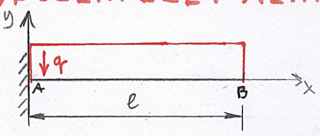
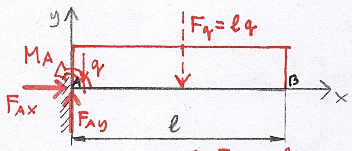
RÚDSZERKEZET ALAKVÁLTOZÁSA



$q = 0{,}04\ \frac{N}{mm}$ $I_z = 6\cdot 10^4\ mm^4$

$\ell = 1000\ mm$ $E = 2\cdot 10^5\ MPa$

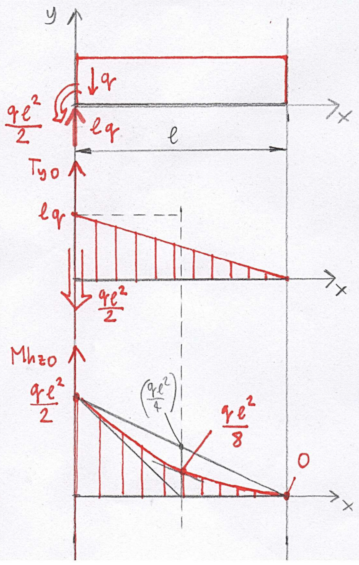
EREDETI TERHELÉS (ER I)



$F_y = 0 = F_{Ay} - \ell q \Rightarrow F_{Ay} = \ell q$

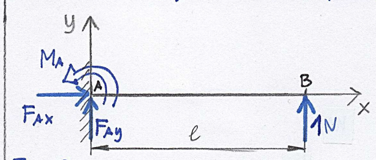
$M_A = 0 = M_A - \frac{\ell}{2}\cdot \ell q \Rightarrow M_A = \frac{q\ell^2}{2}$

$F_x = 0 = F_{Ax}$



B pont függőleges elmozdulása

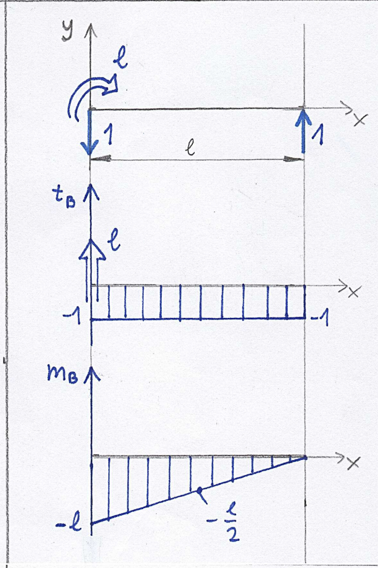
EGYSÉGNYI F_{By} TERHELÉS (ER II)



$F_y = 0 = F_{Ay} + 1 \Rightarrow F_{Ay} = -1$

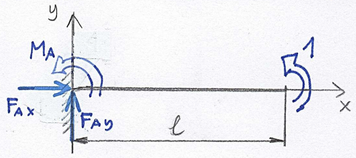
$M_a = 0 = M_A + \ell\cdot 1 \Rightarrow M_A = -\ell$

$F_x = 0 = F_{Ax}$



B k.m. szögelfordulása

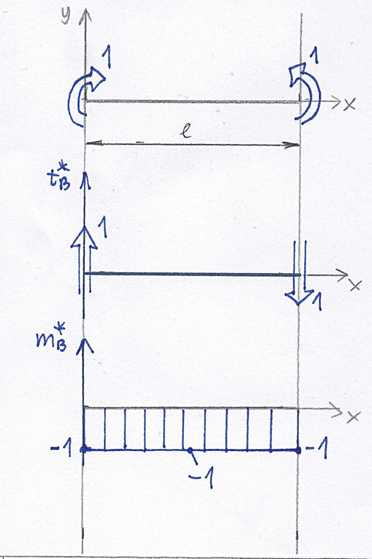
EGYSÉGNYI M_{Bz} TERHELÉS (ER II)



$F_x = 0 = F_{Ax}$

$F_y = 0 = F_{Ay}$

$M_a = 0 = M_A + 1 \Rightarrow M_A = -1$



• Betti szerint:

ER I hajlító igénybevétele: $M_{hz}' = M_{hz0}$

ER II hajlító igénybevétele: $M_{hz}'' = F_{By} m_B$

$$W_{21} = U_{12}$$

$$F_{By}\, v_B = \int_0^\ell \frac{M_{hz}' M_{hz}''}{I_z E}\,dx$$

$$\cancel{F_{By}}\, v_B = \frac{1}{I_z E}\int_0^\ell M_{hz0}\, \cancel{F_{By}}\, m_B\, dx$$

$$v_B = \frac{1}{I_z E}\int_0^\ell M_{hz0}\, m_B\, dx$$

• Betti szerint:

ER I hajlító igénybevétele: $M_{hz}' = M_{hz0}$

ER II hajlító igénybevétele: $M_{hz}'' = M_{Bz} m_B^*$

$$W_{21} = U_{12}$$

$$M_{Bz}\, \varphi_B = \int_0^\ell \frac{M_{hz}' M_{hz}''}{I_z E}\,dx$$

$$\cancel{M_{Bz}}\, \varphi_B = \frac{1}{I_z E}\int_0^\ell M_{hz0}\, \cancel{M_{Bz}}\, m_B^*$$

$$\varphi_B = \frac{1}{I_z E}\int_0^\ell M_{hz0}\, m_B^*$$

• Castigliano szerint:

A hajlító igénybevétel: $M_{hz} = M_{hz0} + F_{By} m_B$

alakváltozási energia:

$$U \approx U_H = \frac{1}{2}\int_0^\ell \frac{M_{hz}^2}{I_z E}dx = \frac{1}{2}\int_0^\ell \frac{(M_{hz0} + F_{By} m_B)^2}{I_z E}dx$$

$$v_B = \left.\frac{\partial U}{\partial F_{By}}\right|_{F_{By}=0}$$

$$v_B = \left.\int_0^\ell \frac{(M_{hz0} + F_{By} m_B)\, m_B}{I_z E}dx\right|_{F_{By}=0}$$

$$v_B = \frac{1}{I_z E}\int_0^\ell M_{hz0}\, m_B\, dx$$

• Castigliano szerint:

A hajlító igénybevétel: $M_{hz} = M_{hz0} + M_{Bz} m_B^*$

alakváltozási energia:

$$U \approx U_H = \frac{1}{2}\int_0^\ell \frac{M_{hz}^2}{I_z E}dx = \frac{1}{2}\int_0^\ell \frac{(M_{hz0} + M_{Bz} m_B^*)^2}{I_z E}dx$$

$$\varphi_B = \left.\frac{\partial U}{\partial M_{Bz}}\right|_{M_{Bz}=0}$$

$$\varphi_B = \left.\int_0^\ell \frac{(M_{hz0} + M_{Bz} m_B^*)\, m_B^*}{I_z E}dx\right|_{F_{By}=0}$$

$$\varphi_B = \frac{1}{I_z E}\int_0^\ell M_{hz0}\, m_B^*\, dx$$

elmozdulás kiszámítása Simpsonnal:

$$v_B = \frac{1}{I_z E}\int_0^\ell M_{hz0}\, m_B =$$

$$= \frac{1}{I_z E}\cdot\frac{\ell}{6}\cdot\left(\frac{q\ell^2}{2}\cdot(-\ell) + 4\cdot\frac{q\ell^2}{8}\cdot\left(-\frac{\ell}{2}\right) + 0\cdot 0\right) =$$

$$= \frac{1}{I_z E}\cdot\frac{\ell}{6}\cdot\left(-\frac{2q\ell^3}{4} - \frac{q\ell^3}{4}\right) =$$

$$= \frac{-3q\ell^4}{24 I_z E} = \frac{-3\cdot 0{,}04\cdot 1000}{24\cdot 6\cdot 10^4\cdot 2\cdot 10^5} =$$

$$= -0{,}416\ mm\ (\downarrow)$$

szögelfordulás kiszámítása Simpsonnal:

$$\varphi_B = \frac{1}{I_z E}\cdot\int_0^\ell M_{hz0}\, m_B^*\, dx =$$

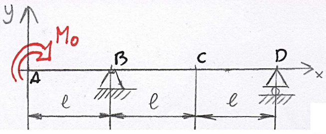
$$= \frac{1}{I_z E}\cdot\frac{\ell}{6}\cdot\left(\frac{q\ell^2}{2}\cdot(-1) + 4\cdot\frac{q\ell^2}{8}\cdot(-1) + 0\cdot(-1)\right)$$

$$= \frac{1}{I_z E}\cdot\frac{\ell}{6}\cdot\left(-\frac{q\ell^2}{2} - \frac{q\ell^2}{2}\right) =$$

$$= -\frac{q\ell^3}{6 I_z E} = -\frac{0{,}04\cdot 1000^3}{6\cdot 6\cdot 10^4\cdot 2\cdot 10^5} =$$

$$= -5{,}5\cdot 10^{-4} = -0{,}0318^\circ\ (\curvearrowright)$$

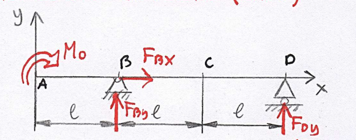
RÚDSZERKEZET ALAKVÁLTOZÁSA



adott: $M_0 = 4{,}8$ kNm, $I_z = 171\cdot10^4\ \text{mm}^4$, $l = 1{,}2$ m, $E = 2{,}1\cdot10^5$ MPa

Feladat: v_C, φ_C

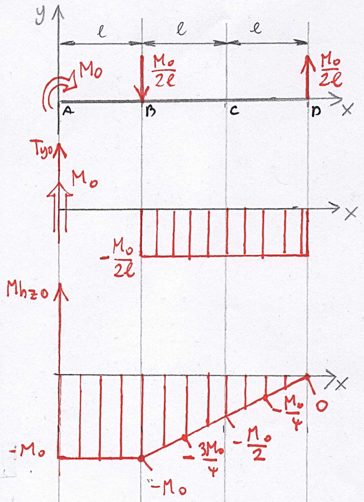
EREDETI TERHELÉS (ER I)



$M_b = 0 = -M_0 + 2l F_{Dy} \Rightarrow F_{Dy} = \frac{M_0}{2l}$

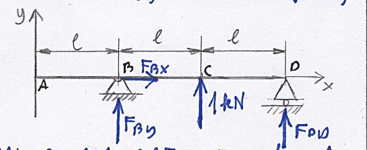
$M_d = 0 = -M_0 - 2l F_{By} \Rightarrow F_{By} = -\frac{M_0}{2l}$

$F_x = 0 = F_{Bx}$



C pont függőleges elmozdulása

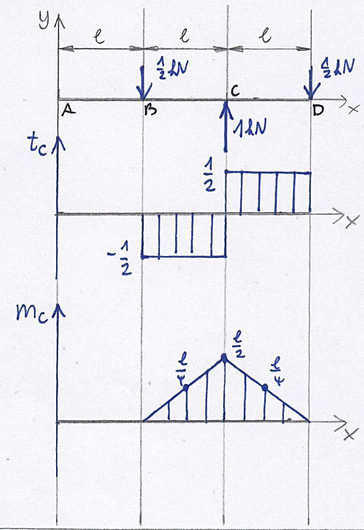
EGYSÉGNYI F_{Cy} TERHELÉS (ER II)



$M_b = 0 = l\cdot1 + 2l F_{Dy} \Rightarrow F_{Dy} = -\frac{l}{2l} = -\frac{1}{2}$

$M_d = 0 = -2l F_{By} - l\cdot1 \Rightarrow F_{By} = -\frac{l}{2l} = -\frac{1}{2}$

$F_x = 0 = F_{Bx}$



- Betti szerint:

ER I hajlító igénybevétele: $M_{hz}' = M_{hz0}$

ER II hajlító igénybevétele: $M_{hz}'' = F_{cy} m_c$

$$W_{21} = U_{12}$$

$$F_{cy} v_c = \int_0^{3l} \frac{M_{hz}' M_{hz}''}{I_z E} dx$$

$$\cancel{F_{cy}}\, v_c = \frac{1}{I_z E}\int_0^{3l} M_{hz0} \cancel{F_{cy}}\, m_c\, dx$$

$$v_c = \frac{1}{I_z E}\int_0^{3l} M_{hz0}\, m_c\, dx$$

- Castigliano szerint:

A hajlító igénybevétel: $M_{hz} = M_{hz0} + F_{cy} m_c$

Az alakváltozási energia:

$$U \approx U_H = \frac{1}{2}\int_0^{3l} \frac{M_{hz}^2}{I_z E} dx = \frac{1}{2}\int_0^{3l} \frac{(M_{hz0} + F_{cy} m_c)^2}{I_z E} dx$$

$$v_c = \left.\frac{\partial U}{\partial F_{cy}}\right|_{F_{cy}=0}$$

$$v_c = \left.\int_0^{3l} \frac{(M_{hz0} + F_{cy} m_c)\cdot m_c}{I_z E} dx\right|_{F_{cy}=0}$$

$$v_c = \frac{1}{I_z E}\int_0^{3l} M_{hz0}\, m_c\, dx$$

elmozdulás kiszámítása Simpsonnal:

$$v_c = \frac{1}{I_z E}\int_0^{3l} M_{hz0}\, m_c\, dx =$$

$$= \frac{1}{I_z E}\left[\int_0^{l} M_{hz0} m_c dx + \int_l^{2l} M_{hz0} m_c dx + \int_{2l}^{3l} M_{hz0} m_c dx\right] =$$

$$= \frac{1}{I_z E}\left[\frac{l}{6}\left(-M_0\cdot0 + 4\cdot\left(-\frac{3M_0}{4}\right)\cdot\frac{l}{4} + \left(-\frac{M_0}{2}\right)\cdot\frac{l}{2}\right) + \frac{l}{6}\left(-\frac{M_0}{2}\cdot\frac{l}{2} + 4\cdot\left(-\frac{M_0}{4}\right)\cdot\frac{l}{4} + 0\cdot0\right)\right] =$$

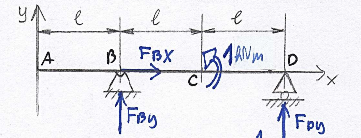
$$= \frac{1}{I_z E}\cdot\frac{l}{6}\cdot\left(-\frac{3M_0 l}{4} - \frac{M_0 l}{4} - \frac{M_0 l}{4} - \frac{M_0 l}{4}\right) =$$

$$= \frac{1}{I_z E}\cdot\frac{l}{6}\cdot\frac{-6M_0 l}{4} = -\frac{M_0 l^2}{4 I_z E} = -\frac{4{,}8\cdot10^6\cdot1200^2}{4\cdot171\cdot10^4\cdot2{,}1\cdot10^5} =$$

$$= -4{,}812\ \text{mm} \downarrow$$

C k.m. szögelfordulása

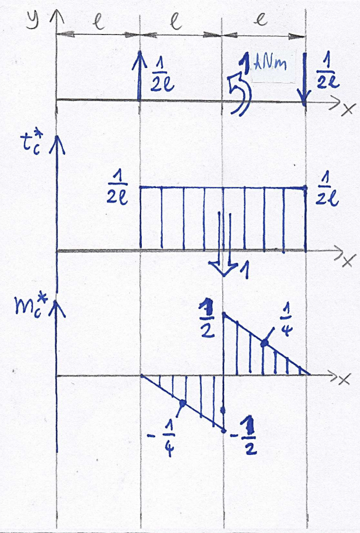
EGYSÉGNYI M_{cz} TERHELÉS (ER II)



$M_b = 0 = 1 + 2l F_{Dy} \Rightarrow F_{Dy} = -\frac{1}{2l}$

$M_d = 0 = 1 - 2l F_{By} \Rightarrow F_{By} = \frac{1}{2l}$

$F_x = 0 = F_{Bx}$



- Betti szerint:

ER I hajlító igénybevétele: $M_{hz}' = M_{hz0}$

ER II hajlító igénybevétele: $M_{hz}'' = M_{cz} m_c^*$

$$W_{21} = U_{12}$$

$$M_{cz}\varphi_c = \int_0^{3l} \frac{M_{hz}' M_{hz}''}{I_z E} dx$$

$$\cancel{M_{cz}}\,\varphi_c = \frac{1}{I_z E}\int_0^{3l} M_{hz0} \cancel{M_{cz}}\, m_c^*\, dx$$

$$\varphi_c = \frac{1}{I_z E}\int_0^{3l} M_{hz0}\, m_c^*\, dx$$

- Castigliano szerint:

A hajlító igénybevétel: $M_{hz} = M_{hz0} + M_{cz} m_c^*$

Az alakváltozási energia:

$$U \approx U_H = \frac{1}{2}\int_0^{3l} \frac{M_{hz}^2}{I_z E} dx = \frac{1}{2}\int_0^{3l} \frac{(M_{hz0} + M_{cz} m_c)^2}{I_z E} dx$$

$$\varphi_c = \left.\frac{\partial U}{\partial M_{cz}}\right|_{M_{cz}=0}$$

$$\varphi_c = \left.\int_0^{3l} \frac{(M_{hz0} + M_{cz} m_c^*)\, m_c^*}{I_z E} dx\right|_{M_{cz}=0}$$

$$\varphi_c = \frac{1}{I_z E}\int_0^{3l} M_{hz0}\, m_c^*\, dx$$

szögelfordulás kiszámítása Simpsonnal:

$$\varphi_c = \frac{1}{I_z E}\cdot\int_0^{3l} M_{hz0}\, m_c^*\, dx =$$

$$= \frac{1}{I_z E}\left[\int_0^{l} M_{hz0} m_c^* dx + \int_l^{2l} M_{hz0} m_c^* dx + \int_{2l}^{3l} M_{hz0} m_c^* dx\right] =$$

$$= \frac{1}{I_z E}\left[\frac{l}{6}\left(-M_0\cdot0 + 4\cdot\left(-\frac{3M_0}{4}\right)\cdot\left(-\frac{1}{4}\right) + \left(-\frac{M_0}{2}\right)\cdot\left(-\frac{1}{2}\right)\right) + \frac{l}{6}\left(-\frac{M_0}{2}\cdot\frac{1}{2} + 4\cdot\left(-\frac{M_0}{4}\right)\cdot\frac{1}{4} + 0\cdot0\right)\right] =$$

$$= \frac{1}{I_z E}\cdot\frac{l}{6}\cdot\left(\frac{3M_0}{4} + \frac{M_0}{4} - \frac{M_0}{4} - \frac{M_0}{4}\right) = \frac{1}{I_z E}\cdot\frac{l}{6}\cdot\frac{M_0}{2} =$$

$$= \frac{M_0 l}{12 I_z E} = \frac{4{,}8\cdot10^6\cdot1200}{12\cdot171\cdot10^4\cdot2{,}1\cdot10^5} = 0{,}001337$$

$$\Downarrow$$

$0{,}0766°$ ↺