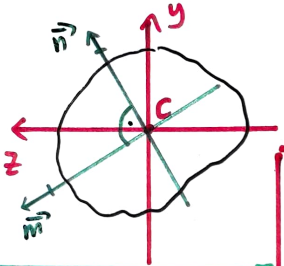
SECOND MOMENTS OF AREA (= AREA MOMENTS OF INERTIA)

• TENSOR OF SECOND MOMENTS



$$\underset{yz}{\underline{\underline{I_c}}} = \begin{bmatrix} I_y & -I_{yz} \\ -I_{zy} & I_z \end{bmatrix}$$

$$I_y = \int_{(A)} z^2 \, dA$$

$$I_z = \int_{(A)} y^2 \, dA$$

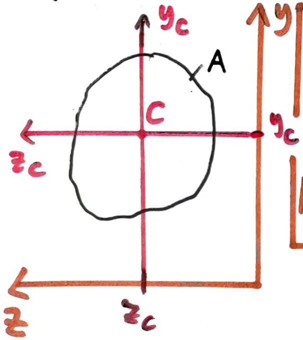
$$I_{yz} = I_{zy} = \int_{(A)} yz \, dA$$

$$I_n = \vec{n} \cdot \underset{yz}{\underline{\underline{I_c}}} \cdot \vec{n}$$

$$I_m = \vec{m} \cdot \underset{yz}{\underline{\underline{I_c}}} \cdot \vec{m}$$

$$-I_{mn} = -I_{nm} = \vec{n} \cdot \underset{yz}{\underline{\underline{I_c}}} \cdot \vec{m} \Rightarrow \underset{nm}{\underline{\underline{I_c}}} = \begin{bmatrix} I_n & -I_{nm} \\ -I_{mn} & I_m \end{bmatrix}$$

• PARALELL AXIS THEOREM (= STEINER'S THEOREM)



$$I_z = I_{z_c} + A y_c^2$$

$$I_y = I_{y_c} + A z_c^2$$

$$I_{yz} = I_{y_c z_c} + A y_c z_c$$

• PRINCIPAL AXES OF THE SECOND MOMENTS

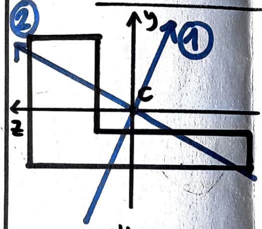
Principal axis ① ⇒ hardest to bend about ⇒ I_1
Principal axis ② ⇒ easiest to bend about ⇒ I_2
} Principal second moments of area

$I_1 > I_2$ $\quad I_{12} = I_{21} = 0$ $\quad \underset{12}{\underline{\underline{I_c}}} = \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix}$

number of principal axis

number of axis of symmetry

number of axis of symmetry	0	1	2	>2
number of principal axis	2	2	2	∞



⇓

eigenvalue problem:

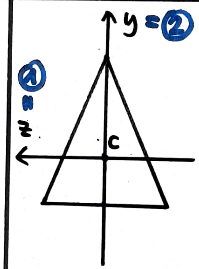
$$\underset{yz}{\underline{\underline{I_c}}} \, \vec{q} = \lambda \vec{q}$$

⇓

$$\left(\underset{yz}{\underline{\underline{I_c}}} - \lambda \underline{\underline{E}}\right) \underset{\neq 0}{\vec{q}} = \vec{0}$$

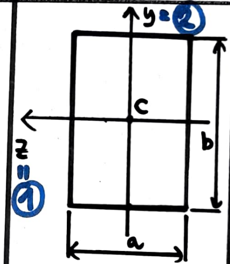
λ → eigenvalues
⇓
principal second moments of area

$\vec{q}$ → eigenvectors
⇓
direction vectors of principal axes



$I_1 = I_z$

$I_2 = I_y$

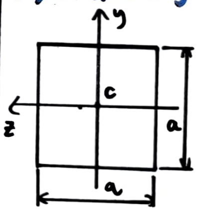


$$I_1 = I_z = \frac{a b^3}{12}$$

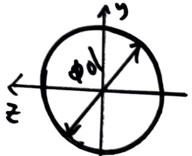
$$I_2 = I_y = \frac{a^3 b}{12}$$

$I_y = I_z = I_1 = I_2 = I_n$

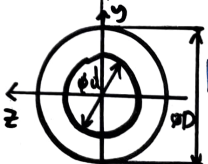
n ⇒ arbitrary axis



$$I_n = \frac{a^4}{12}$$



$$I_n = \frac{d^4 \pi}{64}$$



$$I_n = \frac{(D^4 - d^4)\pi}{64}$$