TRUSSES

- made up of straight rods (= members)
- members are connected with pins
- external loads act only on the pins

$\Rightarrow$ only axial (norma) Force will occur in the members

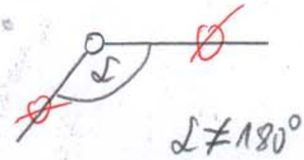
tension: | compression: | zero force member:

$N>0$ | $N<0$ | $N=0$

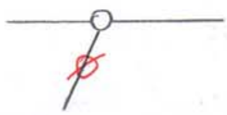
- goal: Find the axial forces in each member / in specific members
- STEP I: Calculate reaction forces
- STEP II: Find zero-force members

ZERO FORCE MEMBER IDENTIFICATION RULES

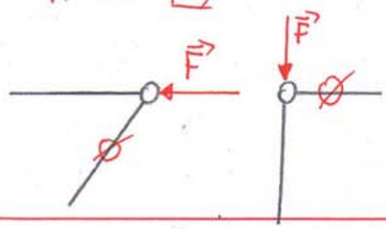
RULE $\boxed{1}$



RULE $\boxed{2}$



RULE $\boxed{3}$



- STEP III: calculate the axial forces in the non-zero force members

method of joints

1) select a joint

   Note: Do not pick a joint with more than ② unknowns

2) draw the FBD of the joint
3) study the equilibrium of the selected joint

   $\Downarrow$

   ② scalar equations:

   $$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \end{aligned}\right\} \Rightarrow$$ ② unknown axial forces can be calculated

4) go to step 1)

method of sections

1) divide the structure into 2 parts

   Note: • Do not cut through more than ③ unknown members

   • Unkown members should not be connected to the same joint

2) keep one part and leave the other

   Hint: keep the easier part!

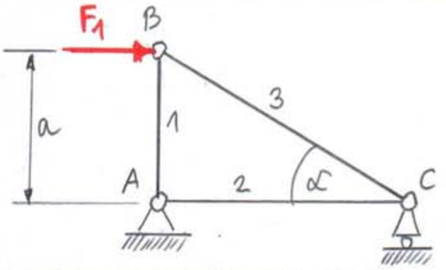
3) draw the FBD of the kept part
4) study the equilibrium of the kept part

   $\Downarrow$

   ③ scalar equations

   $$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_{A_z} &= 0 \end{aligned}\right\}$$ ③ unkown axial forces can be calculated

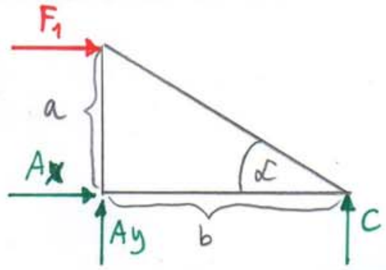
EXERCISE 9.1



given:
$a = 2m$
$\alpha = 30^{\circ}$
$F_1 = 500N$

STEP I : reaction forces

1) FBD:



$\bullet \tan\alpha = \frac{a}{b} \Rightarrow b = \frac{a}{\tan\alpha} = \frac{2}{\tan 30^{\circ}} = 3.464m$

2) EE: $\Sigma F_x = 0: \; 1)\; F_1 + A_x = 0$

$\Sigma F_y = 0: \; 2)\; A_y + C = 0$

$\Sigma M_{A_z} = 0: \; 3)\; b \cdot C - a F_1 = 0$

3) SOLUTION: $1) \Rightarrow A_x = -F_1 = -500N \; (\leftarrow)$

$3) \Rightarrow C = \frac{a F_1}{b} = \frac{2 \cdot 500}{3.464} = 288.7N \; (\uparrow)$

$2) \Rightarrow A_y = -C = -288.7N \; (\downarrow)$

4) RESULTS:

$\vec{A} = A_x \vec{i} + A_y \vec{j} = (-500\vec{i} - 288.7\vec{j})N$

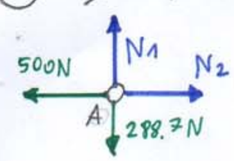
$\vec{C} = C\vec{j} = (288.7\vec{j})N$

STEP II : find zero force members

There are no zero force members

STEP III : axial forces

Ⓐ $\Rightarrow N_1, N_2$

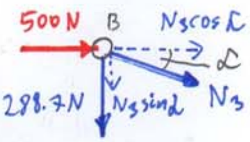


2 equations, 2 unknowns

$\Sigma F_x: \; 1)\; N_2 - 500 = 0 \;\Rightarrow\; \boxed{N_2 = 500N}$

$\Sigma F_y: \; 2)\; N_1 - 288.7 = 0 \;\Rightarrow\; \boxed{N_1 = 288.7N}$

Ⓑ $\Rightarrow N_3$

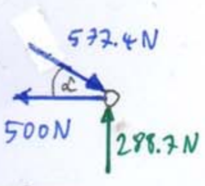


2 equations, 1 unknowns

$\Sigma F_x: \; 1)\; 500 + N_3 \cos\alpha = 0 \;\Rightarrow\; \boxed{N_3 = -\frac{500}{\cos\alpha} = -577.4N}$

$\Sigma F_y: \; 2)\; -288.7 - N_3 \sin\alpha = 0 \;\left(\Rightarrow N_3 = -\frac{288.7}{\sin\alpha} = -577.4N\right)$

Ⓒ



2 equations, 0 unknowns

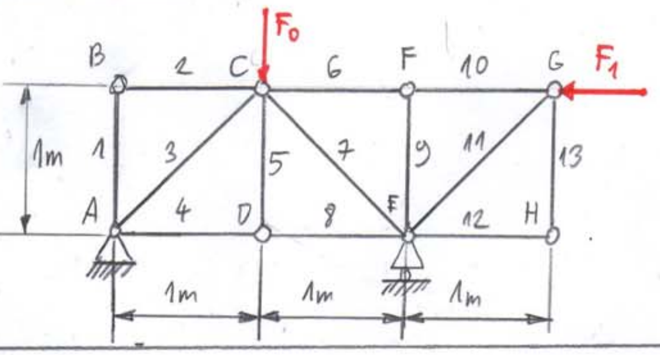
$\Sigma F_x: \; 1)\; -500 + 577.4 \cos\alpha = 0$ ✓

$\Sigma F_y: \; 2)\; 288.7 - 577.4 \sin\alpha = 0$ ✓

• tension ($\leftarrow\square\rightarrow$, $N > 0$) : 1, 2

• compression ($\rightarrow\square\leftarrow$ $N < 0$) : 3

EXERCISE 9.2



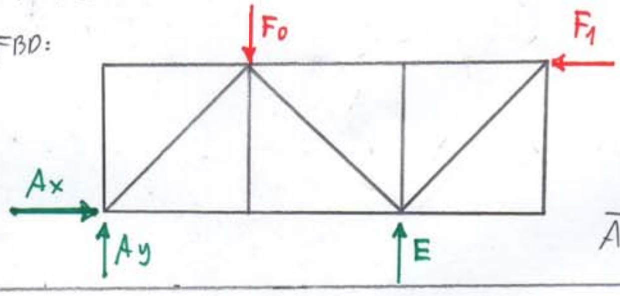
given:

$F_0 = 15\,kN$

$F_1 = 5\,kN$

STEP I : reaction forces

FBD:



ΣF_x: 1) $A_x - F_1 = 0 \Longrightarrow A_x = F_1 = 5\,kN\,(\rightarrow)$

ΣF_y: 2) $A_y + E - F_0 = 0 \Longrightarrow A_y = F_0 - E = 15 - 5 = 10\,kN\,(\uparrow)$

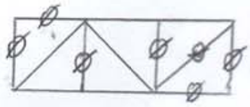
ΣM_{Az}: 3) $2E - 1F_0 + 1F_1 = 0 \Rightarrow E = \frac{F_0 - F_1}{2} = 5\,kN\,(\uparrow)$

$\vec{A} = (5\vec{i} + 10\vec{j})\,kN \quad \vec{E} = (5\vec{j})\,kN$

STEP II : find zero force members

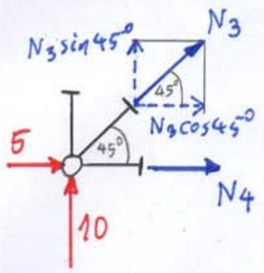
$\boxed{1} \rightarrow$ B $\Rightarrow \boxed{N_1 = 0}$; $\boxed{N_2 = 0}$ $\quad \boxed{2} \rightarrow$ D $\Rightarrow \boxed{N_5 = 0}$ $\quad \boxed{3} \rightarrow$ G $\Rightarrow \boxed{N_{11} = 0}$

$\rightarrow$ H $\Rightarrow \boxed{N_{12} = 0}$; $\boxed{N_{13} = 0}$ $\quad \rightarrow$ F $\Rightarrow \boxed{N_9 = 0}$



STEP III : axial forces

A $\Rightarrow N_3, N_4$



ΣF_x: 1) $5 + N_4 + N_3 \cos 45° = 0$

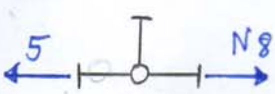
ΣF_y: 2) $10 + N_3 \sin 45° = 0$

2) $\Rightarrow \boxed{N_3} = \frac{-10}{\sin 45°} = \boxed{-14.14\,kN}$

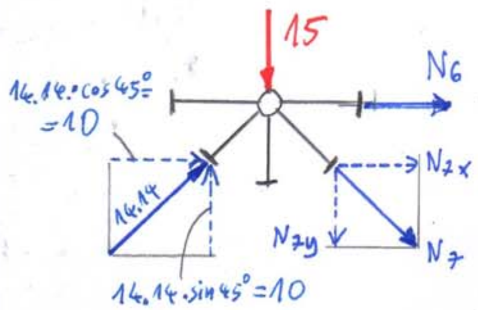
1) $\Rightarrow \boxed{N_4} = -5 - N_3 \cos 45° = -5 - (-14.14)\cos 45° = \boxed{5\,kN}$

D $\Rightarrow N_8$

ΣF_x: $N_8 - 5 = 0 \Rightarrow \boxed{N_8 = 5\,kN}$



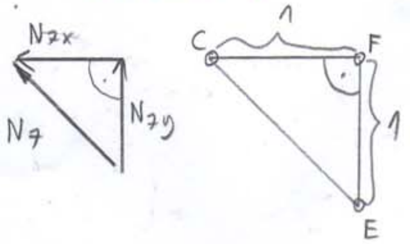
Ⓒ => N_6, N_7



ΣF_x: 1) $10 + N_{7x} + N_6 = 0$

ΣF_y: 2) $10 - N_{7y} - 15 = 0$

(+) $\frac{N_{7y}}{N_{7x}} = \frac{EF}{CF} = \frac{1}{1} = 1$



2) => $N_{7y} = -15 + 10 = -5$ kN (↑)

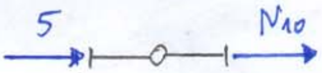
(+) => $N_{7x} = \frac{N_{7y}}{1} = \frac{-5}{1} = -5$ kN (←)

$\boxed{N_7} = -\sqrt{N_{7x}^2 + N_{7y}^2} = -\sqrt{5^2 + 5^2} = \boxed{-7.07 \text{ kN}}$ ($\overset{7}{\rightarrow\square\leftarrow}$)

1) => $\boxed{N_6} = -10 - N_{7x} = -10 - (-5) = \boxed{-5 \text{ kN}}$ ($\overset{6}{\rightarrow\square\leftarrow}$)

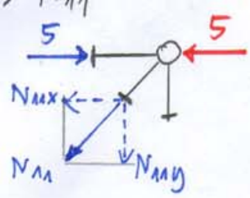
Note: for calculation of N_7, we could of course have used the same procedure as for N_3 at point A (with sin45° and cos45°)

Ⓕ => N_{10}



ΣF_x: $5 + N_{10} = 0$ => $\boxed{N_{10} = -5 \text{ kN}}$ ($\overset{10}{\rightarrow\square\leftarrow}$)

Ⓖ => N_{11}



ΣF_x: 1) $5 - 5 - N_{11x} = 0$ => $N_{11x} = 0$

ΣF_y: 2) $-N_{11y} = 0$ => $N_{11y} = 0$

=> $\boxed{N_{11} = 0}$

Note: This was already stated in STEP II

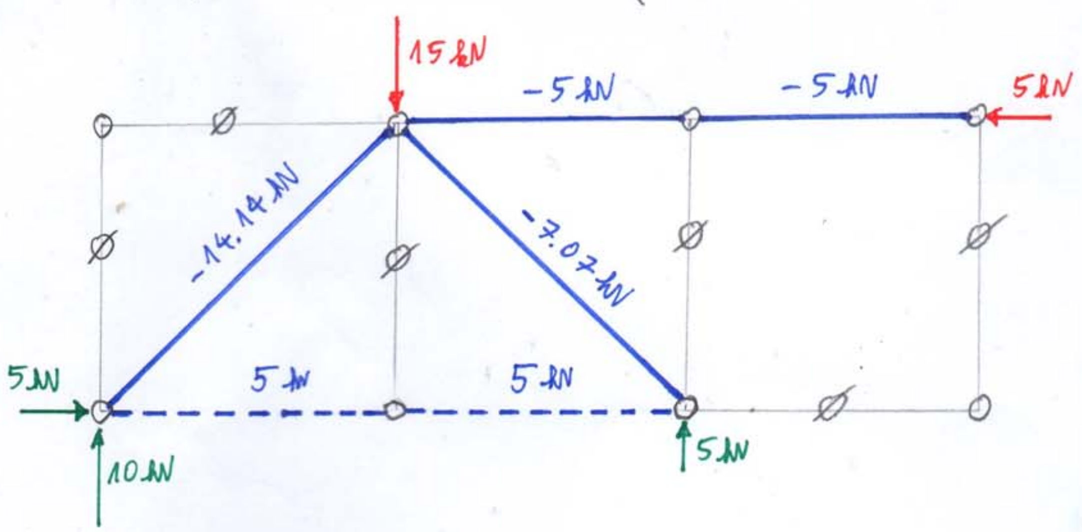
---

results:

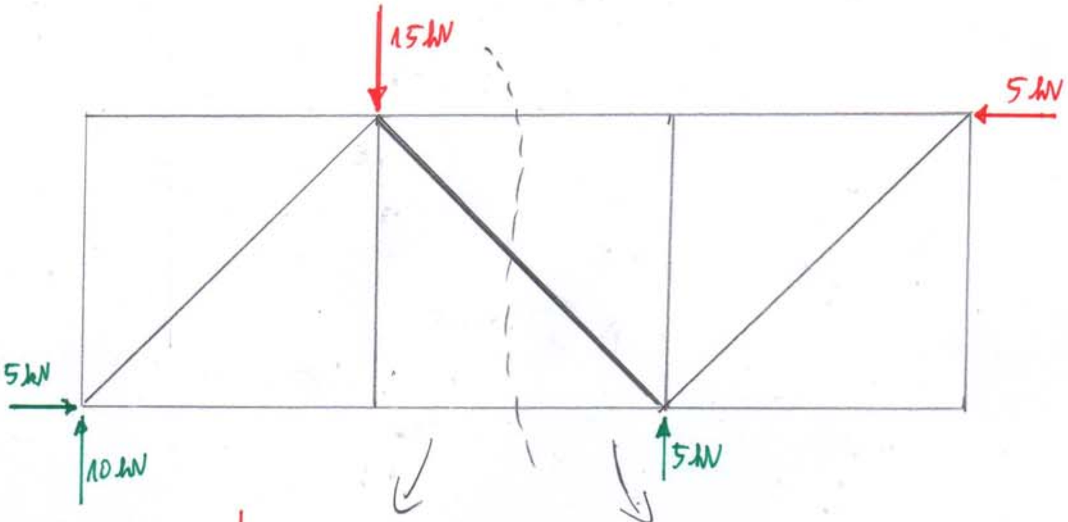
—— : compression ($\rightarrow\square\leftarrow$ $N<0$) : members 3, 5, 6, 10

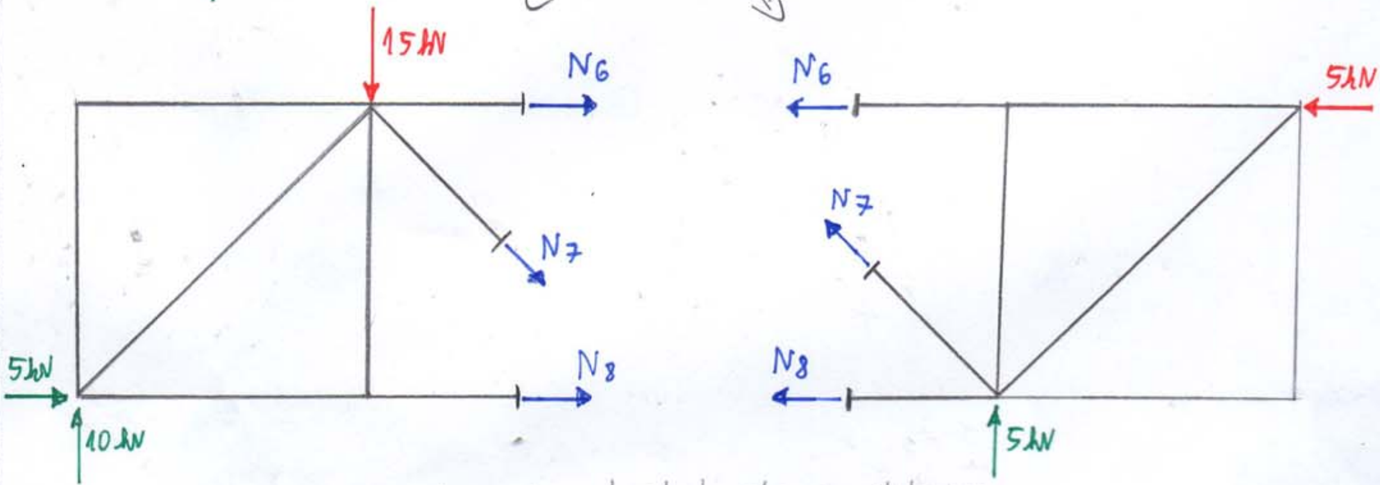
- - - - : tension ($\leftarrow\square\rightarrow$ $N>0$) : members 4, 8

—∅— : zero force member ($N=0$) : members 1, 2, 5, 9, 12, 13

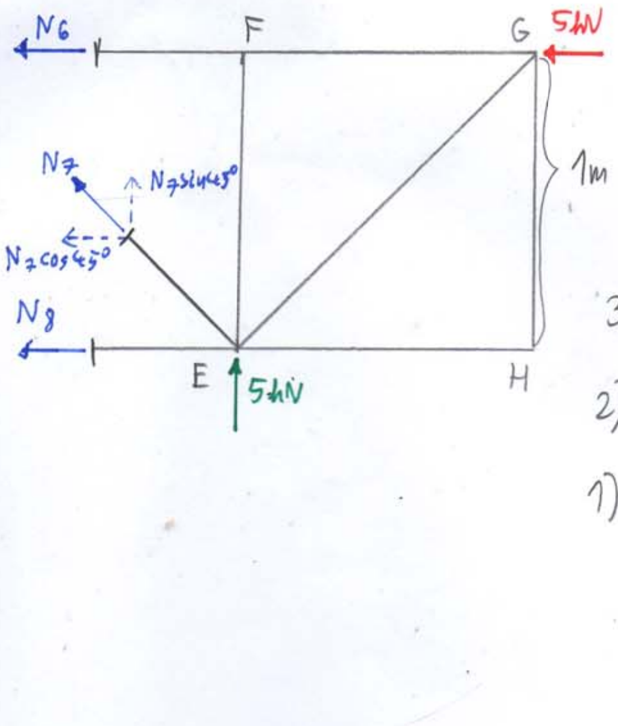


Calculation of N_7 with method of sections:





• Let's keep the right part and study its equilibrium



$$\Sigma F_x: \; 1) \; -N_6 - N_8 - N_7 \cos 45^\circ - 5 = 0$$

$$\Sigma F_y: \; 2) \; N_7 \sin 45^\circ + 5 = 0$$

$$\Sigma M_{E_z}: \; 3) \; 1 \cdot N_6 + 1 \cdot 5 = 0$$

$$3) \Rightarrow \boxed{N_6 = -5\,kN} \quad (\rightarrow \overset{6}{\square} \leftarrow)$$

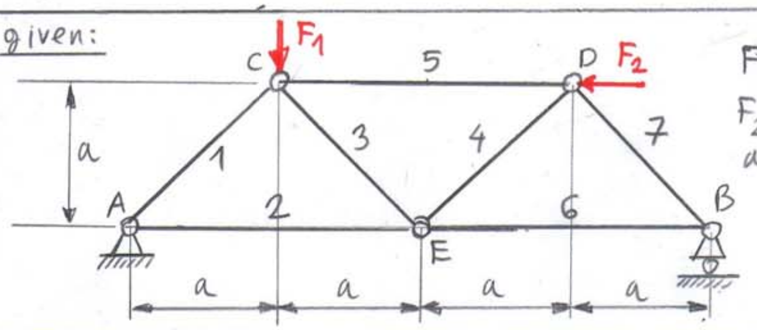
$$2) \Rightarrow \boxed{N_7} = \frac{-5}{\sin 45^\circ} = \boxed{-7.07\,kN} \quad (\searrow \overset{7}{\square} \nwarrow)$$

$$1) \Rightarrow \boxed{N_8} = -N_6 - N_7 \cos 45^\circ - 5 =$$

$$= -(-5) - (-7.07)\cos 45^\circ - 5 = \boxed{5\,kN} \quad (\leftarrow \overset{8}{\square} \rightarrow)$$

EXERCISE 9.3

given:



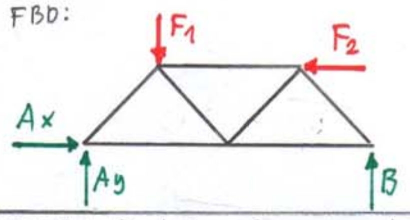
$F_1 = 400N$

$F_2 = 200N$

$a = 1m$

STEP I: reaction forces

FBD:



ΣF_x: 1) $A_x - F_2 = 0 \Rightarrow A_x = F_2 = 200N (\rightarrow)$

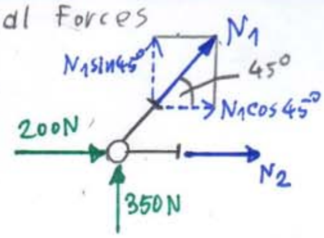
ΣF_y: 2) $A_y + B - F_1 = 0 \Rightarrow A_y = F_1 - B = 400 - 50 = 350N (\uparrow)$

ΣM_{Az}: 3) $4a\,B + a\,F_2 - a\,F_1 = 0 \Rightarrow B = \frac{F_1 - F_2}{4} = \frac{400-200}{4} = 50N (\uparrow)$

STEP II: Find zero force members —

STEP III: axial forces

(A) $\Rightarrow N_1, N_2$



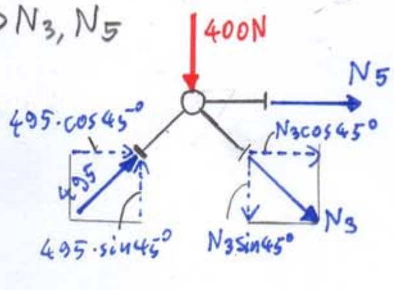
ΣF_x: 1) $200 + N_2 + N_1 \cos 45° = 0$

ΣF_y: 2) $350 + N_1 \sin 45° = 0$

2) $\Rightarrow \boxed{N_1} = \frac{-350}{\sin 45°} = \boxed{-495N}$

1) $\Rightarrow \boxed{N_2} = -200 - N_1 \cos 45° = -200 - (-495)\cos 45° = \boxed{150N}$

(C) $\Rightarrow N_3, N_5$



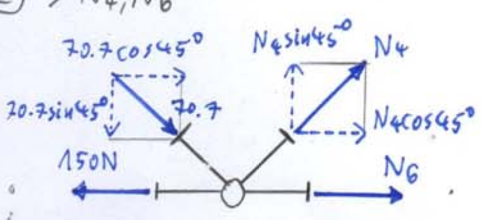
ΣF_x: 1) $495 \cos 45° + N_3 \cos 45° + N_5 = 0$

ΣF_y: 2) $495 \sin 45° - N_3 \sin 45° - 400 = 0$

2) $\Rightarrow \boxed{N_3} = \frac{495 \sin 45° - 400}{\sin 45°} = \boxed{-70.7N}$

1) $\Rightarrow \boxed{N_5} = -495 \cos 45° - N_3 \cos 45° = -495 \cos 45° - (-70.7)\cos 45° =$
$= \boxed{-300N}$

(E) $\Rightarrow N_4, N_6$



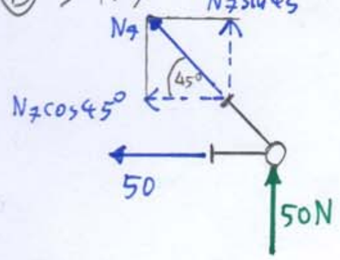
ΣF_x: 1) $-150 + 70.7 \cos 45° + N_4 \cos 45° + N_6 = 0$

ΣF_y: 2) $-70.7 \sin 45° + N_4 \sin 45° = 0$

2) $\Rightarrow \boxed{N_4} = \frac{70.7 \sin 45°}{\sin 45°} = \boxed{70.7N}$

1) $\Rightarrow \boxed{N_6} = 150 - 70.7 \cos 45° - 70.7 \cos 45° = \boxed{50N}$

(B) $\Rightarrow N_7$



ΣF_x: 1) $-50 - N_7 \cos 45° = 0$ ← 2 equations, 1 unknown

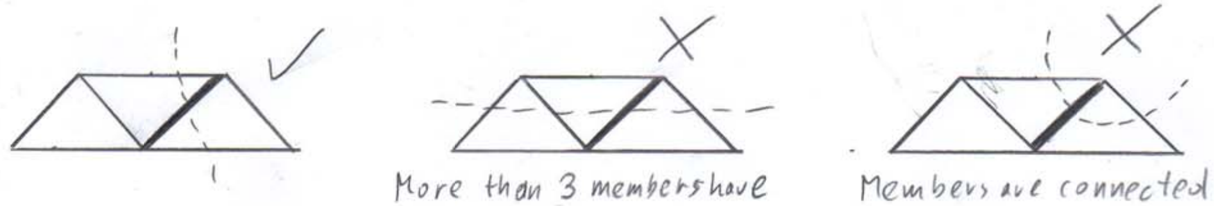
ΣF_y: 2) $50 + N_7 \sin 45° = 0$

1) $\Rightarrow \boxed{N_7} = \frac{-50}{\cos 45°} = \boxed{-70.7N}$

(2) $\Rightarrow N_7 = \frac{-50}{\sin 45°} = -70.7N$)

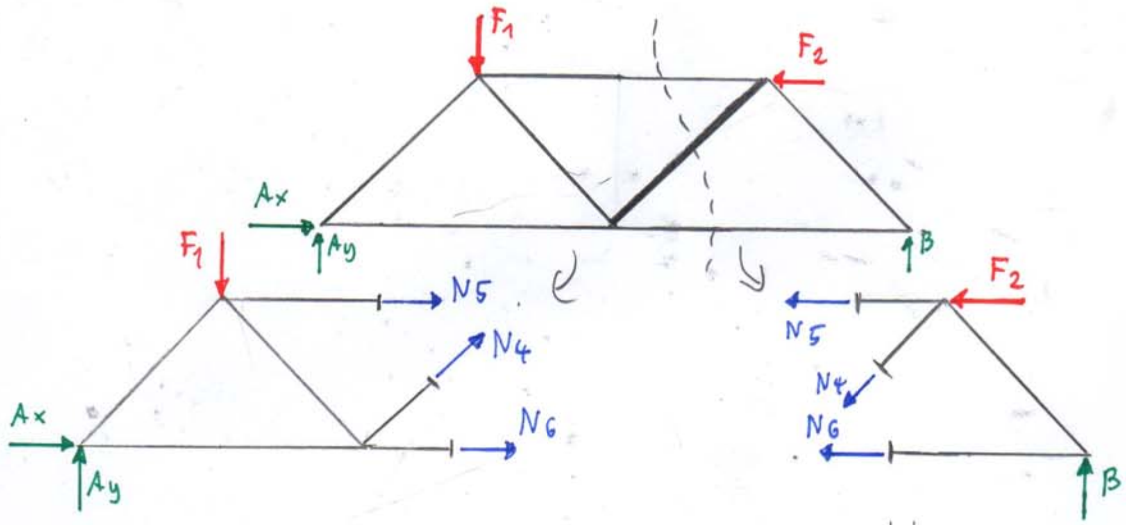
• Calculation of N_4 with method of sections

How to cut the truss?

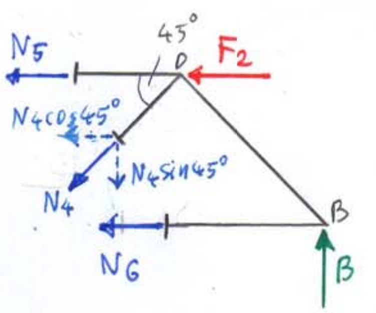


More than 3 members have been cut

Members are connected to 1 common point



Let's keep the right part and study its equilibrium



$$\Sigma F_x:\ 1)\ -N_5 - N_4\cos 45° - N_6 - F_2 = 0$$

$$\Sigma F_y:\ 2)\ -N_4 \sin 45° + B = 0$$

$$M_{0z}:\ 3)\ -a\,N_6 + a\,B = 0$$

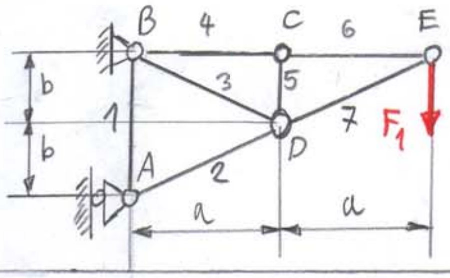
$$3) \Rightarrow \boxed{N_6} = \frac{a\,B}{a} = B = \boxed{50N}$$

$$2) \Rightarrow \boxed{N_4} = \frac{B}{\sin 45°} = \frac{50}{\sin 45°} = \boxed{70.7N}$$

$$1) \Rightarrow \boxed{N_5} = -N_4\cos 45° - N_6 - F_2 =$$

$$= -70.7\cos 45° - 50 - 200 = \boxed{-300N}$$

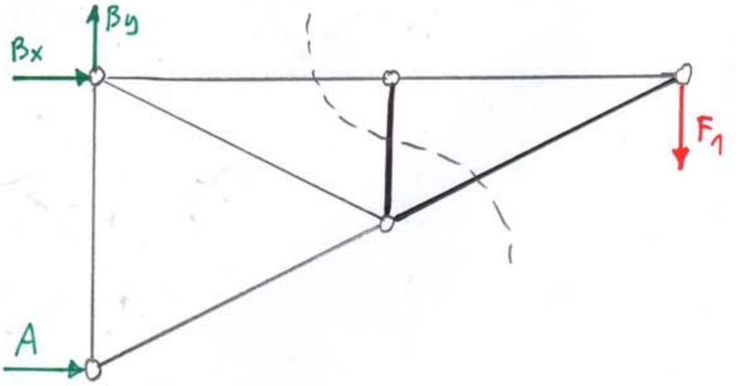
EXERCISE 9.4



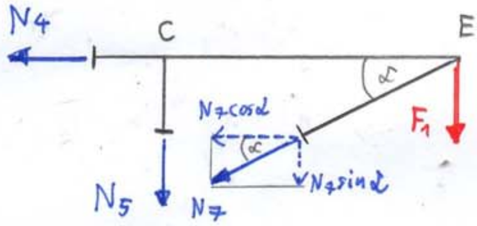
$F_1 = 6\ kN$

$a = 1m$

$b = 0.5m$



• Let's keep the right part and study its equilibrium



$\tan\alpha = \frac{b}{a} = \frac{0.5}{1} = 0.5$

$\alpha = \arctan(0.5) = 26.6°$

ΣF_x: 1) $-N_4 - N_7 \cos\alpha = 0$

ΣF_y: 2) $-N_5 - N_7 \sin\alpha - F_1 = 0$

ΣM_{Ez}: 3) $a \cdot N_5 = 0$

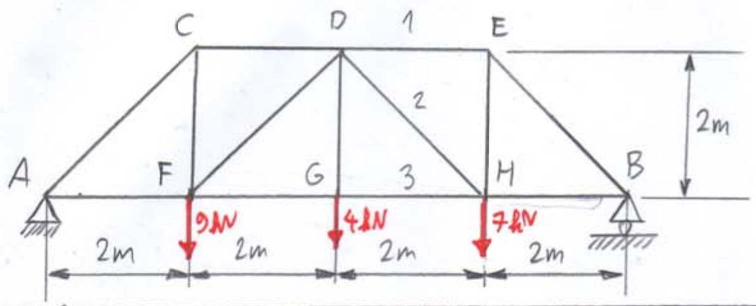
3) ⇒ $\boxed{N_5 = 0}$ (zero force member)

2) ⇒ $\boxed{N_7} = \frac{-N_5 - F_1}{\sin\alpha} = \frac{-0-6}{\sin 26.6°} = \boxed{-13.4\ kN}$ (→▭←)

1) ⇒ $\boxed{N_4} = -N_7 \cos\alpha = -(-13.4)\cos 26.6° =$

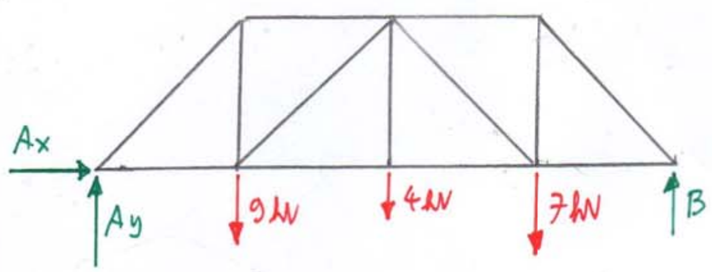
$= \boxed{12\ kN}$ (←▭→)

EXERCISE 9.5



• reaction forces:

FBD:



ΣF_x: 1) $A_x = 0$

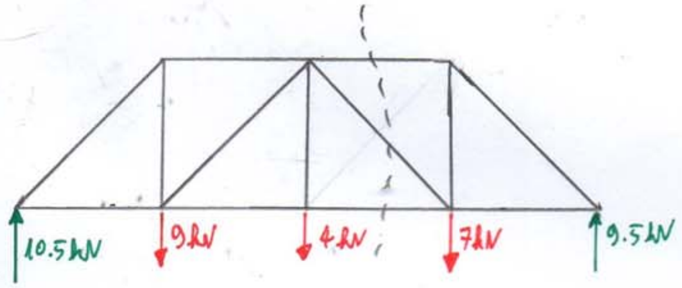
ΣF_y: 2) $A_y + B - 9 - 4 - 7 = 0$

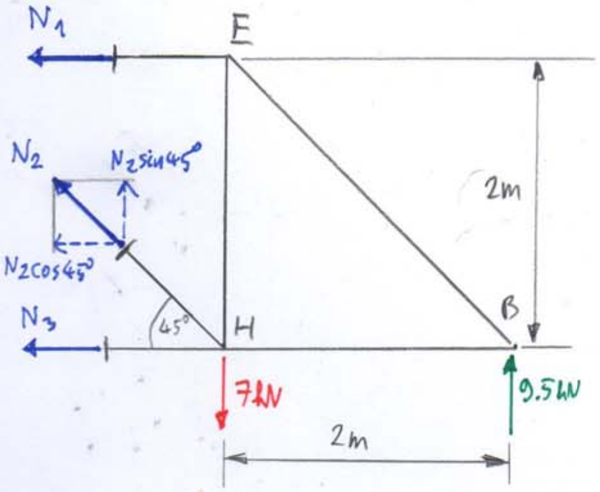
ΣM_{A_z}: 3) $-2 \cdot 9 - 4 \cdot 4 - 6 \cdot 7 + 8 \cdot B = 0$

3) $\Rightarrow B = \dfrac{2 \cdot 9 + 4 \cdot 4 + 6 \cdot 7}{8} = 9.5\,kN\,(\uparrow)$

1) $\Rightarrow A_x = 0$

2) $\Rightarrow A_y = 9 + 4 + 7 - B = 9 + 4 + 7 - 9.5 =$
$= 10.5\,kN\,(\uparrow)$





ΣF_x: 1) $-N_3 - N_2 \cos 45° - N_1 = 0$

ΣF_y: 2) $N_2 \sin 45° - 7 + 9.5 = 0$

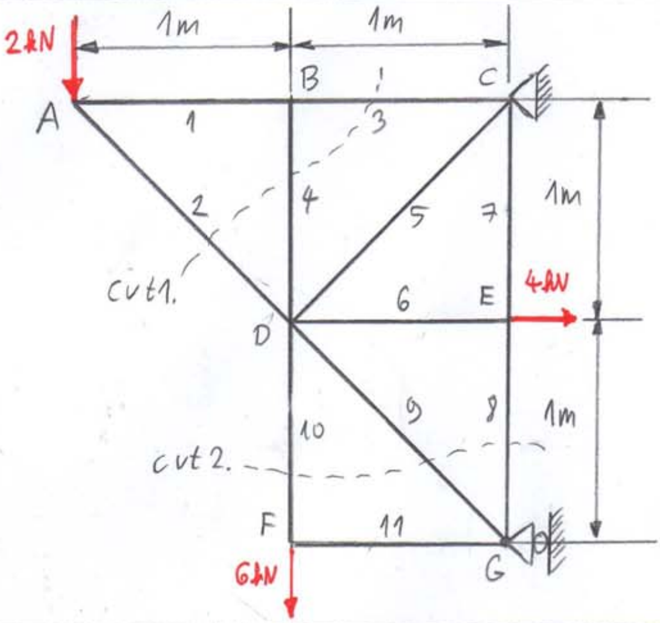
ΣM_{H_z}: 3) $2 \cdot N_1 + 2 \cdot 9.5 = 0$

3) $\Rightarrow \boxed{N_1} = \dfrac{-2 \cdot 9.5}{2} = \boxed{-9.5\,kN}$

2) $\Rightarrow \boxed{N_2} = \dfrac{7 - 9.5}{\sin 45°} = \boxed{-3.54\,kN}$

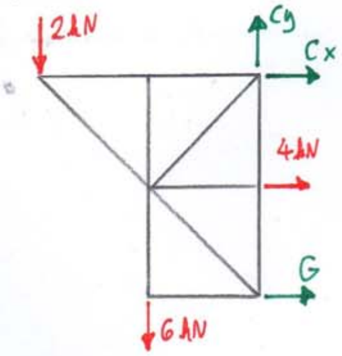
1) $\Rightarrow \boxed{N_3} = -N_2 \cos 45° - N_1 =$
$= -(-3.54)\cos 45° - (-9.5) =$
$= \boxed{12\,kN}$

EXERCISE 9.6



STEP I reaction Forces

FBD:



ΣF_x: 1) $C_x + G + 4 = 0$

ΣF_y: 2) $C_y - 2 - 6 = 0$

ΣM_{Cz}: 3) $2 \cdot 2 + 1 \cdot 4 + 1 \cdot 6 + 2 \cdot G = 0$

3) $\Rightarrow G = \dfrac{-2 \cdot 2 - 1 \cdot 4 - 1 \cdot 6}{2} = -7\,kN\,(\leftarrow)$

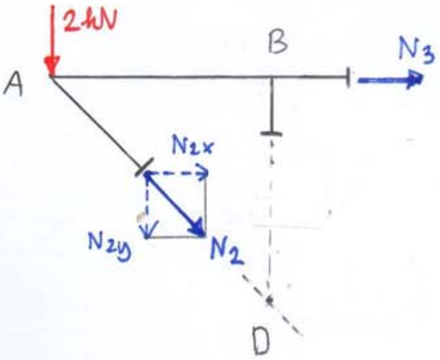
2) $\Rightarrow C_y = 2 + 6 = 8\,kN\,(\uparrow)$

1) $\Rightarrow C_x = -4 - G = -4 - (-7) = 3\,kN\,(\rightarrow)$

STEP II: Find zero force members

$\boxed{2} \to (B) \Rightarrow \boxed{N_4 = 0}$; $\boxed{3} \to (F) \Rightarrow \boxed{N_{11} = 0}$

cut1. $\Rightarrow N_2, N_3$, N_4



ΣF_x: 1) $N_3 + N_{2x} = 0$

ΣF_y: 2) $-N_{2y} - 2 = 0$

ΣM_{Dz}: 3) $1 \cdot 2 - 1 \cdot N_3 = 0$

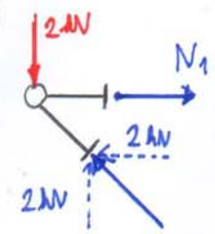
3) $\Rightarrow \boxed{N_3} = \dfrac{2}{1} = \boxed{2\,kN}$

2) $\Rightarrow N_{2y} = -2\,kN\ (\uparrow)$

1) $\Rightarrow N_{2x} = -N_3 = -2\,kN\ (\leftarrow)$

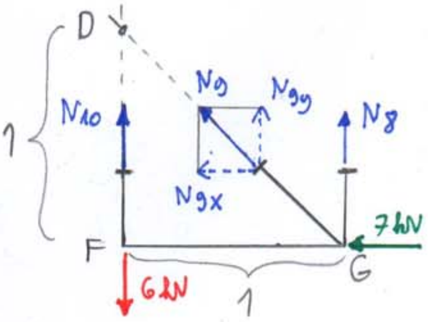
$\boxed{N_2} = -\sqrt{N_{2x}^2 + N_{2y}^2} = \sqrt{2^2 + 2^2} = \boxed{-2.83\,kN}$

(A) $\Rightarrow N_1$



$$\Sigma F_x: -2 + N_1 = 0 \Rightarrow \boxed{N_1 = 2\ kN}$$

(cut 2.) $\Rightarrow N_8, N_9, N_{10}$



$$\Sigma F_x: 1)\ -7 - N_{9x} = 0$$

$$\Sigma F_y: 2)\ N_{10} + N_{9y} + N_8 - 6 = 0$$

$$\Sigma M_{Gz}: 3)\ 1 \cdot 6 - 1 \cdot N_{10} = 0$$

$$\Sigma M_{Dz}: 4)\ 1 \cdot N_8 - 1 \cdot 7 = 0$$

$$4) \Rightarrow \boxed{N_8 = \frac{7}{1} = 7\ kN}$$

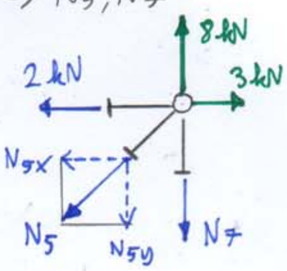
$$3) \Rightarrow \boxed{N_{10} = \frac{6}{1} = 6\ kN}$$

$$1) \Rightarrow N_{9x} = -7\ kN$$

$$2) \Rightarrow N_{9y} = -N_{10} - N_8 + 6 = -7 - 6 + 6 = -7\ kN$$

$$\boxed{N_9 = -\sqrt{N_{9x}^2 + N_{9y}^2} = -\sqrt{7^2 + 7^2} = -9{,}9\ kN}$$

(C) $\Rightarrow N_5, N_7$



$$\Sigma F_x: 1)\ -2 - N_{5x} + 3 = 0$$

$$\Sigma F_y: 2)\ -N_{5y} - N_7 + 8 = 0$$

$$4)\ \frac{N_{5y}}{N_{5x}} = \frac{BD}{BC} = \frac{1}{1} = 1$$

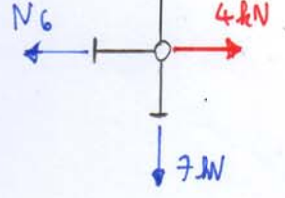
$$1) \Rightarrow N_{5x} = -2 + 3 = 1\ kN$$

$$4) \Rightarrow N_{5y} = 1 \cdot N_{5x} = 1 \cdot 1 = 1\ kN$$

$$\boxed{N_5 = \sqrt{N_{5x}^2 + N_{5y}^2} = \sqrt{1^2 + 1^2} = 1.41\ kN}$$

$$2) \Rightarrow \boxed{N_7 = 8 - N_{5y} = 8 - 1 = 7\ kN}$$

(E) $\Rightarrow N_6$



$$\Sigma F_x: 4 - N_6 = 0 \Rightarrow \boxed{N_6 = 4\ kN}$$