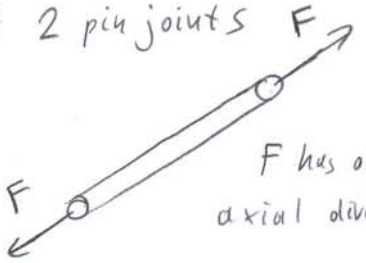
two-force member

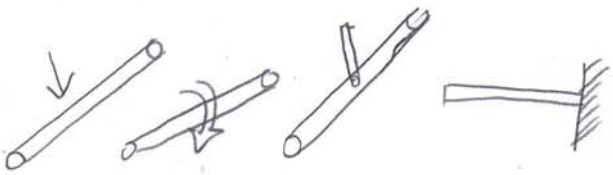
a member that is loaded only at 2 pin joints



F has an axial direction!

multi-force member

a member that is loaded at MORE than 2 points



STRUCTURES

FRAMES

- assembly of members of which at least one is a multi-force member

1): FBD for each multi-force member

2) EE for each multi force member

3) solving of EE

(4) Final results in vector form)

three pinned structures

Compound beam (Gerber beam)

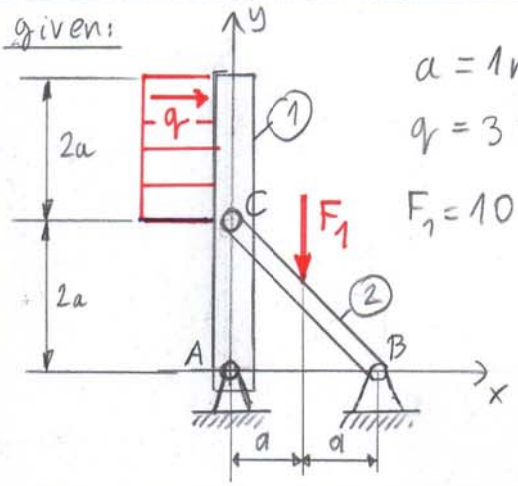
other Frames

TRUSSES

- assembly of members of which each member is a two-force member

EXERCISE 8.1

given:



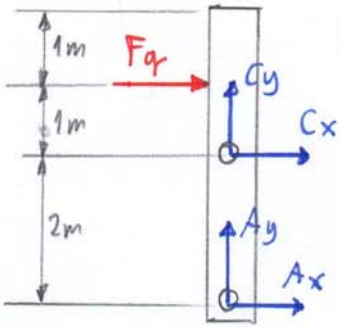
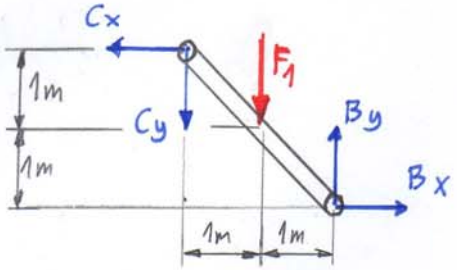
$a = 1\,m$

$q = 3\,\frac{kN}{m}$

$F_1 = 10\,kN$

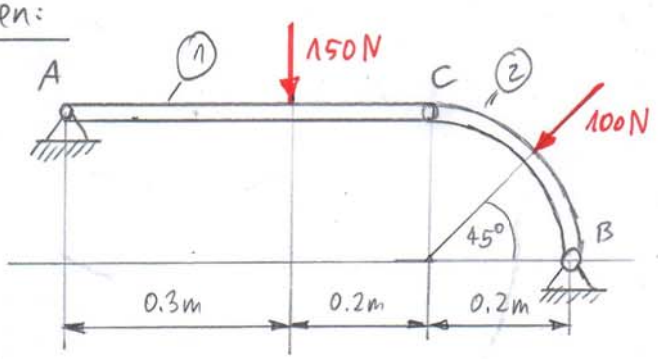
task:

- reaction forces at points A and B
- internal force at point C

	body ①	body ②
1. FBD	 $F_q = 2 \cdot 3 = 6\,kN(\rightarrow)$	
2. EE	$\Sigma F_x = 0$: 1) $A_x + C_x + F_q = 0$ $\Sigma F_y = 0$: 2) $A_y + C_y = 0$ $\Sigma M_{Az} = 0$: 3) $-2C_x - 3F_q = 0$	$\Sigma F_x = 0$: 4) $B_x - C_x = 0$ $\Sigma F_y = 0$: 5) $B_y - C_y - F_1 = 0$ $\Sigma M_{Bz} = 0$: 6) $2C_x + 2C_y + 1 \cdot F_1 = 0$
3. EE SOL.	3) $\Rightarrow C_x = \frac{-3F_q}{2} = \frac{-3 \cdot 6}{2} = -9\,kN\,(\leftarrow\rightarrow)$ 6) $\Rightarrow C_y = \frac{-2C_x - 1 \cdot F_1}{2} = \frac{-2 \cdot (-9) - 1 \cdot 10}{2} = 4\,kN\,(1\downarrow)$ 1) $\Rightarrow A_x = -C_x - F_q = -(-9) - 6 = 3\,kN\,(\rightarrow)$ 2) $\Rightarrow A_y = -C_y = -4\,kN\,(\downarrow)$ 3) $\Rightarrow B_x = C_x = -9\,kN\,(\leftarrow)$ 4) $\Rightarrow B_y = C_y + F_1 = 4 + 10 = 14\,kN\,(\uparrow)$	
4. RES.	$\vec{A} = A_x\vec{i} + A_y\vec{j} = (3\vec{i} - 4\vec{j})\,kN$ $\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (-9\vec{i} + 4\vec{j})\,kN$	$\vec{B} = B_x\vec{i} + B_y\vec{j} = (-9\vec{i} + 14\vec{j})\,kN$ $\vec{C}_{21} = -C_x\vec{i} - C_y\vec{j} = (9\vec{i} - 4\vec{j})\,kN$

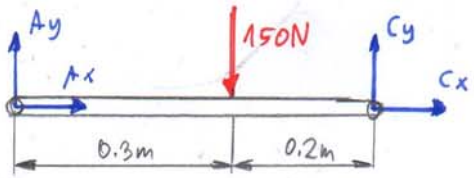
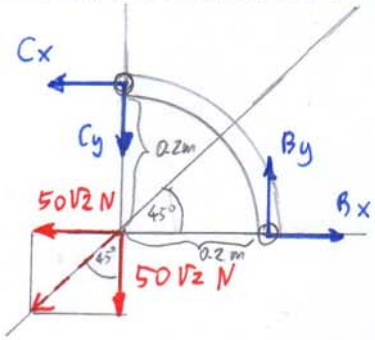
EXERCISE 8.2

given:

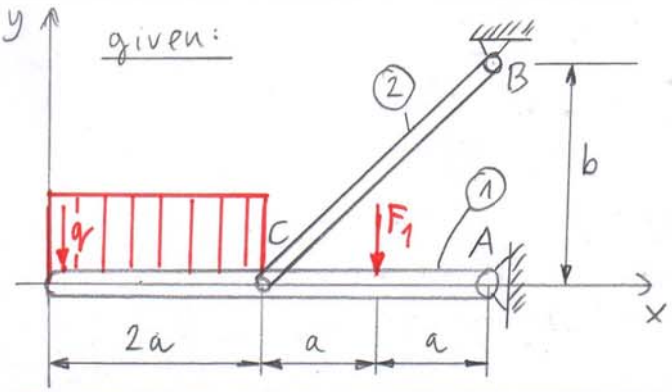


task:

- reaction forces at points A and B
- internal force at point C

	body ①	body ②
1 FBD		
2 EE	$\Sigma F_x=0$: 1) $A_x + C_x = 0$ $\Sigma F_y=0$: 2) $A_y + C_y - 150 = 0$ $\Sigma M_{A_z}=0$: 3) $-0.3\cdot 150 + 0.5\cdot C_y = 0$	$\Sigma F_x=0$: 4) $B_x - C_x - 50\sqrt{2} = 0$ $\Sigma F_y=0$: 5) $B_y - C_y - 50\sqrt{2} = 0$ $\Sigma M_{B_z}=0$: 6) $0.2C_x + 0.2C_y + 0.2\,50\sqrt{2} = 0$
3 EE SOL	3) $\Rightarrow C_y = \frac{0.3\cdot 150}{0.5} = 90\,N\ (\uparrow\downarrow)$ 6) $\Rightarrow C_x = \frac{-0.2C_y - 0.2\cdot 50\sqrt{2}}{0.2} = -C_y - 50\sqrt{2} = -90 - 50\sqrt{2} = -160.7\,N\ (\leftarrow\rightarrow)$ 1) $\Rightarrow A_x = -C_x = -(-160.7) = 160.7\,N\ (\rightarrow)$ 2) $\Rightarrow A_y = 150 - C_y = 150 - 90 = 60\,N\ (\uparrow)$ 4) $\Rightarrow B_x = 50\sqrt{2} + C_x = 50\sqrt{2} - 160.7 = -90\,N\ (\leftarrow)$ 5) $\Rightarrow B_y = 50\sqrt{2} + C_y = 50\sqrt{2} + 90 = 160.7\,N\ (\uparrow)$	
4 RES	$\vec{A} = A_x\vec{i} + A_y\vec{j} = (160.7\vec{i} + 60\vec{j})\,N$ $\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (-160.7\vec{i} + 90\vec{j})\,N$	$\vec{B} = B_x\vec{i} + B_y\vec{j} = (-90\vec{i} + 160.7\vec{j})\,N$ $\vec{C}_{12} = -C_x\vec{i} - C_y\vec{j} = (160.7\vec{i} - 90\vec{j})\,N$

EXERCISE 8.3



given:

$a = 200\,mm$

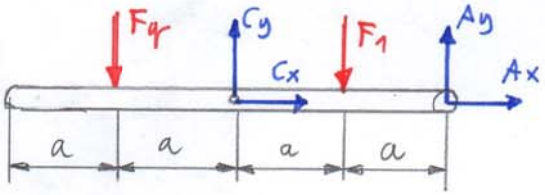
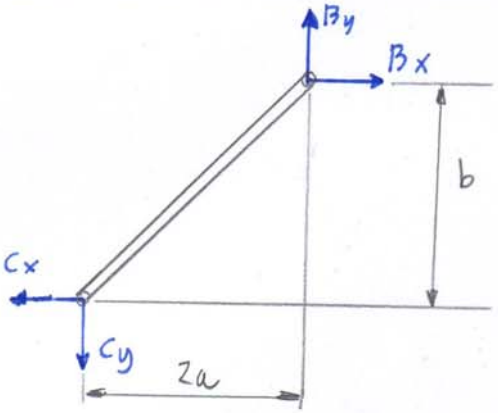
$b = 400\,mm$

$q = 500\,\frac{N}{m}$

$F_1 = 50\,N$

task:

- reaction Forces at points A and B
- internal Force at point C

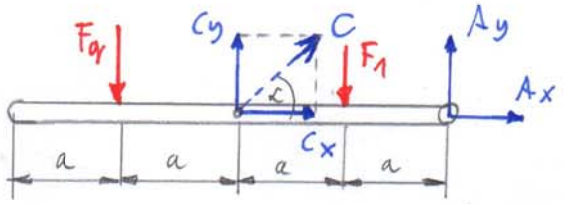
	body ①	body ②
1 FBD	$F_q = 2aq = 2 \cdot 200 \cdot 0.5 = 200N\,(\downarrow)$ (q in $\frac{N}{mm}$!) 	
2 EE	$\Sigma F_x = 0$: 1) $C_x + A_x = 0$ $\Sigma F_y = 0$: 2) $C_y + A_y - F_q - F_1 = 0$ $\Sigma M_{Az} = 0$: 3) $3a F_q - 2a C_y + a F_1 = 0$	$\Sigma F_x = 0$: 4) $B_x - C_x = 0$ $\Sigma F_y = 0$: 5) $B_y - C_y = 0$ $\Sigma M_{Bz} = 0$: 6) $-b C_x + 2a C_y = 0$
3. EE SOL	3) $\Rightarrow C_y = \frac{a F_1 + 3a F_q}{2a} = \frac{F_1 + 3F_q}{2} = \frac{50 + 3 \cdot 200}{2} = 325N\,(\uparrow\downarrow)$ 6) $\Rightarrow C_x = \frac{2a C_y}{b} = \frac{2 \cdot 200 \cdot 325}{400} = 325N\,(\rightarrow\leftarrow)$ 1) $\Rightarrow A_x = -C_x = -325N\,(\leftarrow)$ 2) $\Rightarrow A_y = F_1 + F_q - C_y = 50 + 200 - 325 = -75N\,(\downarrow)$ 4) $\Rightarrow B_x = C_x = 325N\,(\rightarrow)$ 5) $\Rightarrow B_y = C_y = 325N\,(\uparrow)$	
4. RES	$\vec{A} = A_x \vec{i} + A_y \vec{j} = (-325\vec{i} - 75\vec{j})N$ $\vec{C}_{21} = C_x \vec{i} + C_y \vec{j} = (325\vec{i} + 325\vec{j})N$	$\vec{B} = B_x \vec{i} + B_y \vec{j} = (325\vec{i} + 325\vec{j})N$ $\vec{C}_{12} = -C_x \vec{i} - C_y \vec{j} = (-325\vec{i} - 325\vec{j})N$

Note: As rod ② is unloaded it can be considered as a supporting rod (usually called as 2 force member)

⇓

The exercise can be solved as the equilibrium of one rigid body: rod ①

1) free body diagram:



2) equations of equilibrium:

$\Sigma F_x = 0 \Rightarrow$ 1) $C_x + A_x = 0$

$\Sigma F_y = 0 \Rightarrow$ 2) $C_y + A_y - F_q - F_1 = 0$

$\Sigma M_{A_z} = 0 \Rightarrow$ 3) $3a\,F_q - 2a\,C_y + a\,F_1 = 0$

$\oplus \quad +\gamma\alpha = \frac{C_y}{C_x} = \frac{b}{2a} = \frac{400}{2 \cdot 200} = 1$

3) solution of the equations of equilibrium:

3) $\Rightarrow C_y = \frac{a\,F_1 + 3a\,F_q}{2a} = \frac{F_1 + 3F_q}{2} = \frac{50 + 3 \cdot 200}{2} = 325\,N\,(\uparrow)$

$\oplus \Rightarrow C_x = \frac{C_y}{1} = 325\,N\,(\rightarrow)$

1) $\Rightarrow A_x = -C_x = -325\,N\,(\leftarrow)$

2) $\Rightarrow A_y = F_1 + F_q - C_y = 50 + 200 - 325 = -75\,N\,(\downarrow)$

4) results in vector form:

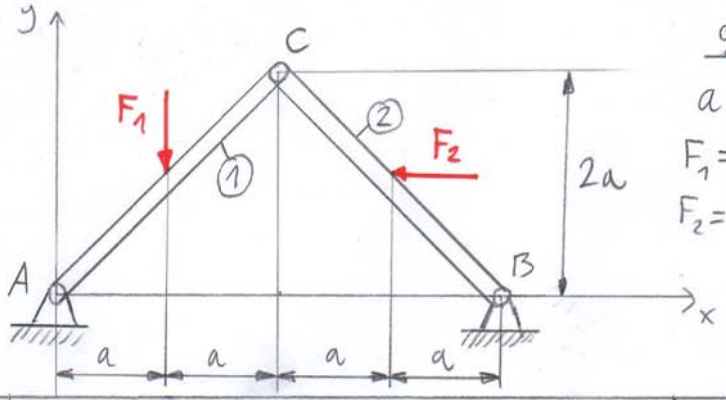
$\vec{A} = A_x\vec{i} + A_y\vec{j} = (-325\vec{i} - 75\vec{j})\,N$

$\vec{C} = \vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (325\vec{i} + 325\vec{j})\,N$

$\vec{B} = \vec{C} = (325\vec{i} + 325\vec{j})\,N$

$\vec{C}_{11} = -\vec{C}_{21} = (-325\vec{i} - 325\vec{j})\,N$

EXERCISE 8.4



given:

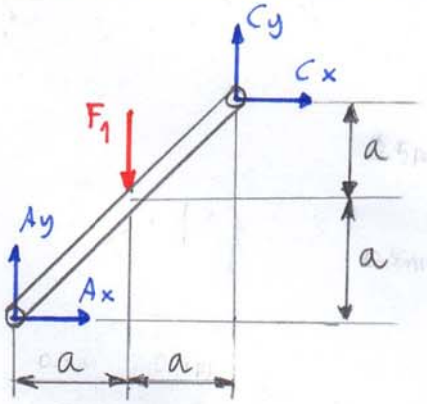
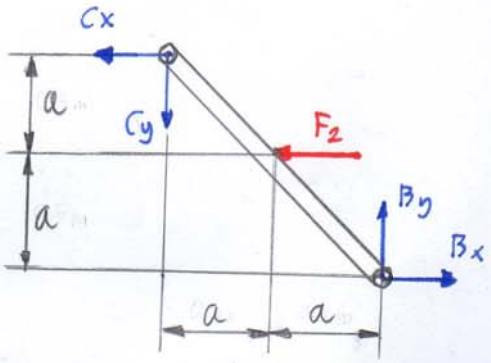
$a = 0{,}5\,m$

$F_1 = 200\,N$

$F_2 = 400\,N$

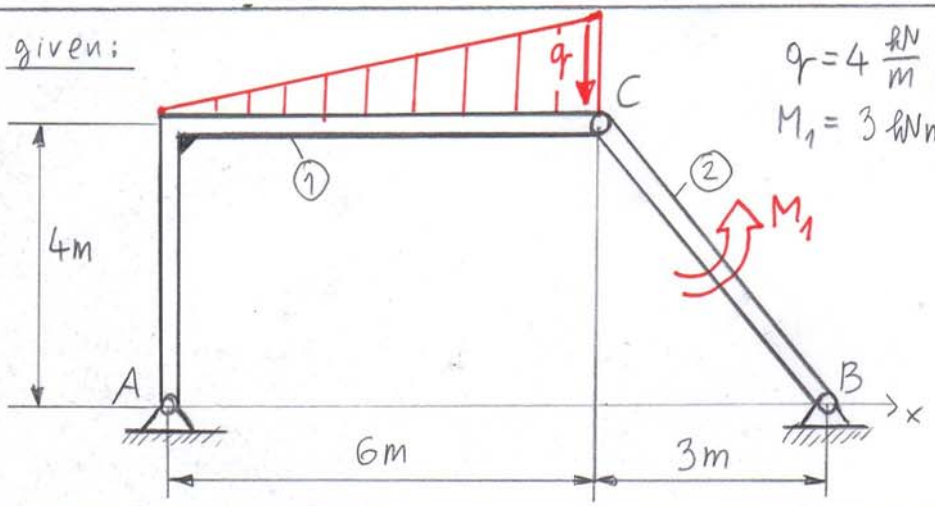
task:

- reaction forces at points A and B
- internal force at point C

	body ①	body ②
1 FBD		
2 EE	$\Sigma F_x = 0$: 1) $A_x + C_x = 0$ $\Sigma F_y = 0$: 2) $A_y + C_y - F_1 = 0$ $\Sigma M_{Az} = 0$: 3) $-a F_1 + 2a C_y - 2a C_x = 0$	$\Sigma F_x = 0$: 4) $B_x - C_x - F_2 = 0$ $\Sigma F_y = 0$: 5) $B_y - C_y = 0$ $\Sigma M_{Bz} = 0$: 6) $a F_2 + 2a C_x + 2a C_y$
3 EE SOL	3) + 6) $\Rightarrow a F_2 - a F_1 + 4a C_y = 0 \Rightarrow C_y = \frac{F_1 - F_2}{4} = \frac{200-400}{4} = -50\,N\,(\downarrow\uparrow)$ 3) $\Rightarrow C_x = \frac{2a C_y - a F_1}{2a} = \frac{2C_y - F_1}{2} = \frac{2\cdot(-50)-200}{2} = -150\,N\,(\leftarrow\rightarrow)$ 1) $\Rightarrow A_x = -C_x = -(-150) = 150\,N\,(\rightarrow)$ 2) $\Rightarrow A_y = F_1 - C_y = 200 - (-50) = 250\,N\,(\uparrow)$ 4) $\Rightarrow B_x = C_x + F_2 = -150 + 400 = 250\,N\,(\rightarrow)$ 5) $\Rightarrow B_y = C_y = -50\,N\,(\downarrow)$	
4 RES	$\vec{A} = A_x\vec{i} + A_y\vec{j} = (150\vec{i} + 250\vec{j})\,N$ $\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (-150\vec{i} - 50\vec{j})\,N$	$\vec{B} = B_x\vec{i} + B_y\vec{j} = (250\vec{i} - 50\vec{j})\,N$ $\vec{C}_{12} = -C_x\vec{i} - C_y\vec{j} = (150\vec{i} + 50\vec{j})\,N$

EXERCISE 8.5

given:

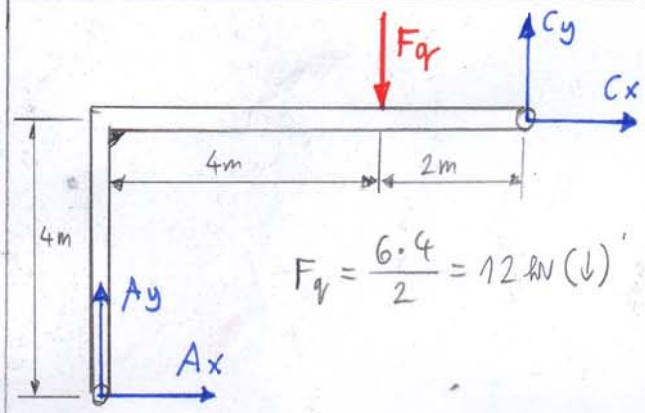
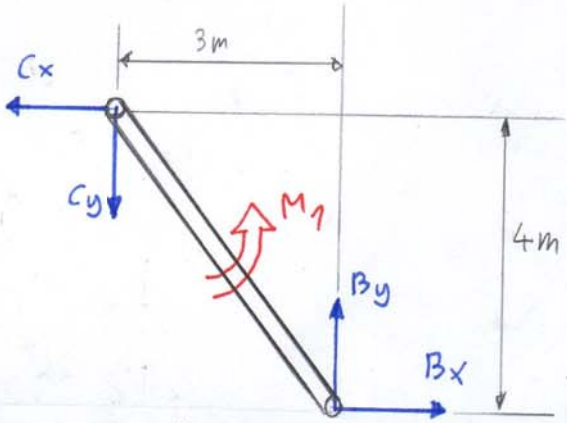


$q = 4 \frac{kN}{m}$

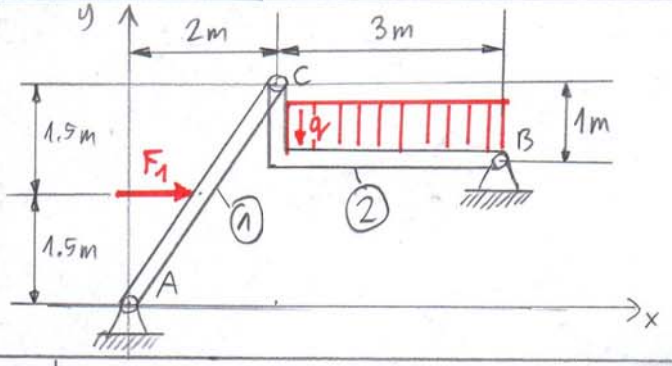
$M_1 = 3\ kNm$

task:

- reaction forces at points A and B
- internal force at point C

	body ①	body ②
1. FBD	 $F_q = \frac{6 \cdot 4}{2} = 12\ kN\ (\downarrow)$	
2. EE	$\Sigma F_x = 0$: 1) $A_x + C_x = 0$ $\Sigma F_y = 0$: 2) $A_y + C_y - F_q = 0$ $\Sigma M_{Az} = 0$: 3) $-4F_q + 6C_y - 4C_x = 0$	$\Sigma F_x = 0$: 4) $-C_x + B_x = 0$ $\Sigma F_y = 0$: 5) $-C_y + B_y = 0$ $\Sigma M_{Bz} = 0$: 6) $4C_x + 3C_y + M_1 = 0$
3. EE SOL	3) + 6) $\Rightarrow -4F_q + 9C_y + M_1 = 0 \Rightarrow C_y = \frac{4F_q - M_1}{9} = \frac{4 \cdot 12 - 3}{9} = 5\ kN\ (\uparrow\downarrow)$ 6) $\Rightarrow C_x = \frac{-M_1 - 3C_y}{4} = \frac{-3 - 3 \cdot 5}{4} = -4.5\ kN\ (\leftarrow\rightarrow)$ 1) $\Rightarrow A_x = -C_x = -(-4.5) = 4.5\ kN\ (\rightarrow)$ 2) $\Rightarrow A_y = F_q - C_y = 12 - 5 = 7\ kN\ (\uparrow)$ 3) $\Rightarrow B_x = C_x = -4.5\ kN\ (\leftarrow)$ 4) $\Rightarrow B_y = C_y = 5\ kN\ (\uparrow)$	
4. RES.	$\vec{A} = A_x\vec{i} + A_y\vec{j} = (4.5\vec{i} + 7\vec{j})\ kN$ $\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (-4.5\vec{i} + 5\vec{j})\ kN$	$\vec{B} = B_x\vec{i} + B_y\vec{j} = (-4.5\vec{i} + 5\vec{j})\ kN$ $\vec{C}_{12} = -C_x\vec{i} - C_y\vec{j} = (4.5\vec{i} - 5\vec{j})\ kN$

EXERCISE 8.6



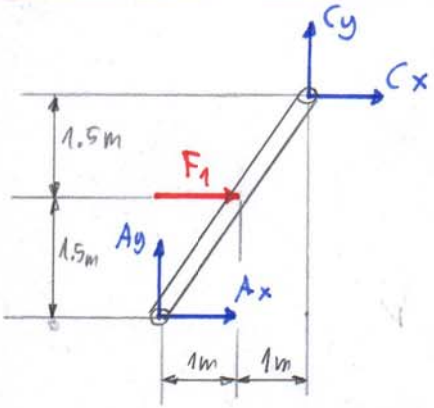
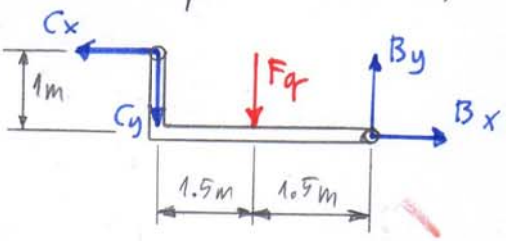
given:

$F_1 = 7\,kN$

$q = 2\,\frac{kN}{m}$

task:

- reaction forces at points A and B
- reaction force at point C

	body ①	body ②
1. FBD		$F_q = 3 \cdot 2 = 6\,kN(\downarrow)$ 
2. EE	$\Sigma F_x = 0$: 1) $A_x + C_x + F_1 = 0$ $\Sigma F_y = 0$: 2) $A_y + C_y = 0$ $\Sigma M_{A_z} = 0$: 3) $-1.5F_1 - 3C_x + 2C_y = 0$	$\Sigma F_x = 0$: 4) $B_x - C_x = 0$ $\Sigma F_y = 0$: 5) $B_y - C_y - F_q = 0$ $\Sigma M_{B_z} = 0$: 6) $1 \cdot C_x + 3 \cdot C_y + 1.5F_q = 0$

3. EE SOL

$6) \Rightarrow C_x = -3C_y - 1.5F_q$

$6) \to 3) \Rightarrow -1.5F_1 - 3(-3C_y - 1.5F_q) + 2C_y = 0$

$-1.5F_1 + 9C_y + 4.5F_q + 2C_y = 0$

$11C_y = 1.5F_1 - 4.5F_q$

$C_y = \frac{1.5F_1 - 4.5F_q}{11} = \frac{1.5 \cdot 7 - 4.5 \cdot 6}{11} = -1.5\,kN \quad (\downarrow\uparrow)$

$6) \Rightarrow C_x = -3C_y - 1.5F_q = -3(-1.5) - 1.5 \cdot 6 = -4.5\,kN \quad (\leftarrow\rightarrow)$

$1) \Rightarrow A_x = -C_x - F_1 = -(-4.5) - 7 = -2.5\,kN\,(\leftarrow)$

$2) \Rightarrow A_y = -C_y = -(-1.5) = 1.5\,kN\,(\uparrow)$

$4) \Rightarrow B_x = C_x = -4.5\,kN\,(\leftarrow)$

$5) \Rightarrow B_y = C_y + F_q = -1.5 + 6 = 4.5\,kN\,(\uparrow)$

4. RES.

$\vec{A} = A_x\vec{i} + A_y\vec{j} = (-2.5\vec{i} + 1.5\vec{j})\,kN$

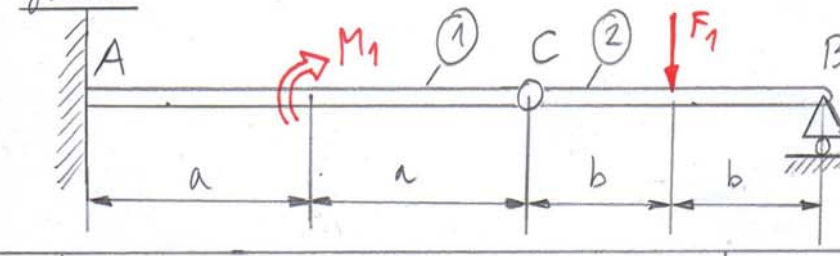
$\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (-4.5\vec{i} - 1.5\vec{j})\,kN$

$\vec{B} = B_x\vec{i} + B_y\vec{j} = (-4.5\vec{i} + 4.5\vec{j})\,kN$

$\vec{C}_{12} = -C_x\vec{i} - C_y\vec{j} = (4.5\vec{i} + 1.5\vec{j})\,kN$

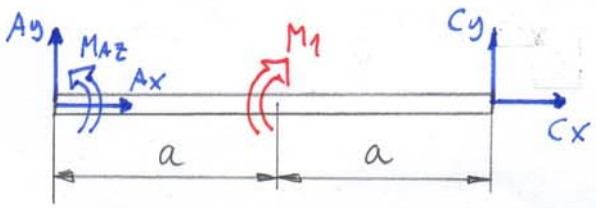
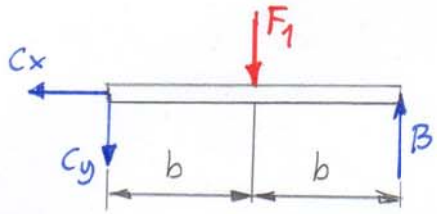
EXERCISE 8.7

given:



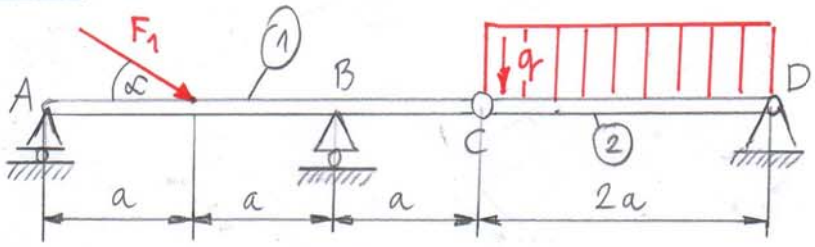
$a = 3m$
$b = 2m$
$F_1 = 6kN$
$M_1 = 4kNm$

task:
- reaction forces at points A and B
- internal force at point C

	body (1)	body (2)
1 FBD		
2 EE	$\Sigma F_x = 0$: 1) $A_x + C_x = 0$ $\Sigma F_y = 0$: 2) $A_y + C_y = 0$ $\Sigma M_{Az} = 0$: 3) $M_{Az} - M_1 + 2a C_y = 0$	$\Sigma F_x = 0$: 4) $-C_x = 0$ $\Sigma F_y = 0$: 5) $B - C_y - F_1 = 0$ $\Sigma M_{Bz} = 0$: 6) $2b C_y + b F_1 = 0$
3 EE SOL	6) $\Rightarrow C_y = \frac{-bF_1}{2b} = -\frac{F_1}{2} = -\frac{6}{2} = -3\,kN\,(\downarrow\uparrow)$ 3) $\Rightarrow M_{Az} = M_1 - 2a C_y = 4 - 2\cdot 3\cdot(-3) = 22\,kNm\,(\circlearrowleft)$ 4) $\Rightarrow C_x = 0$ 1) $\Rightarrow A_x = -C_x = 0$ 2) $\Rightarrow A_y = -C_y = -(-3) = 3\,kN\,(\uparrow)$ 5) $\Rightarrow B = F_1 + C_y = 6 + (-3) = 3\,kN\,(\uparrow)$	
4 RES	$\vec{A} = A_x\vec{i} + A_y\vec{j} = (0\vec{i} + 3\vec{j})\,kN$ $\vec{M}_A = M_{Az}\vec{k} = (22\vec{k})\,kNm$ $\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (0\vec{i} - 3\vec{j})\,kN$	$\vec{B} = B\vec{j} = (3\vec{j})\,kN$ $\vec{C}_{12} = -C_x\vec{i} - C_y\vec{j} = (0\vec{i} + 3\vec{j})\,kN$

EXERCISE 8.8

given:



$F_1 = 12\,kN$

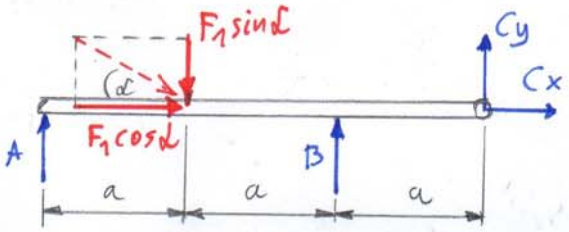
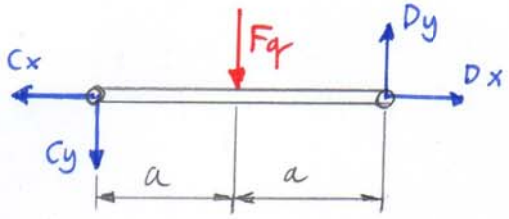
$q = 6\,\frac{kN}{m}$

$a = 1\,m$

$\alpha = 30°$

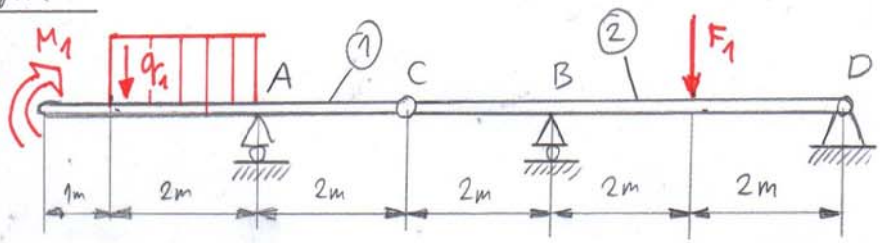
task:

- reaction forces at points A, B and D
- internal force at point C

	body ①	body ②
1 FBD		$F_q = 2a\,q = 2 \cdot 1 \cdot 6 = 12\,kN(\downarrow)$ 
2 EE	$\Sigma F_x = 0$: 1) $F_1 \cos\alpha + C_x = 0$ $\Sigma F_y = 0$: 2) $A + B + C_y - F_1 \sin\alpha = 0$ $\Sigma M_{Az} = 0$: 3) $-a F_1 \sin\alpha + 2a B + 3a C_y = 0$	$\Sigma F_x = 0$: 4) $D_x - C_x = 0$ $\Sigma F_y = 0$: 5) $D_y - C_y - F_q = 0$ $\Sigma M_{Dz} = 0$: 6) $2a C_y + a F_q = 0$
3 EE SOL.	6) $\Rightarrow C_y = \frac{-a F_q}{2a} = -\frac{F_q}{2} = -\frac{12}{2} = -6\,kN\,(\downarrow\uparrow)$ 3) $\Rightarrow B = \frac{a F_1 \sin\alpha - 3a C_y}{2a} = \frac{F_1 \sin\alpha - 3C_y}{2} = \frac{12 \sin 30° - 3 \cdot (-6)}{2} = 12\,kN(\uparrow)$ 1) $\Rightarrow C_x = -F_1 \cos\alpha = -12 \cos 30° = -10.4\,kN(\leftarrow\rightarrow)$ 2) $\Rightarrow A = F_1 \sin\alpha - B - C_y = 12 \sin 30° - 12 - (-6) = 0\,kN$ 4) $\Rightarrow D_x = C_x = -10.4\,kN(\leftarrow)$ 5) $\Rightarrow D_y = F_q + C_y = 12 + (-6) = 6\,kN(\uparrow)$	
4 RES.	$\vec{A} = A\vec{j} = \vec{0}$ $\vec{B} = B\vec{j} = (12\vec{j})\,kN$ $\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (-10.4\vec{i} - 6\vec{j})\,kN$	$\vec{D} = D_x\vec{i} + D_y\vec{j} = (-10.4\vec{i} + 6\vec{j})\,kN$ $\vec{C}_{12} = -C_x\vec{i} - C_y\vec{j} = (10.4\vec{i} + 6\vec{j})\,kN$

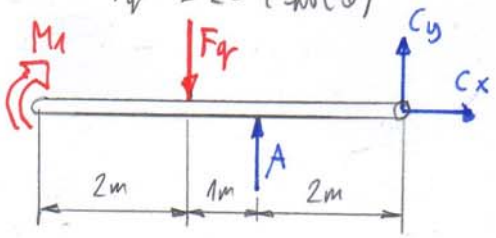
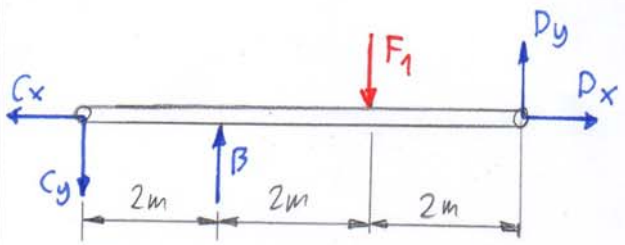
EXERCISE 8.9

given:



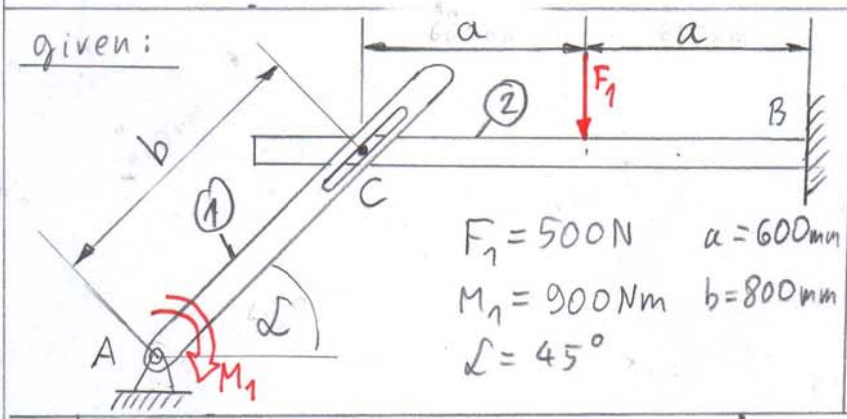
$M_1 = 8\ kNm$
$F_1 = 4\ kN$
$q_1 = 2\ \frac{kN}{m}$

task:
- reaction forces at points A, B and D
- internal force at point C

	body ①	body ②
1 FBD	$F_q = 2 \cdot 2 = 4\ kN (\downarrow)$ 	
2 EE	$\Sigma F_x = 0$: 1) $C_x = 0$ $\Sigma F_y = 0$: 2) $A + C_y - F_q = 0$ $\Sigma M_{Az} = 0$: 3) $-M_1 + 1 \cdot F_q + 2 \cdot C_y = 0$	$\Sigma F_x = 0$: 4) $-C_x + D_x = 0$ $\Sigma F_y = 0$: 5) $B - C_y - F_1 + D_y = 0$ $\Sigma M_{Dz} = 0$: 6) $6C_y - 4B + 2F_1 = 0$
3 EE SOL	3) $\Rightarrow C_y = \frac{M_1 - F_q}{2} = \frac{8-4}{2} = 2\ kN\ (\uparrow\downarrow)$ 6) $\Rightarrow B = \frac{6C_y + 2F_1}{4} = \frac{6 \cdot 2 + 2 \cdot 4}{4} = 5\ kN\ (\uparrow)$ 1) $\Rightarrow C_x = 0$ 2) $\Rightarrow A = F_q - C_y = 4 - 2 = 2\ kN\ (\uparrow)$ 4) $\Rightarrow D_x = C_x = 0$ 5) $\Rightarrow D_y = F_1 + C_y - B = 4 + 2 - 5 = 1\ kN\ (\uparrow)$	
4 RES	$\vec{A} = A\vec{j} = (2\vec{j})\ kN$ $\vec{C}_{21} = C_x\vec{i} + C_y\vec{j} = (0\vec{i} + 2\vec{j})\ kN$	$\vec{B} = B\vec{j} = (5\vec{j})\ kN$ $\vec{D} = D_x\vec{i} + D_y\vec{j} = (0\vec{i} + 1\vec{j})\ kN$ $\vec{C}_{12} = -C_x\vec{i} - C_y\vec{j} = (0\vec{i} - 2\vec{j})\ kN$

EXERCISE 8.10

given:



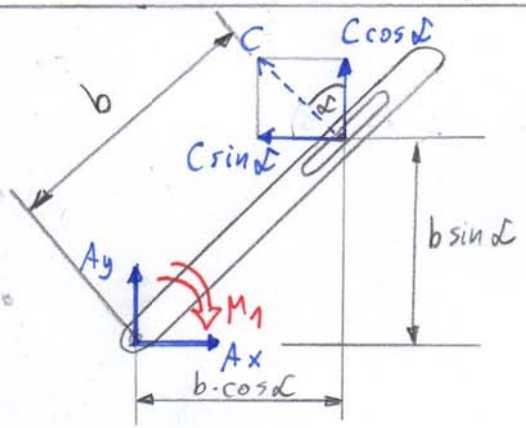
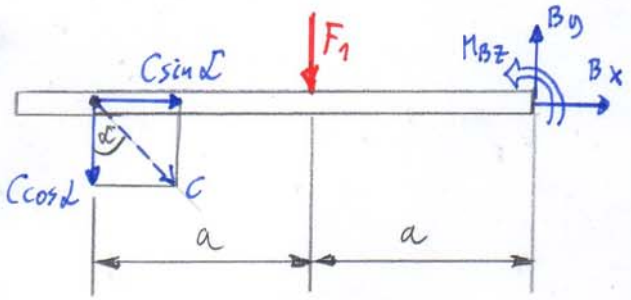
$F_1 = 500\,N$ $\quad a = 600\,mm$

$M_1 = 900\,Nm$ $\quad b = 800\,mm$

$\alpha = 45°$

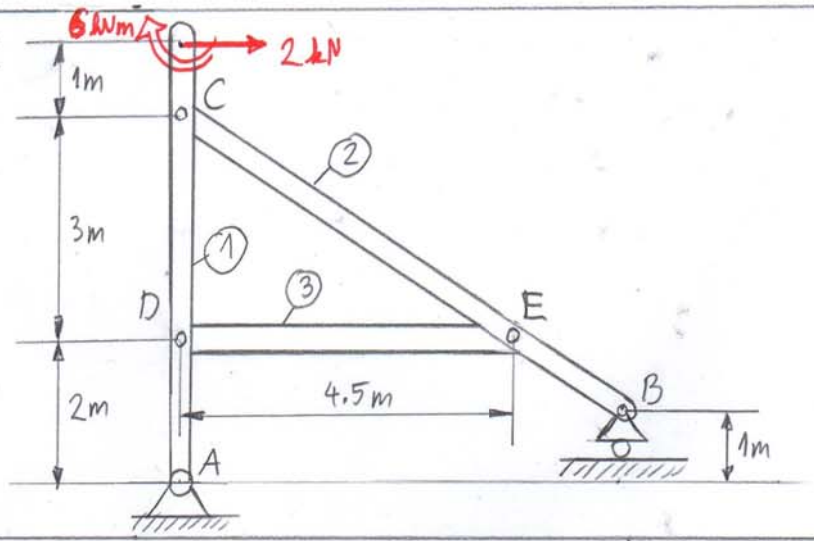
task:

- reaction forces at points A and B
- internal force at slider C

	body ①	body ②
1 FBD		
2. EE	$\Sigma F_x = 0$: 1) $A_x - C\sin\alpha = 0$ $\Sigma F_y = 0$: 2) $A_y + C\cos\alpha = 0$ $\Sigma M_{Az} = 0$: 3) $-M_1 + bC = 0$	$\Sigma F_x = 0$: 4) $C\sin\alpha + B_x = 0$ $\Sigma F_y = 0$: 5) $-C\cos\alpha + B_y - F_1 = 0$ $\Sigma M_{Bz} = 0$: 6) $2aC\cos\alpha + aF_1 + M_{Bz} = 0$
3. EE SOL	3) $\Rightarrow C = \frac{M_1}{b} = \frac{900}{0.8} = 1125\,N\ (\nwarrow\searrow)$ 6) $\Rightarrow M_{Bz} = -aF_1 - 2aC\cos\alpha = -0.6\cdot 500 - 2\cdot 0.6\cdot 1125\cos 45° = -1254.6\,Nm\ (\circlearrowleft)$ 1) $\Rightarrow A_x = C\sin\alpha = 1125\sin 45° = 795.5\,N\ (\rightarrow)$ 2) $\Rightarrow A_y = -C\cos\alpha = -1125\cos 45° = -795.5\,N\ (\downarrow)$ 4) $\Rightarrow B_x = -C\sin\alpha = -1125\sin 45° = -795.5\,N\ (\leftarrow)$ 5) $\Rightarrow B_y = F_1 + C\cos\alpha = 500 + 1125\cos 45° = 1295.5\,N\ (\uparrow)$	
4. RES	$\vec{A} = A_x\vec{i} + A_y\vec{j} = (795.5\vec{i} - 795.5\vec{j})\,N$ $\vec{C}_{21} = -C\sin\alpha\,\vec{i} + C\cos\alpha\,\vec{j} =$ $= -1125\sin 45°\,\vec{i} + 1125\cos 45°\,\vec{j} =$ $= (-795.5\vec{i} + 795.5\vec{j})\,N$	$\vec{B} = B_x\vec{i} + B_y\vec{j} = (-795.5\vec{i} + 1295.5\vec{j})\,N$ $\vec{M}_B = M_{Bz}\vec{k} = (-1254.6\vec{k})\,Nm$ $\vec{C}_{12} = C\sin\alpha\,\vec{i} - C\cos\alpha\,\vec{j} =$ $= (795.5\vec{i} - 795.5\vec{j})\,N$

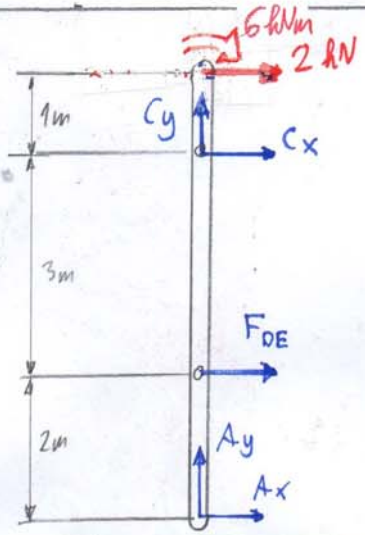
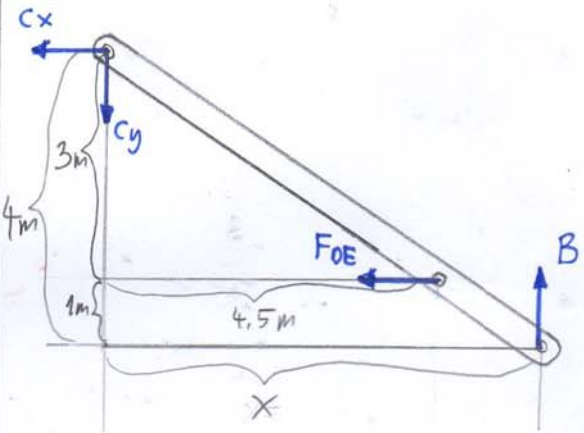
EXERCISE 8.11

given:

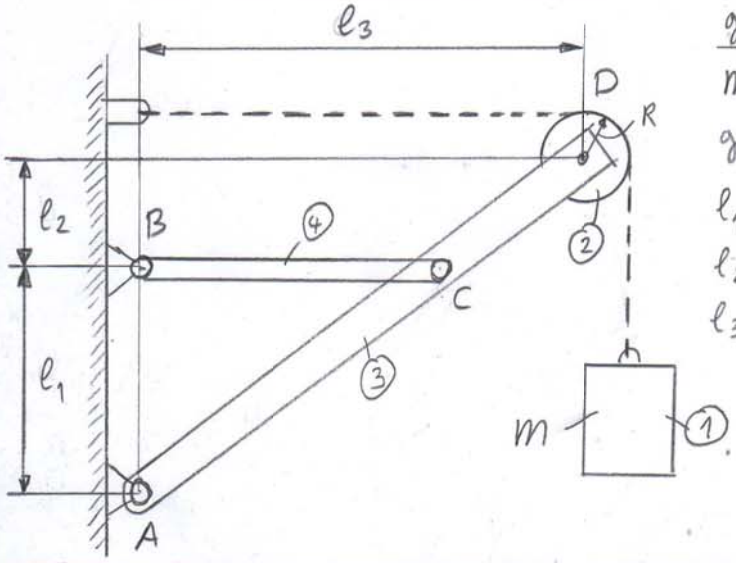


task:

- reaction forces at points A and B
- internal forces at points C, D and E

	body ①	body ②
1 FBD		 $\frac{X}{4}=\frac{4.5}{3} \Rightarrow X=\frac{4\cdot 4.5}{3}=6\,m$
2 EE	$\Sigma F_x=0$: 1) $A_x+F_{DE}+C_x+2=0$ $\Sigma F_y=0$: 2) $A_y+C_y=0$ $\Sigma M_{A_z}=0$: 3) $-2F_{DE}-5C_x-6\cdot 2-6=0$	$\Sigma F_x=0$: 4) $-C_x-F_{DE}=0$ $\Sigma F_y=0$: 5) $B-C_y=0$ $\Sigma M_{B_z}=0$: 6) $1\cdot F_{DE}+4C_x+6C_y=0$
3 EE SOL	4) $\Rightarrow C_x=-F_{DE}$ 4) → 3) $\Rightarrow -2F_{DE}-5(-F_{DE})-6\cdot 2-6=0 \Rightarrow F_{DE}=6\,kN$ 4) $\Rightarrow C_x=-F_{DE}=-6\,kN$ 6) $\Rightarrow C_y=\frac{-F_{DE}-4C_x}{6}=\frac{-6-4\cdot(-6)}{6}=3\,kN$ 1) $\Rightarrow A_x=-2-F_{DE}-C_x=-2-6-(-6)=-2\,kN$ 2) $\Rightarrow A_y=-C_y=-3\,kN$ 5) $\Rightarrow B=C_y=3\,kN$	
4 RES	$\vec{A}=A_x\vec{i}+A_y\vec{j}=(-2\vec{i}-3\vec{j})\,kN$ $\vec{C}_{21}=C_x\vec{i}+C_y\vec{j}=(-6\vec{i}+3\vec{j})\,kN$ $\vec{D}_{31}=F_{DE}\vec{i}=(6\vec{i})\,kN$	$\vec{B}=B\vec{j}=(3\vec{j})\,kN$ $\vec{C}_{12}=-C_x\vec{i}-C_y\vec{j}=(6\vec{i}-3\vec{j})\,kN$ $\vec{E}_{32}=-F_{DE}\vec{i}=(-6\vec{i})\,kN$

EXERCISE 8.12



given:

$m = 50\,kg$

$g = 9.81\,\frac{m}{s^2}$

$\ell_1 = 0.6\,m$

$\ell_2 = 0.3\,m$

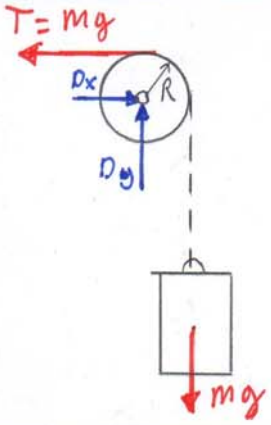
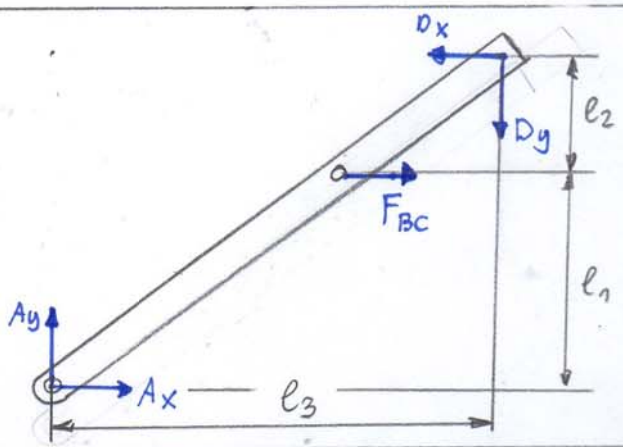
$\ell_3 = 1.2\,m$

task:

- reaction forces at points A and B

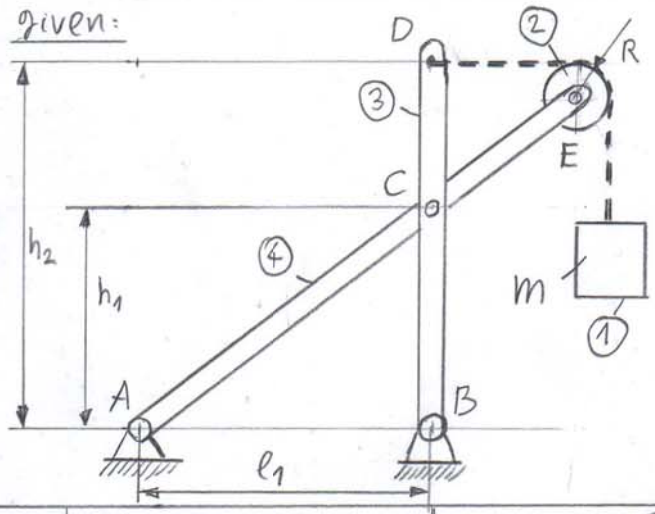
Note: rod ④ is a two-force member ⇓ we don't have to analyze it separately as:

$\vec{B} = \vec{C} = \vec{F}_{BC}$

	body ①+②	body ③
1 FBD		
2 EE	$\Sigma F_x = 0$: 1) $-mg + D_x = 0$ $\Sigma F_y = 0$: 2) $-mg + D_y = 0$ $\Sigma M_{D_z} = 0$: 3) $Rmg - Rmg = 0$ ✓	$\Sigma F_x = 0$: 4) $A_x + F_{BC} - D_x = 0$ $\Sigma F_y = 0$: 5) $A_y - D_y = 0$ $\Sigma M_{A_z} = 0$: 6) $-\ell_1 F_{BC} + (\ell_1+\ell_2) D_x - \ell_3 D_y = 0$
3 EE SOL	1) $\Rightarrow D_x = mg = 50 \cdot 9.81 = 490.5\,N\ (\rightarrow\leftarrow)$ 2) $\Rightarrow D_y = mg = 50 \cdot 9.81 = 490.5\,N\ (\uparrow\downarrow)$ 5) $\Rightarrow A_y = D_y = 490.5\,N\ (\uparrow)$ 6) $\Rightarrow F_{BC} = \frac{(\ell_1+\ell_2)D_x - \ell_3 D_y}{\ell_1} = \frac{(0.6+0.3)\,490.5 - 1.2 \cdot 490.5}{0.6} = -245.25\,N\ (\leftarrow)$ 4) $\Rightarrow A_x = D_x - F_{BC} = 490.5 - (-245.25) = 735.75\,N\ (\rightarrow)$	
4 RES		$\vec{A} = A_x\vec{i} + A_y\vec{j} = (735.75\vec{i} + 490.5\vec{j})\,N$ $\vec{B} = F_{BC}\vec{i} = (-245.25\vec{i})\,N$

EXERCISE 8.13

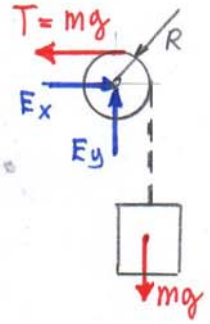
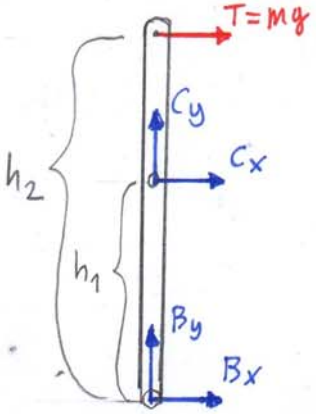
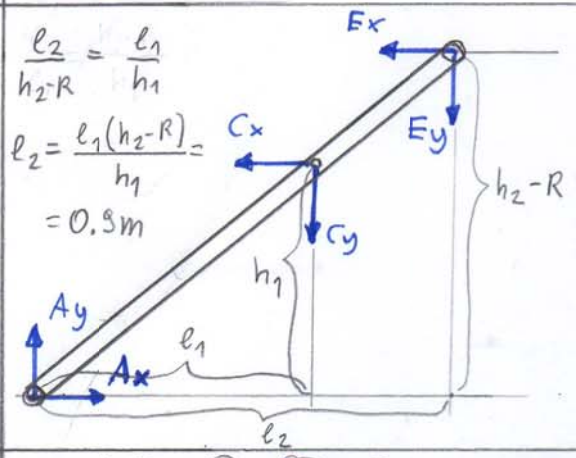
given:



$h_1 = 0.45m$
$h_2 = 0.75m$
$\ell_1 = 0.6m$
$R = 0.075m$
$m = 100kg$
$g = 9.81 \frac{m}{s^2}$

task:

- reaction forces at points A and B
- internal forces at points C and E

	body ①+②	body ③	body ④
1 FBD			
2 EE	ΣF_x: 1) $E_x - mg = 0$ ΣF_y: 2) $E_y - mg = 0$ ΣM_{Ez}: 3) $R\,mg - R\,mg = 0$	ΣF_x: 4) $B_x + C_x + mg = 0$ ΣF_y: 5) $B_y + C_y = 0$ ΣM_{Bz}: 6) $-h_1 C_x - h_2 mg = 0$	ΣF_x: 7) $A_x - C_x - E_x = 0$ ΣF_y: 8) $A_y - C_y - E_y = 0$ ΣM_{Az}: 9) $h_1 C_x - \ell_1 C_y + (h_2-R) E_x - \ell_2 E_y = 0$

3 EE SOL.

1) $\Rightarrow E_x = mg = 100 \cdot 9.81 = 981\,N$

2) $\Rightarrow E_y = mg = 100 \cdot 9.81 = 981\,N$

6) $\Rightarrow C_x = \frac{-h_2 mg}{h_1} = \frac{-0.75 \cdot 100 \cdot 9.81}{0.45} = -1635N$

9) $\Rightarrow C_y = \frac{h_1 C_x + (h_2-R) E_x - \ell_2 E_y}{\ell_1} = \frac{0.45 \cdot (-1635) + (0.75-0.075) \cdot 981 - 0.9 \cdot 981}{0.6} = -1594.1N$

4) $\Rightarrow B_x = -mg - C_x = -100 \cdot 9.81 - (-1635) = 654\,N\ (\rightarrow)$

5) $\Rightarrow B_y = -C_y = -(-1594.1) = 1594.1\,N\ (\uparrow)$

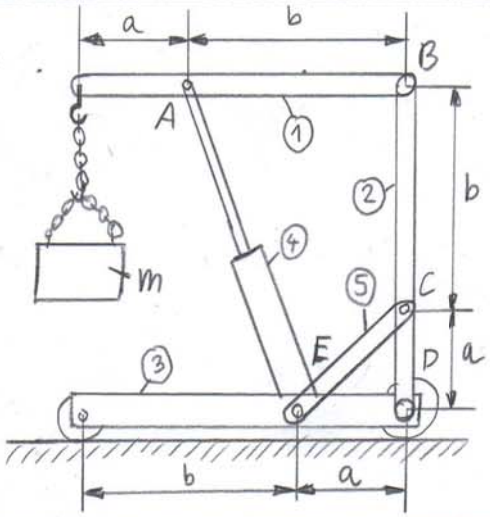
7) $\Rightarrow A_x = C_x + E_x = -1635 + 981 = -654N\ (\leftarrow)$

8) $\Rightarrow A_y = C_y + E_y = -1594.1 + 981 = -613.1\,N\ (\downarrow)$

4 RES			
	$\vec{E}_{42} = E_x \vec{i} + E_y \vec{j} = (981\vec{i} + 981\vec{j})N$	$\vec{B} = B_x \vec{i} + B_y \vec{j} = (654\vec{i} + 1594.1\vec{j})N$ $\vec{C}_{43} = C_x \vec{i} + C_y \vec{j} = (-1635\vec{i} - 1594.1\vec{j})N$	$\vec{A} = A_x \vec{i} + A_y \vec{j} = (-654\vec{i} - 613.1\vec{j})N$ $\vec{C}_{34} = -C_x \vec{i} - C_y \vec{j} = (1635\vec{i} + 1594.1\vec{j})N$ $\vec{E}_{24} = -E_x \vec{i} - E_y \vec{j} = (-981\vec{i} - 981\vec{j})N$

EXERCISE 8.14

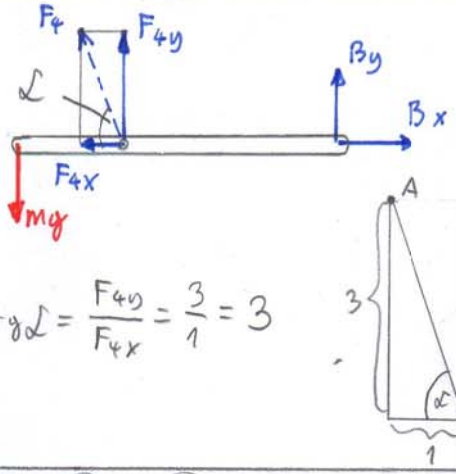
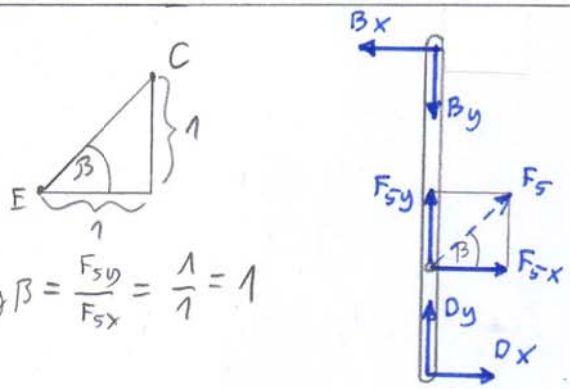
given:



$a = 1\,m$
$b = 2\,m$
$m = 125\,kg$
$g = 9.81\,\frac{m}{s^2}$

task:

- forces arising in member AE and member EC

	body ①	body ②
1 FBD	 (+1) $+_y \alpha = \frac{F_{4y}}{F_{4x}} = \frac{3}{1} = 3$	 (+2) $+_y \beta = \frac{F_{5y}}{F_{5x}} = \frac{1}{1} = 1$
2 EE	$\Sigma F_x = 0$: 1) $B_x - F_{4x} = 0$ $\Sigma F_y = 0$: 2) $B_y + F_{4y} - mg = 0$ $\Sigma M_{A_z} = 0$: 3) $a\,mg + b\,B_y = 0$	$\Sigma F_x = 0$: 4) $D_x + F_{5x} - B_x = 0$ $\Sigma F_y = 0$: 5) $D_y + F_{5y} - B_y = 0$ $\Sigma M_{C_z} = 0$: 6) $a\,D_x + b\,B_x = 0$
3 EE SOL	3) $\Rightarrow B_y = \frac{-amg}{b} = \frac{-1 \cdot 125 \cdot 9.81}{2} = -613.1\,N\ (\downarrow)$ 2) $\Rightarrow F_{4y} = mg - B_y = 125 \cdot 9.81 - (-613.1) = 1839.4\,N\ (\uparrow)$ (+1) $\Rightarrow F_{4x} = \frac{F_{4y}}{3} = \frac{1839.4}{3} = 613.1\,N\ (\leftarrow)$ 1) $\Rightarrow B_x = F_{4x} = 613.1\,N\ (\rightarrow)$ 6) $\Rightarrow D_x = \frac{-bB_x}{a} = \frac{-2 \cdot 613.1}{1} = -1226.2\,N\ (\leftarrow)$ 4) $\Rightarrow F_{5x} = B_x - D_x = 613.1 - (-1226.2) = 1839.4\,N\ (\rightarrow)$ (+2) $\Rightarrow F_{5y} = 1 \cdot F_{5x} = 1839.4\,N\ (\uparrow)$	
4 RES.	$F_{AE} = F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = \sqrt{613.1^2 + 1839.4^2} = 1938.9\,N$ $F_{EC} = F_5 = \sqrt{F_{5x}^2 + F_{5y}^2} = \sqrt{1839.4^2 + 1839.4^2} = 2601.3\,N$	