot 60 = 20\,mm$

$A_3 = \frac{20^2 \pi}{2} = 628.3\,mm^2$

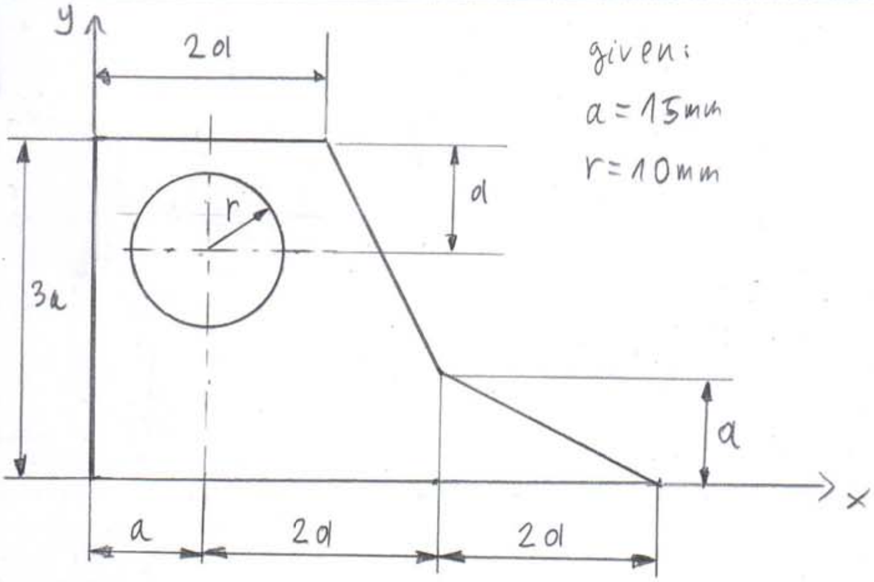
$x_{c_3} = 90 - \frac{4 \cdot 20}{3\pi} = 81.5\,mm$

$y_{c_3} = 10 + \frac{40}{2} = 30\,mm$

$$x_c = \frac{A_1 x_{c_1} + A_2 x_{c_2} - A_3 x_{c_3}}{A_1 + A_2 - A_3} = \frac{3600 \cdot 60 + 900 \cdot 20 - 628.3 \cdot 81.5}{3600 + 900 - 628.3} = 47.2\,mm$$

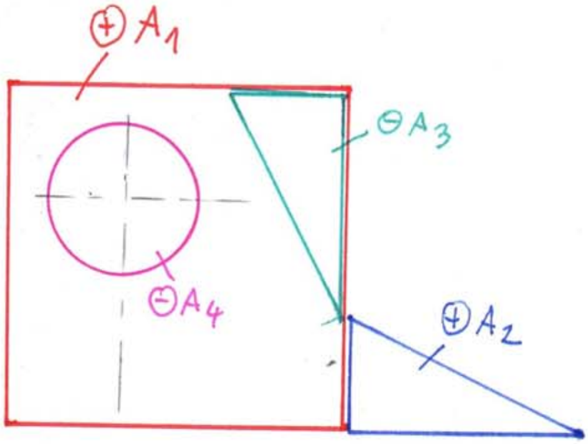
$$y_c = \frac{A_1 y_{c_1} + A_2 y_{c_2} - A_3 y_{c_3}}{A_1 + A_2 - A_3} = \frac{3600 \cdot 30 + 900 \cdot 20 - 628.3 \cdot 30}{3600 + 900 - 628.3} = 27.7\,mm$$

EXERCISE 7.3



given:
$a = 15\,mm$
$r = 10\,mm$

task:
center of gravity of the plate.



$A_1 = (3a)^2 = 2025\,mm^2$

$x_{c_1} = 1.5a = 22.5\,mm$

$y_{c_1} = 1.5a = 22.5\,mm$

$A_2 = \frac{2a \cdot a}{2} = 225\,mm^2$

$x_{c_2} = 3a + \frac{1}{3} \cdot 2a = 55\,mm$

$y_{c_2} = \frac{1}{3}a = 5\,mm$

$A_3 = \frac{2a \cdot a}{2} = 225\,mm^2$

$x_{c_3} = 2a + \frac{2}{3}a = 40\,mm$

$y_{c_3} = a + \frac{2}{3}(2a) = 35\,mm$

$A_4 = r^2\pi = 314.16\,mm^2$

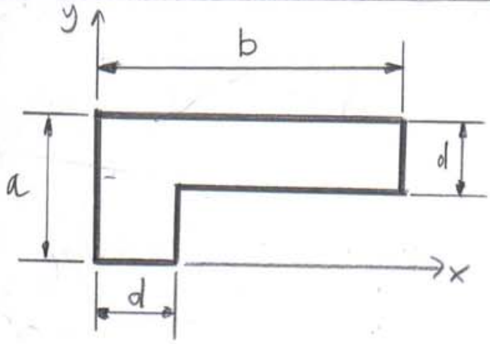
$x_{c_4} = a = 15\,mm$

$y_{c_4} = 2a = 30\,mm$

$$x_c = \frac{A_1 x_{c_1} + A_2 x_{c_2} - A_3 x_{c_3} - A_4 x_{c_4}}{A_1 + A_2 - A_3 - A_4} = 25.85\,mm$$

$$y_c = \frac{A_1 y_{c_1} + A_2 y_{c_2} - A_3 y_{c_3} - A_4 y_{c_4}}{A_1 + A_2 - A_3 - A_4} = 17.18\,mm$$

EXERCISE 7.4



$a = 20\,mm$

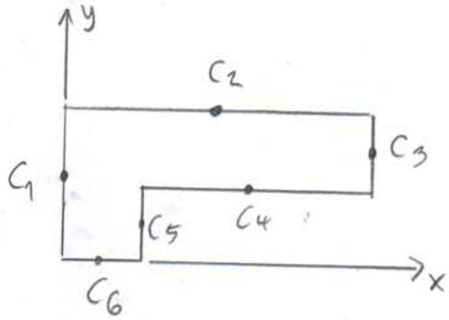
$b = 40\,mm$

$d = 10\,mm$

wire of constant thickness

task:

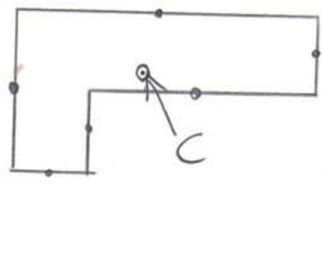
where is the center of gravity?



$\ell_1 = 20\,mm$	$\ell_2 = 40\,mm$	$\ell_3 = 10\,mm$	$\ell_4 = 30\,mm$	$\ell_5 = 10\,mm$	$\ell_6 = 10\,mm$
$x_{c_1} = 0\,mm$	$x_{c_2} = 20\,mm$	$x_{c_3} = 40\,mm$	$x_{c_4} = 25\,mm$	$x_{c_5} = 10\,mm$	$x_{c_6} = 5\,mm$
$y_{c_1} = 10\,mm$	$y_{c_2} = 20\,mm$	$y_{c_3} = 15\,mm$	$y_{c_4} = 10\,mm$	$y_{c_5} = 5\,mm$	$y_{c_6} = 0\,mm$

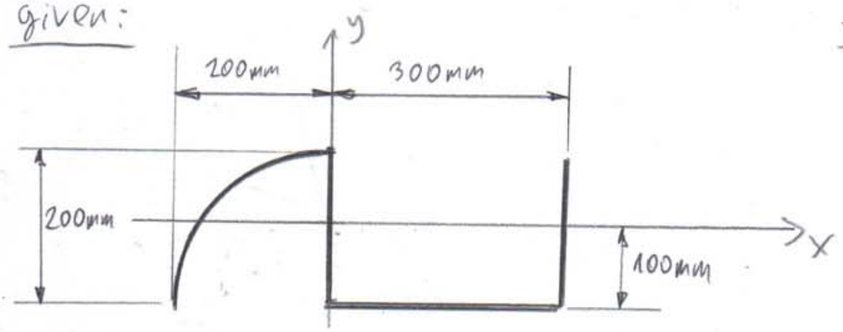
$$x_c = \frac{\sum_{i=1}^{6} \ell_i x_{c_i}}{\sum_{i=1}^{6} \ell_i} = \frac{20\cdot 0 + 40\cdot 20 + 10\cdot 40 + 30\cdot 25 + 10\cdot 10 + 10\cdot 5}{20+40+10+30+10+10} = 17.5\,mm$$

$$y_c = \frac{\sum_{i=1}^{6} \ell_i y_{c_i}}{\sum_{i=1}^{6} \ell_i} = \frac{20\cdot 10 + 4020 + 10\cdot 15 + 30\,10 + 10\cdot 5 + 10\cdot 0}{20+40+10+30+10+10} = 12.5\,mm$$

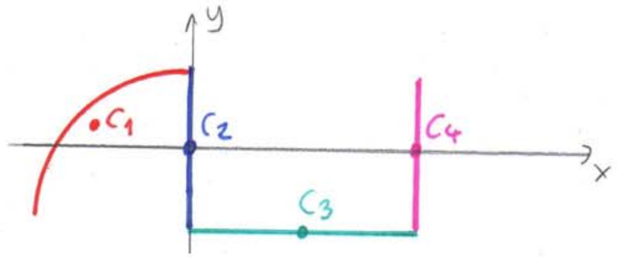


EXERCISE 7.5

given:



task: COG of the wire



$$\ell_1 = \frac{200\pi}{2} = 314.16\,mm$$

$$x_{c_1} = -\frac{2\cdot 200}{\pi} = -127.32\,mm$$

$$y_{c_1} = -100 + \frac{2\cdot 200}{\pi} = 27.32\,mm$$

$\ell_2 = 200\,mm$ $\quad$ $\ell_3 = 300\,mm$ $\quad$ $\ell_4 = 200\,mm$

$x_{c_2} = 0\,mm$ $\quad$ $x_{c_3} = 150\,mm$ $\quad$ $x_{c_4} = 300\,mm$

$y_{c_2} = 0\,mm$ $\quad$ $y_{c_3} = -100\,mm$ $\quad$ $y_{c_4} = 0\,mm$

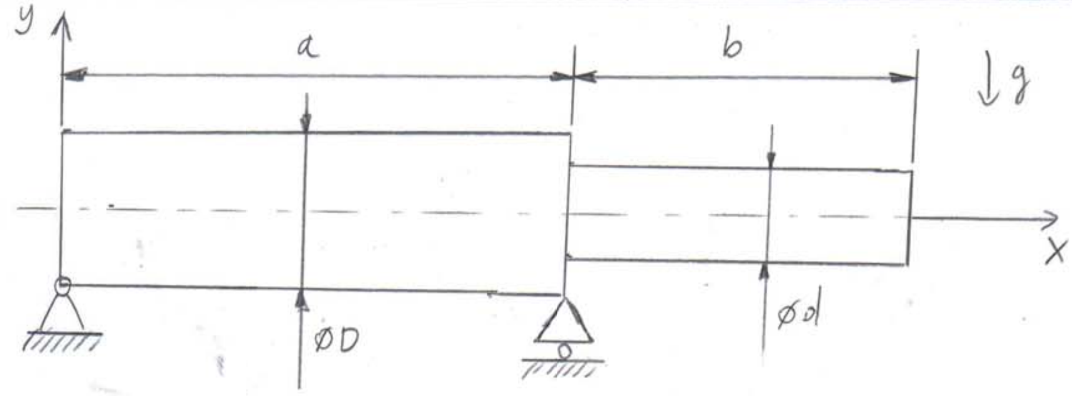
$$x_c = \frac{\sum_{i=1}^{4} \ell_i x_{c_i}}{\sum_{i=1}^{4} \ell_i} = \frac{\ell_1 x_{c_1} + \ell_2 x_{c_2} + \ell_3 x_{c_3} + \ell_4 x_{c_4}}{\ell_1 + \ell_2 + \ell_3 + \ell_4} =$$

$$= \frac{314.16\cdot(-127.32) + 200\cdot 0 + 300\cdot 15 + 200\cdot 300}{314.16 + 200 + 300 + 200} = 64.1\,mm$$

$$y_c = \frac{\sum_{i=1}^{4} \ell_i y_{c_i}}{\sum_{i=1}^{4} \ell_i} = \frac{\ell_1 y_{c_1} + \ell_2 y_{c_2} + \ell_3 y_{c_3} + \ell_4 y_{c_4}}{\ell_1 + \ell_2 + \ell_3 + \ell_4} =$$

$$= \frac{314.16\cdot 27.32 + 200\cdot 0 + 300\cdot(-100) + 200\cdot 0}{314.16 + 200 + 300 + 200} = -21.1\,mm$$

EXERCISE 7.6



$a = 300\,mm$

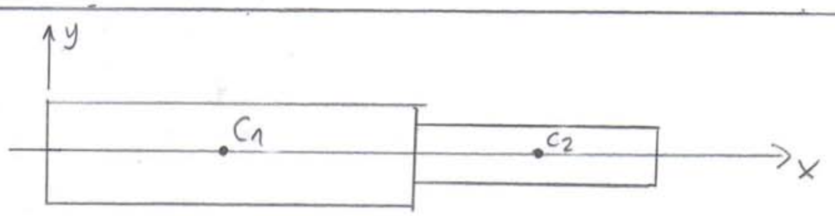
$b = 200\,mm$

$ØD = 40\,mm$

$Ød = 25\,mm$

$\rho = 7800\,\frac{kg}{m^3}$

task: • where is the center of gravity?
• reaction forces

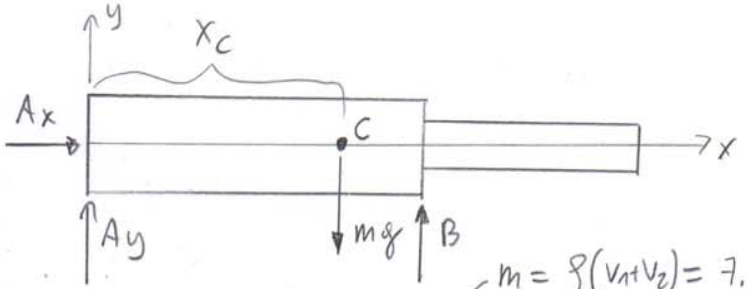


$x_{C_1} = \frac{a}{2} = 150\,mm$ $\qquad$ $x_{C_2} = a + \frac{b}{2} = 400\,mm$

$y_{C_1} = 0\,mm$ $\qquad$ $y_{C_2} = 0\,mm$

$V_1 = \frac{D^2\pi}{4} \cdot a = 376991\,mm^3$ $\qquad$ $V_2 = \frac{d^2\pi}{4} \cdot b = 98175\,mm^3$

$$x_C = \frac{x_{C_1} V_1 + x_{C_2} V_2}{V_1 + V_2} = \frac{150 \cdot 376991 + 400 \cdot 98175}{376991 + 98175} = 201.7\,mm$$



$\Sigma F_x = 0: \quad 1)\ A_x = 0$

$\Sigma F_y = 0: \quad 2)\ A_y + B - mg = 0$

$\Sigma M_{Az} = 0: \quad 3)\ -x_C\, mg + a\,B = 0$

$m = \rho(V_1 + V_2) = 7.8 \cdot 10^{-6}(376991 + 98175) = 3.706\,kg$

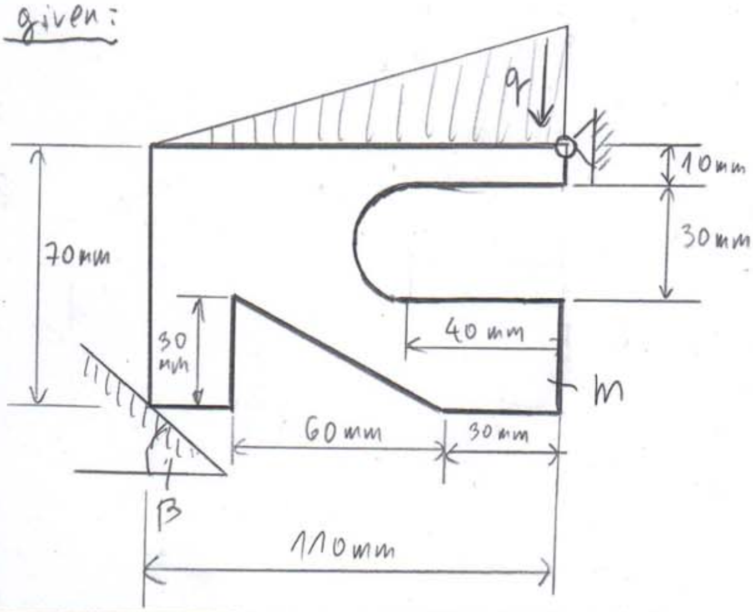
$1) \Rightarrow A_x = 0$

$3) \Rightarrow B = \frac{x_C\, mg}{a} = \frac{201.7 \cdot 3.706 \cdot 9.81}{300} = 24.4\,N\ (\uparrow)$

$2) \Rightarrow A_y = mg - B = 3.706 \cdot 9.81 - 24.4 = 11.9\,N\ (\uparrow)$

EXERCISE 7.7

given:



given:

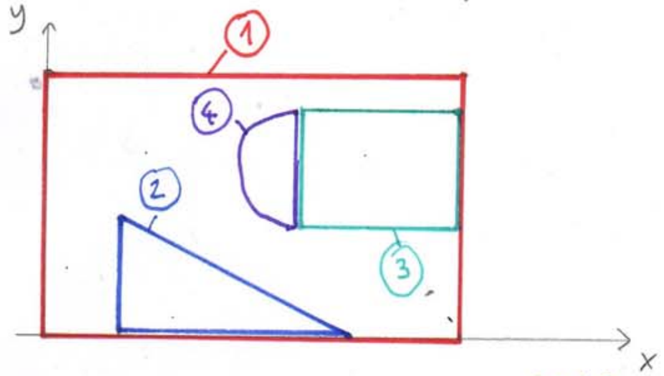
$q = 50 \frac{N}{mm}$

$m = 20\ kg$

$g = 9.81 \frac{m}{s^2}$

task: reaction forces

a.) Calculation of center of gravity (COG)



$A_1 = 110 \cdot 70 = 7700\ mm^2$

① $x_{c1} = 55\ mm$

$y_{c1} = 35\ mm$

$A_2 = \frac{60 \cdot 30}{2} = 900 mm^2$

② $x_{c2} = 20 + \frac{1}{3} \cdot 60 = 40\ mm$

$y_{c2} = \frac{1}{3} \cdot 30 = 10\ mm$

$A_3 = 30 \cdot 40 = 1200\ mm^2$

③ $x_{c3} = 70 + \frac{40}{2} = 90\ mm$

$y_{c3} = 70 - 10 - \frac{30}{2} = 45 mm$

$A_4 = \frac{15^2 \pi}{2} = 353{,}43\ mm^2$

④ $x_{c4} = 70 - \frac{4 \cdot 15}{3\pi} = 63.63 mm$

$y_{c4} = 70 - 10 - \frac{30}{2} = 45\ mm$

$$X_c = \frac{A_1 x_{c1} - A_2 x_{c2} - A_3 x_{c3} - A_4 x_{c4}}{A_1 - A_2 - A_3 - A_4} =$$

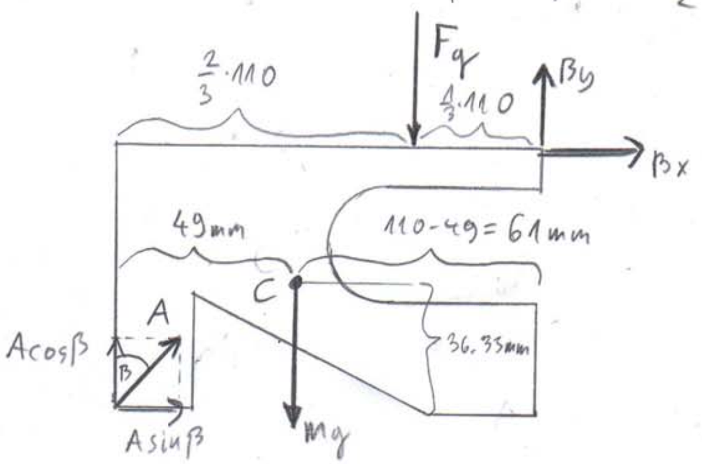
$$= \frac{7700 \cdot 55 - 900 \cdot 40 - 1200 \cdot 90 - 353.43 \cdot 63.63}{7700 - 900 - 1200 - 353.43} = 49 mm.$$

$$y_c = \frac{A_1 y_{c1} - A_2 y_{c2} - A_3 y_{c3} - A_4 y_{c4}}{A_1 - A_2 - A_3 - A_4} =$$

$$= \frac{7700 \cdot 35 - 900 \cdot 10 - 1200 \cdot 45 - 353.43 \cdot 45}{7700 - 900 - 1200 - 353.43} = 36.35\ mm$$

1.) free body diagram

$$\to F_q = \frac{110 \cdot 50}{2} = 2750\,N\,(\downarrow)$$



2.) equations of equilibrium

$$\Sigma F_x = 0: \quad 1)\ A\sin\beta + B_x = 0$$

$$\Sigma F_y = 0: \quad 2)\ A\cos\beta - mg - F_q + B_y = 0$$

$$\Sigma M_{Bz} = 0: \quad 3)\ -110\,A\cos\beta + 70\,A\sin\beta + 61\,mg + \frac{1}{3}\cdot 110\,F_q = 0$$

3.) solution of the equation of equilibrium

$$3) \Rightarrow A(-110\cos\beta + 70\sin\beta) = -61\,mg - \frac{1}{3}\cdot 110\,F_q$$

$$A = \frac{-61\,mg - \frac{1}{3}\,110\,F_q}{-110\cos\beta + 70\sin\beta} = \frac{-61\cdot 20\cdot 9.81 - \frac{1}{3}\cdot 110\cdot 2750}{-110\cos 45° - 70\sin 45°} = 886.3\,N\,(\nearrow)$$

$$1) \Rightarrow B_x = -A\sin\beta = -886.3\sin 45° = -626.7\,N\,(\leftarrow)$$

$$2) \Rightarrow B_y = mg + F_q - A\cos\beta = 20\cdot 9.81 + 2750 - 886.3\cdot\cos 45° = 2319{,}5\,N\,(\uparrow)$$

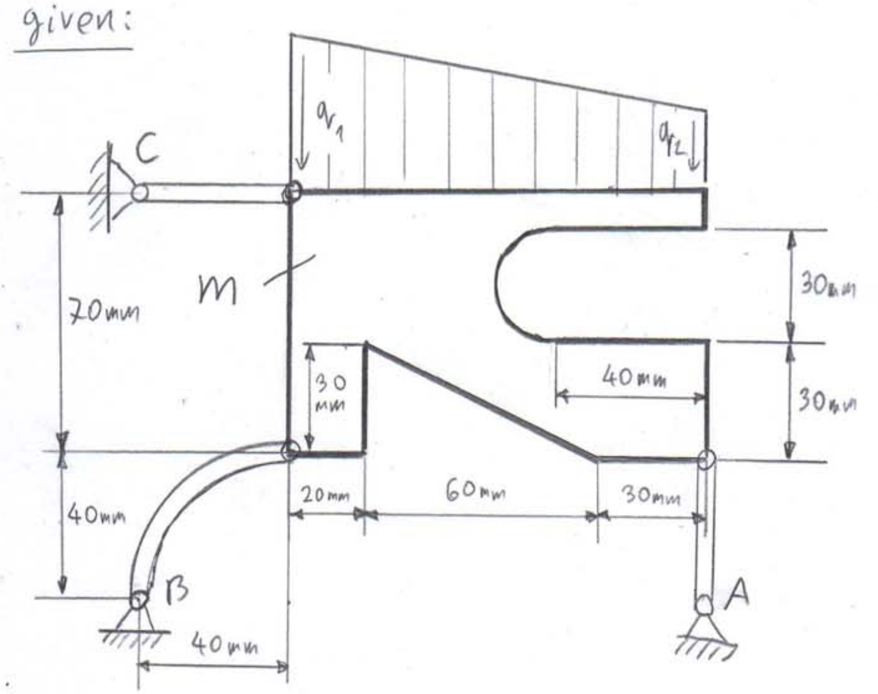
4.) results in vector form

$$\vec{A} = A\sin\beta\,\vec{i} + A\cos\beta\,\vec{j} = 886.3\sin 45°\,\vec{i} + 886.3\cos 45°\,\vec{j} =$$
$$= (626.7\,\vec{i} + 626.7\,\vec{j})\,N$$

$$\vec{B} = B_x\,\vec{i} + B_y\,\vec{j} = (-626.7\,\vec{i} + 2319.5\,\vec{j})\,N$$

EXERCISE 7.8

given:



$m = 20\ kg$

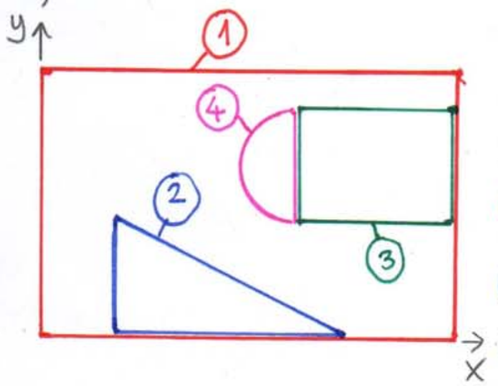
$q_1 = 31\ \frac{N}{mm}$

$q_2 = 10\ \frac{N}{mm}$

$g = 9.81\ \frac{m}{s^2}$

task: reaction forces

0.) Calculation of center of gravity



(1)

$A_1 = 110 \cdot 70 = 7700\ mm^2$

$x_{c_1} = 55\ mm$

$y_{c_1} = 35\ mm$

(2)

$A_2 = \frac{60 \cdot 30}{2} = 900\ mm^2$

$x_{c_2} = 20 + \frac{1}{3} \cdot 60 = 40\ mm$

$y_{c_2} = \frac{1}{3} \cdot 30 = 10\ mm$

(3)

$A_3 = 30 \cdot 40 = 1200\ mm^2$

$x_{c_3} = 110 - \frac{40}{2} = 90\ mm$

$y_{c_3} = 70 - 10 - \frac{30}{2} = 45\ mm$

(4)

$A_4 = \frac{15^2 \pi}{4} = 353.43\ mm^2$

$x_{c_4} = 70 - \frac{4 \cdot 15}{3\pi} = 63.63\ mm$

$y_{c_4} = 70 - 10 - \frac{30}{2} = 45\ mm$

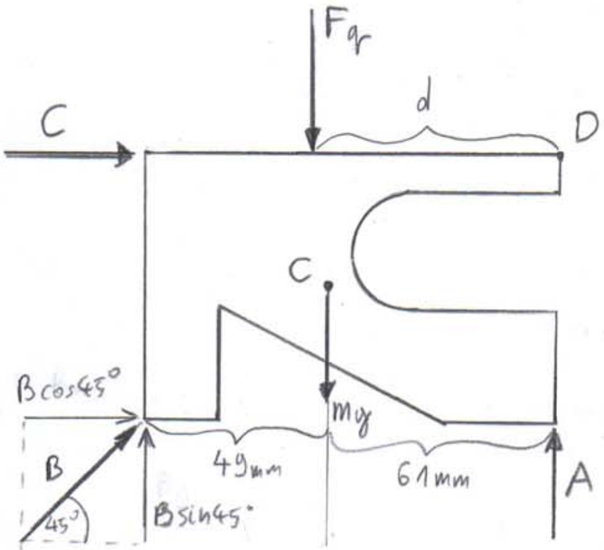
$$x_c = \frac{A_1 x_{c1} - A_2 x_{c2} - A_3 x_{c3} - A_4 x_{c4}}{A_1 - A_2 - A_3 - A_4} =$$

$$= \frac{7700 \cdot 55 - 900 \cdot 40 - 1200 \cdot 90 - 353.43 \cdot 63.63}{7700 - 900 - 1200 - 353.43} = 49\ mm$$

$$y_c = \frac{A_1 y_{c1} - A_2 y_{c2} - A_3 y_{c3} - A_4 y_{c4}}{A_1 - A_2 - A_3 - A_4} =$$

$$= \frac{7700 \cdot 35 - 900 \cdot 10 - 1200 \cdot 45 - 353.43 \cdot 45}{7700 - 900 - 1200 - 353.43} = 36.33\ mm$$

1.) Free body diagram:



$$F_q = \frac{q_1+q_2}{2}\cdot 110 = \frac{34+10}{2}\cdot 110 = 2420N$$

$$d = \frac{2q_1+q_2}{3(q_1+q_2)}\cdot 110 = \frac{2\cdot 34+10}{3(34+10)}\cdot 110 =$$

$$= 65\,mm$$

2.) equations of equilibrium

$\Sigma F_x = 0 \Rightarrow 1)\; C + B\cos 45° = 0$

$\Sigma F_y = 0 \Rightarrow 2)\; B\sin 45° - F_q - mg + A = 0$

$\Sigma M_{Dz} = 0 \Rightarrow 3)\; -110\,B\sin 45° + 70\,B\cos 45° + d\cdot F_q + 61\,mg = 0$

3.) solution of the equations of equilibrium

$$3) \Rightarrow B = \frac{-d\cdot F_q - 61\cdot mg}{-110\sin 45° + 70\cos 45°} = \frac{-65\cdot 2420 - 61\cdot 20\cdot 9.81}{-110\sin 45° + 70\cos 45°} = 1907.6N\,(\nearrow)$$

$$1) \Rightarrow C = -B\cos 45° = -1907.6\cdot\cos 45° = -1348.9N\,(\leftarrow)$$

$$2) \Rightarrow A = F_q + mg - B\sin 45° = 2420 + 20\cdot 9.81 - 1907.6\sin 45° = 1267.3N\,(\uparrow)$$

4.) results in vector form

$$\vec{A} = A\vec{j} = (1267.3\vec{j})N$$

$$\vec{B} = B\cos 45°\vec{i} + B\sin 45°\vec{j} = 1907.6\cos 45°\vec{i} + 1907.6\sin 45°\vec{j} =$$

$$= (1348.9\vec{i} + 1348.9\vec{j})N$$

$$\vec{C} = C\vec{i} = (-1348.9\vec{i})N$$