

FÜGGVÉNYEK

1. Határozza meg a következő összetett függvényeket!

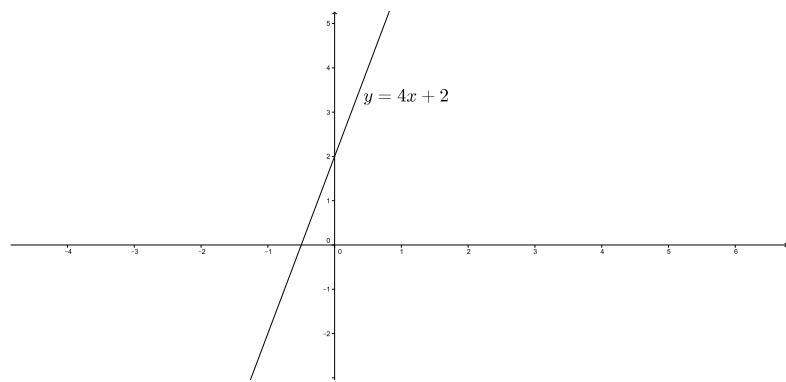
$$[g \circ f = g(f(x)); f \circ g = f(g(x))]$$

- (a) **B** $f(x) = \cos x + x^2; g(x) = \sqrt{x}; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \cos(\sqrt{x}) + (\sqrt{x})^2 = \cos(\sqrt{x}) + x; g(f(x)) = \sqrt{\cos x + x^2}]$
- (b) **B** $f(x) = \sin x; g(x) = x^2; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \sin(x^2) = \sin x^2; g(f(x)) = (\sin x)^2 = \sin^2 x]$
- (c) **B** $f(x) = \frac{x}{1+x^2}; g(x) = \operatorname{tg} x; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \frac{\operatorname{tg} x}{1+(\operatorname{tg} x)^2} = \frac{\operatorname{tg} x}{1+\operatorname{tg}^2 x}; g(f(x)) = \operatorname{tg}\left(\frac{x}{1+x^2}\right)]$
- (d) **B** $f(x) = \ln x + 4x^5; g(x) = e^x; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \ln(e^x) + 4(e^x)^5 = x + 4e^{5x}; g(f(x)) = e^{\ln x + 4x^5}]$
- (e) **B** $f(x) = x^2 - 3x; g(x) = \sqrt{5-2x}; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = (\sqrt{5-2x})^2 - 3\sqrt{5-2x} = 5-2x - 3\sqrt{5-2x};$
 $g(f(x)) = \sqrt{5-2(x^2-3x)} = \sqrt{5-2x^2+6x}]$
- (f) **B** $f(x) = 1-x+x^2; g(x) = e^x; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = 1-e^x+(e^x)^2 = 1-e^x+e^{2x}; g(f(x)) = e^{1-x+x^2}]$
- (g) **B** $f(x) = \cos(7-x); g(x) = x^4 - 3x + 2; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \cos(7-(x^4-3x+2)) = \cos(-x^4+3x+5);$
 $g(f(x)) = (\cos(7-x))^4 - 3\cos(7-x) + 2 = \cos^4(7-x) - 3\cos(7-x) + 2]$
- (i) **B** $f(x) = \sqrt[3]{2-3x}; g(x) = 4x-x^3; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \sqrt[3]{2-3(4x-x^3)} = \sqrt[3]{2-12x+3x^3};$
 $g(f(x)) = 4\sqrt[3]{2-3x} - (\sqrt[3]{2-3x})^3 = 4\sqrt[3]{2-3x} - 2+3x]$
- (h) **B** $f(x) = \sqrt[5]{4-x}; g(x) = x^5 - 3x; f(g(x)) = ?; g(f(x)) = ?$
 $[f(g(x)) = \sqrt[5]{4-(x^5-3x)} = \sqrt[5]{4-x^5+3x};$
 $g(f(x)) = (\sqrt[5]{4-x})^5 - 3\sqrt[5]{4-x} = 4-x - 3\sqrt[5]{4-x}]$

2. Határozza meg a hiányzó függvényeket!

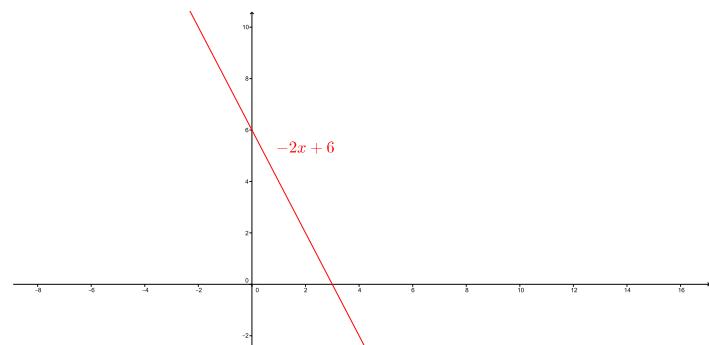
- (a) $f(x) = \sin x; g(x) = ?; f(g(x)) = \sin(x+4)$ $[g(x) = x+4]$
- (b) **B** $f(x) = ?; g(x) = \cos x; f(g(x)) = \cos^4 x + 3 \cos x$ $[f(x) = x^4 + 3x]$
- (c) **B** $f(x) = ?; g(x) = x^2 - e^x; f(g(x)) = x - e^{\sqrt{x}}$ $[f(x) = \sqrt{x}]$
- (d) **B** $f(x) = x^2; g(x) = ?; g(f(x)) = \frac{x}{1+x^4}$ $[g(x) = \frac{\sqrt{x}}{1+x^2}]$

3. Ábrázolja az alábbi $f : R \rightarrow R$ függvényeket, majd olvassa le a függvények értelmezési tartományát és értékkészletét!

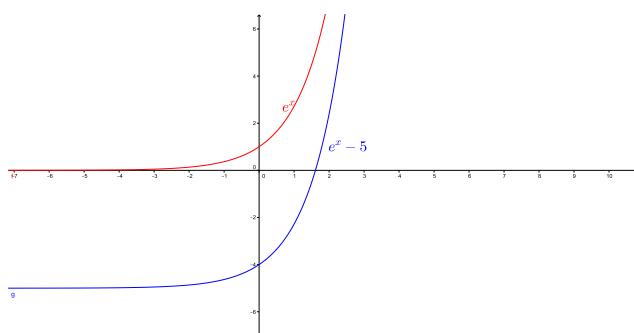


(a) $f(x) = 4x + 2$
 $[D(f) = R; R(f) = R]$

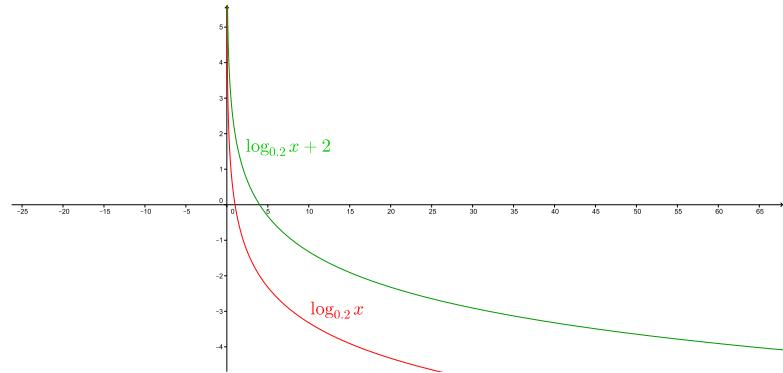
(b) $f(x) = -2x + 6$
 $[D(f) = R; R(f) = R]$



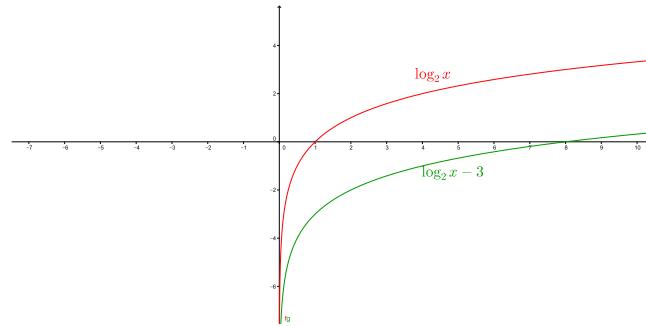
(c) $f(x) = e^x - 5$
 $[D(f) = R; R(f) =] - 5; \infty[$



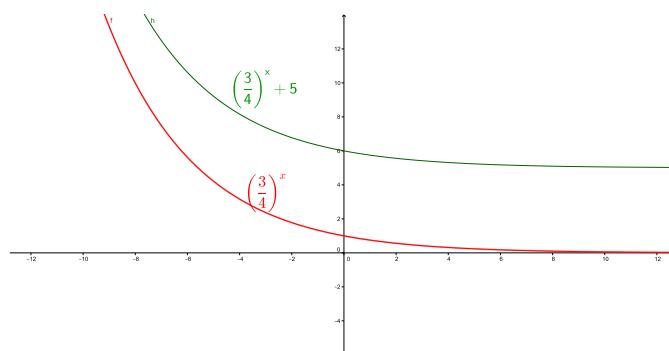
(d) $f(x) = \log_{0,2} x + 2$
 $[D(f) =]0; \infty[; R(f) = R]$



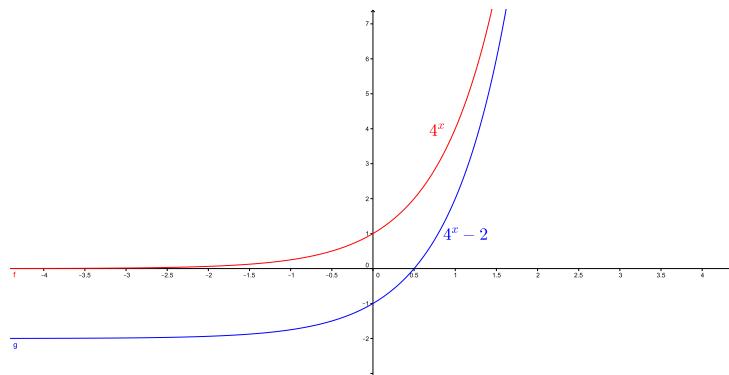
(e) $f(x) = \log_2 x - 3$
 $[D(f) =]0; \infty[; R(f) = R]$



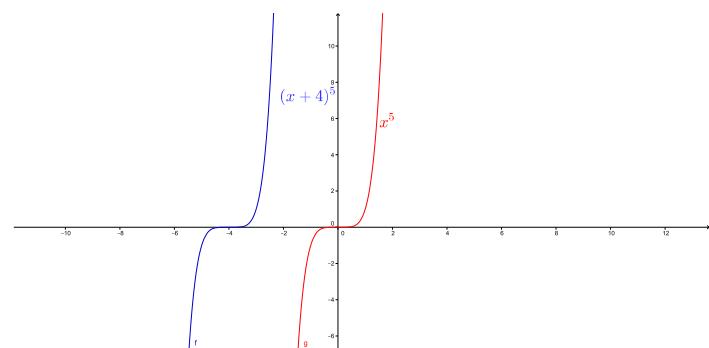
(f) $f(x) = \left(\frac{3}{4}\right)^x + 5$
 $[D(f) = R; R(f) =]5; \infty[$



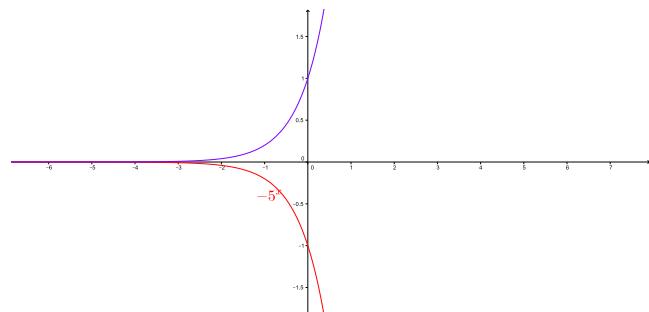
(g) $f(x) = 4^x - 2$
 $[D(f) = R; R(f) =] - 2; \infty[$



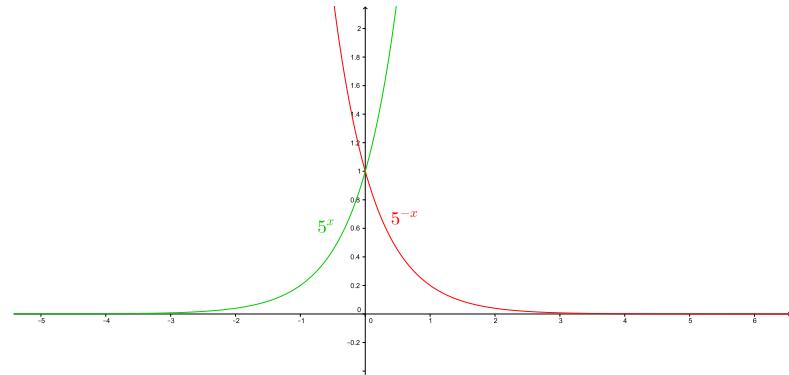
(h) $f(x) = (x + 4)^5$
 $[D(f) = R; R(f) = R]$



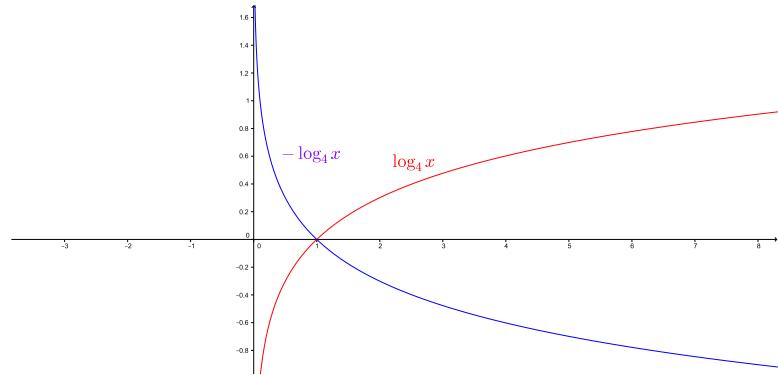
(i) $f(x) = -5^x$
 $[D(f) = R; R(f) =] -\infty; 0[]$



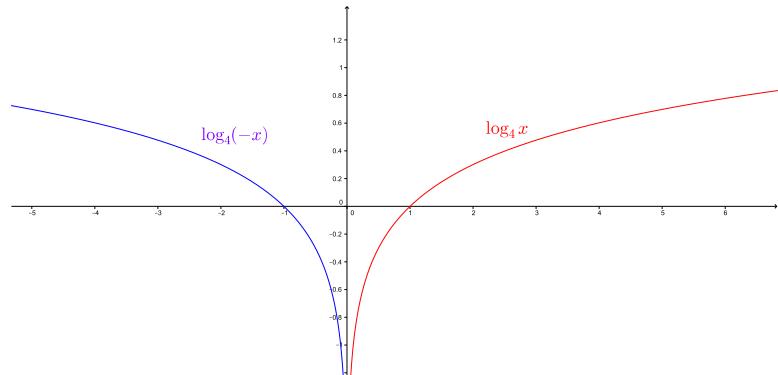
(j) $f(x) = 5^{-x}$
 $[D(f) = R; R(f) =]0; \infty[]$



(k) $f(x) = -\log_4 x$
 $[D(f) =]0; \infty[; R(f) = R]$

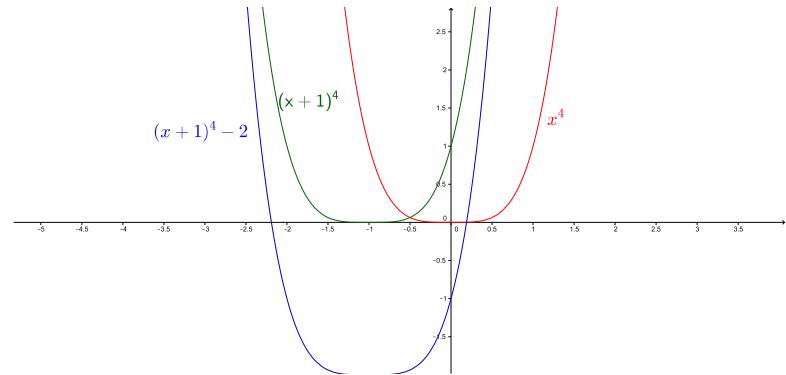


(l) $f(x) = \log_4(-x)$
 $[D(f) =]-\infty; 0[; R(f) = R]$

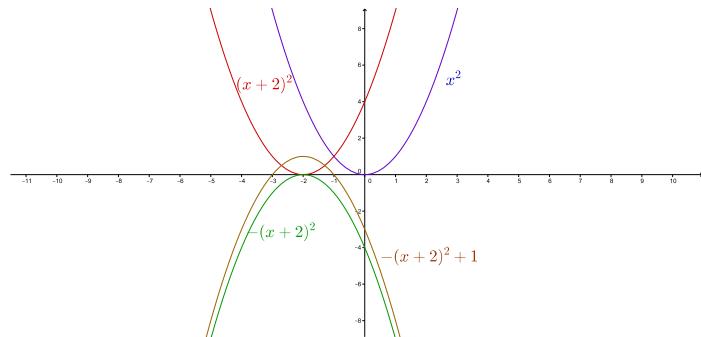


4. Ábrázolja az alábbi $f : R \rightarrow R$ függvényeket, majd olvassa le a függvények értelmezési tartományát és értékkészletét!

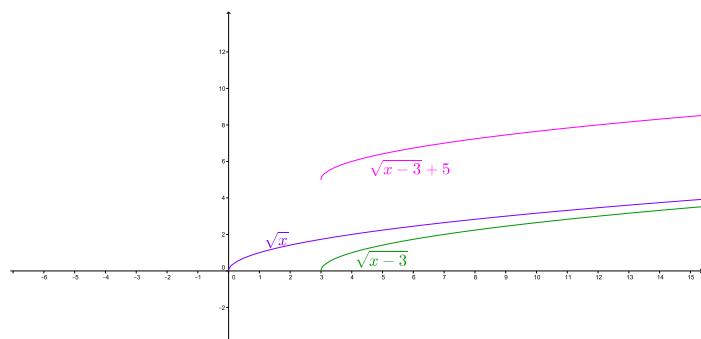
(a) **B** $f(x) = (x+1)^4 - 2$
 $[D(f) = R; R(f) = [-2; \infty[$



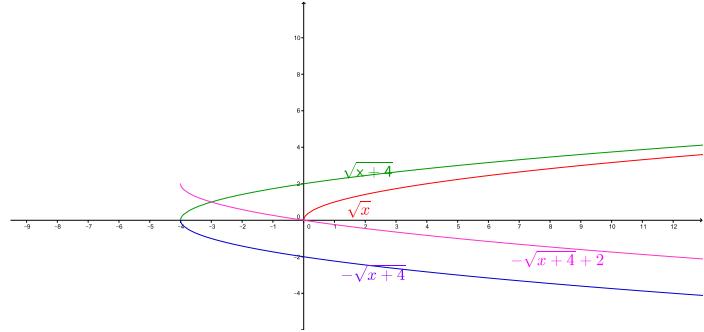
(b) **V** $f(x) = -(x+2)^2 + 1$
 $[D(f) = R; R(f) =] - \infty; 1]$]



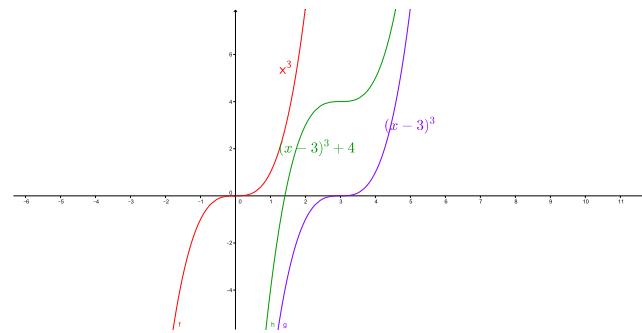
(c) **B** $f(x) = \sqrt{x-3} + 5$
 $[D(f) = [3; \infty[; R(f) = [5; \infty[$]



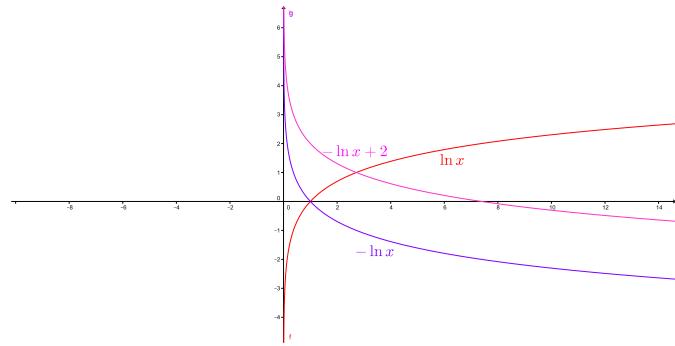
(d) **V** $f(x) = -\sqrt{x+4} + 2$
 $[D(f) = [-4; \infty[; R(f) =] - \infty; 2]$]



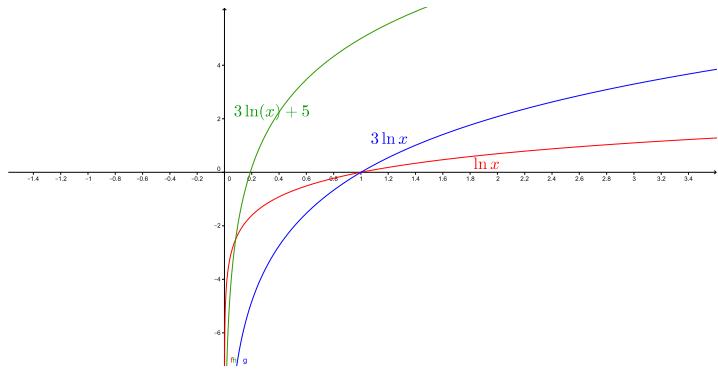
(e) **B** $f(x) = (x-3)^3 + 4$
 $[D(f) = R; R(f) = R]$



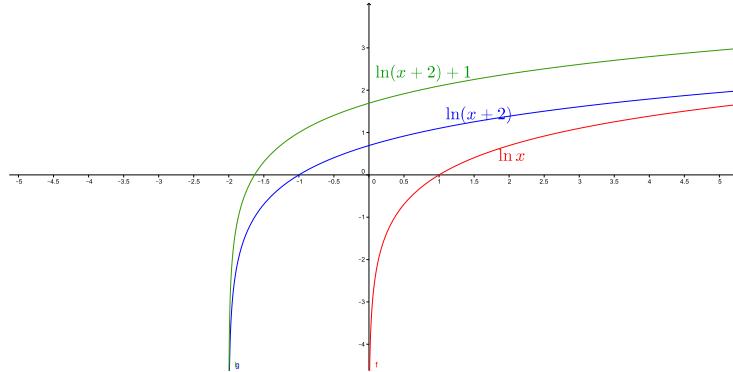
(f) **B** $f(x) = -\ln x + 2$
 $[D(f) =]0; \infty[; R(f) = R]$



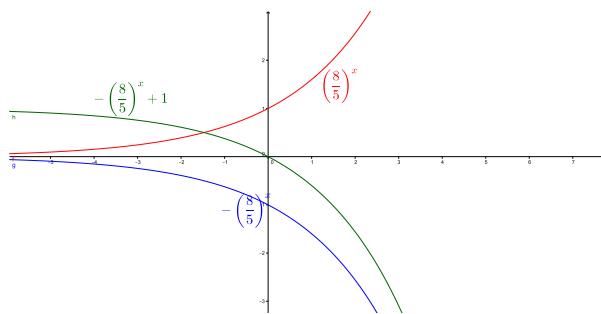
(g) **B** $f(x) = 3 \ln x + 5$
 $[D(f) =]0; \infty[; R(f) = R]$



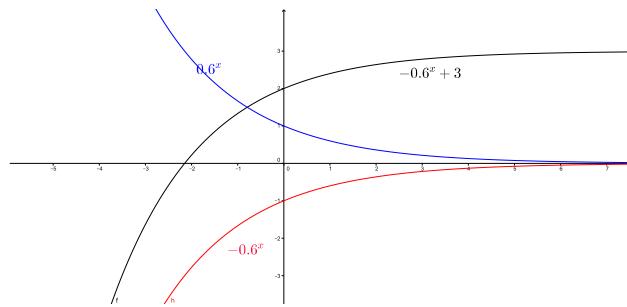
(h) **B** $f(x) = \ln(x+2) + 1$
 $[D(f) =] - 2; \infty[; R(f) = R]$



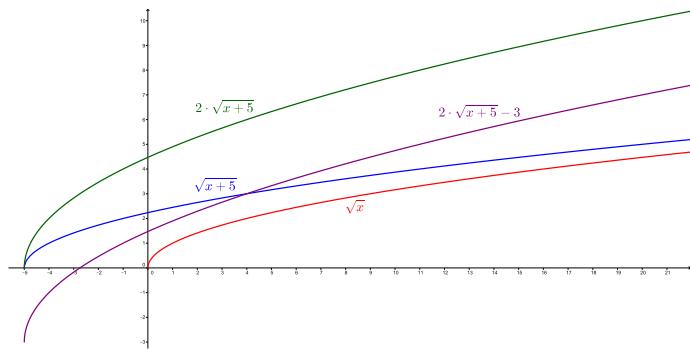
(i) **B** $f(x) = -\left(\frac{8}{5}\right)^x + 1$
 $[D(f) = R; R(f) =] - \infty; 1[]$



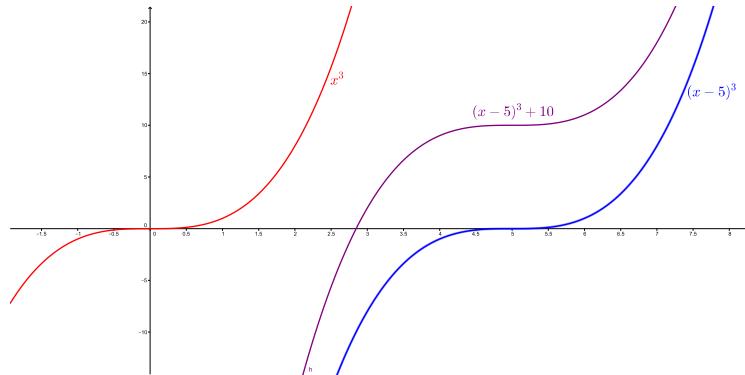
(j) **B** $f(x) = -0,6^x + 3$
 $[D(f) = R; R(f) =] - \infty; 3[]$



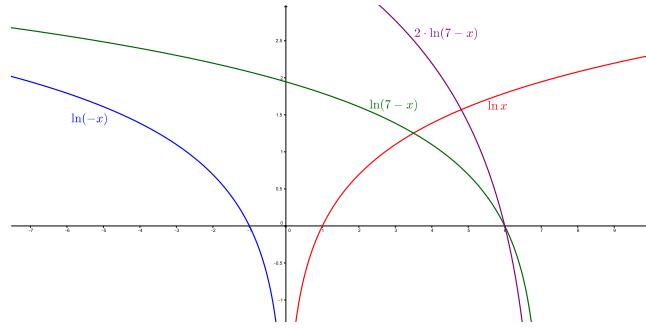
(k) **V** $f(x) = 2\sqrt{x+5} - 3$
 $[D(f) = [-5; \infty[; R(f) = [-3; \infty[]]$



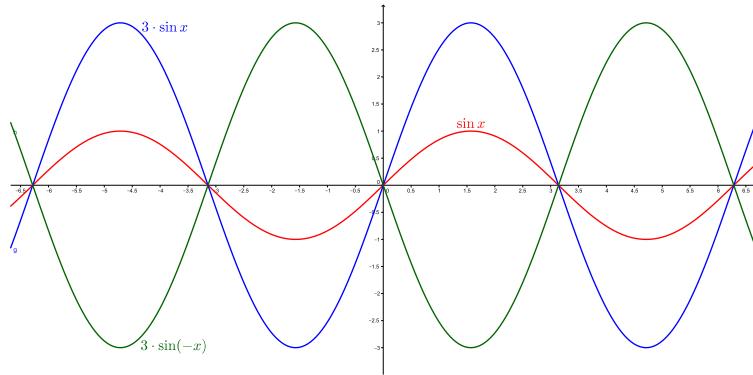
(l) **B** $f(x) = (x-5)^3 + 10$
 $[D(f) = R; R(f) = R]$



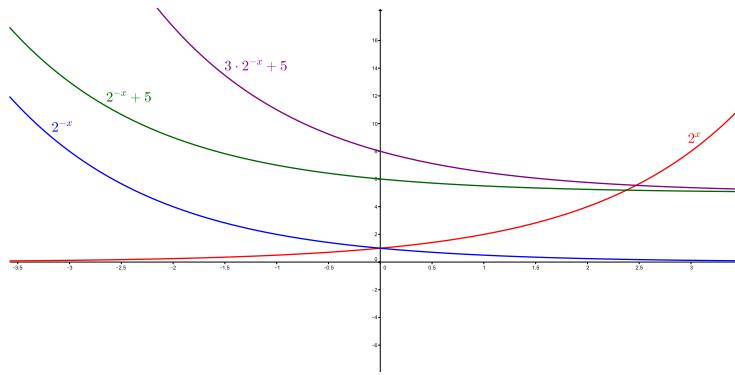
(m) **V** $f(x) = 2 \ln(7-x)$
 $[D(f) =] - \infty; 7[; R(f) = R]$



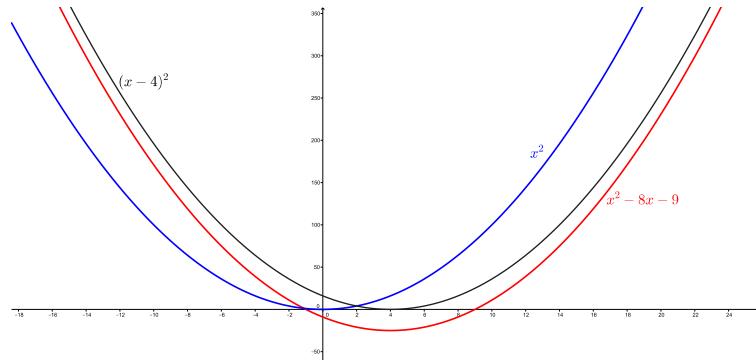
(n) **B** $f(x) = 3 \sin(-x)$
 $[D(f) = R; R(f) = [-3; 3]]$



(o) **V** $f(x) = 3 \cdot 2^{-x} + 5$
 $[D(f) = R; R(f) =]5; \infty[$

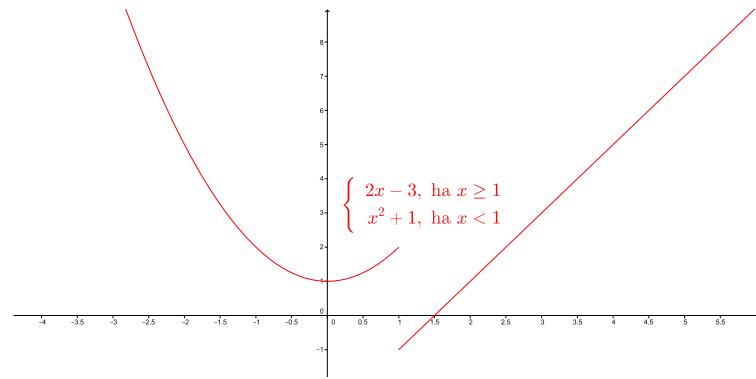


(p) **V** $f(x) = x^2 - 8x - 9$
 $[x^2 - 8x - 9 = (x - 4)^2 - 25; D(f) = R; R(f) = [-25; \infty[$

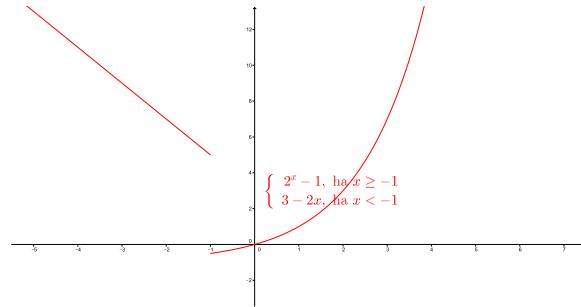


5. Ábrázolja az alábbi $f : R \rightarrow R$ függvényeket, majd olvassa le a függvények értékkészletét!

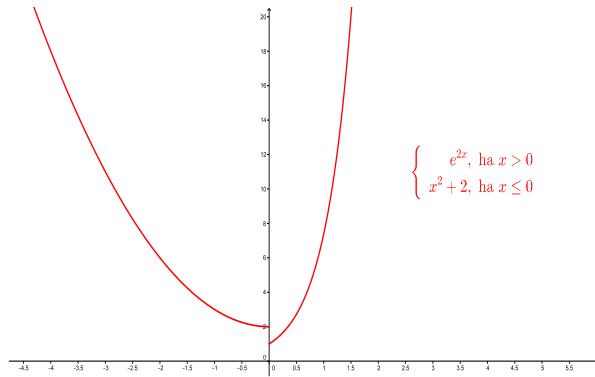
(a) **B** $f(x) = \begin{cases} 2x - 3 & \text{ha } x \geq 1 \\ x^2 + 1 & \text{ha } x < 1 \end{cases}$
 $[R(f)] = [-1; \infty[$



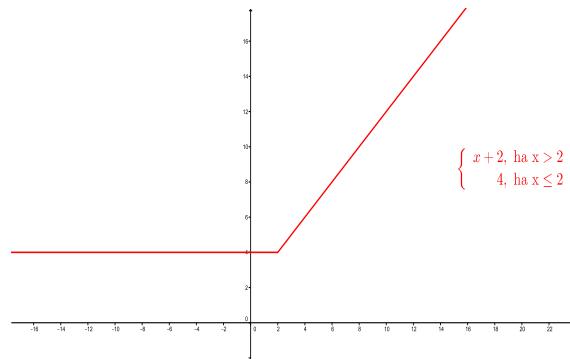
(b) **V** $f(x) = \begin{cases} 2^x - 1 & \text{ha } x \geq -1 \\ 3 - 2x & \text{ha } x < -1 \end{cases}$
 $[R(f)] = [-0,5; \infty[$



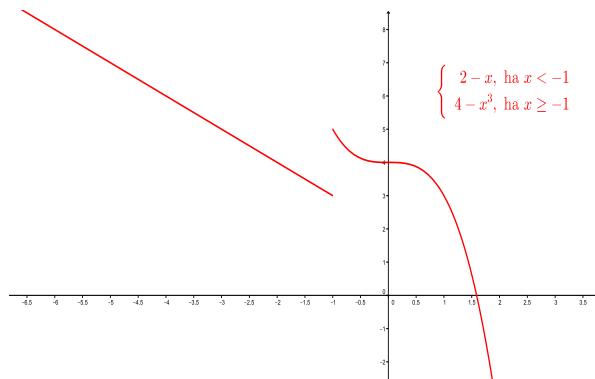
(c) **B** $f(x) = \begin{cases} e^{2x} & \text{ha } x > 0 \\ x^2 + 2 & \text{ha } x \leq 0 \end{cases}$
 $[R(f) =]1; \infty[$



(d) **B** $f(x) = \begin{cases} x + 2 & \text{ha } x > 2 \\ 4 & \text{ha } x \leq 2 \end{cases}$
 $[R(f) = [4; \infty[$



(e) **V** $f(x) = \begin{cases} 2 - x & \text{ha } x < -1 \\ 4 - x^3 & \text{ha } x \geq -1 \end{cases}$
 $[R(f) = R]$



6. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

- (a) $f(x) = \sqrt{4x - 8}$ $[4x - 8 \geq 0; D(f) = [2; \infty[]]$
- (b) $g(x) = \log_8(9 - 5x)$ $[9 - 5x < 0; D(g) =] - \infty; \frac{9}{5}[]$
- (c) $h(x) = \sqrt[7]{4x + 7}$ $[D(h) = R]$
- (d) $f(x) = \frac{3x + 7}{8x + 9}$ $[8x + 9 \neq 0; D(f) = R \setminus \{-\frac{9}{8}\}]$
- (e) $g(x) = e^{6x-7}$ $[D(g) = R]$
- (f) $h(x) = \frac{8x}{\sqrt[5]{4x - 3}}$ $\left[\sqrt[5]{4x - 3} \neq 0; D(h) = R \setminus \{\frac{3}{4}\} \right]$
- (g) $f(x) = \frac{4x - 5}{\sqrt{3x + 6}}$ $\left[\sqrt{3x + 6} \neq 0; 3x + 6 \geq 0; D(f) =] - 2; \infty[\right]$
- (h) $g(x) = \frac{5^{-x+7}}{8 - 5x}$ $\left[8 - 5x \neq 0; D(g) = R \setminus \{\frac{8}{5}\} \right]$
- (i) **B** $h(x) = \sqrt[4]{x^2 + 3x - 10}$ $[x^2 + 3x - 10 \geq 0; D(h) =] - \infty; -5[\cup]2; \infty[]$
- (j) **B** $f(x) = \frac{3x^5 - 2}{\sqrt[3]{x^2 - 2x - 3}}$ $\left[\sqrt[3]{x^2 - 2x - 3} \neq 0; D(f) = R \setminus \{-1; 3\} \right]$
- (k) **B** $f(x) = \frac{2}{\sqrt{x^2 - x - 20}}$ $\left[\sqrt{x^2 - x - 20} \neq 0; x^2 - x - 20 \geq 0; D(f) =] - \infty; -4[\cup]5; \infty[\right]$
- (l) **B** $f(x) = \sqrt{-x^2 + 6x - 8}$ $[-x^2 + 6x - 8 \geq 0; D(f) = [2; 4]]$

7. Határozza meg a következő $f : R \rightarrow R$ függvények legbővebb értelmezési tartományát!

- (a) **B** $f(x) = \log_7(-x^2 + 4x + 12)$ $[-x^2 + 4x + 12 > 0; D(f) =] - 2; 6[]$
- (b) **B** $f(x) = 3^{\frac{4x-3}{-x+7}}$ $[-x + 7 \neq 0; D(f) = R \setminus \{7\}]$
- (c) **B** $f(x) = \frac{6}{\log_3(x + 5)}$ $[x + 5 \geq 0; \log_3(x + 5) \neq 0; D(f) =] - 5; \infty[\setminus \{-4\}]$
- (d) **B** $f(x) = \frac{\sqrt{x}}{x^2 - 5x + 6}$ $[x \geq 0; x^2 - 5x + 6 \neq 0; D(f) = [0; \infty[\setminus \{2; 3\}]$
- (e) **B** $f(x) = -\frac{\ln(3x - 2)}{4x + 7}$ $[3x - 2 > 0; 4x + 7 \neq 0; D(f) =] \frac{2}{3}; \infty[]$
- (f) **B** $f(x) = \ln(3 - 2x - x^2)$ $[3 - 2x - x^2 > 0; D(f) =] - 3; 1[]$
- (g) **B** $f(x) = \frac{\sqrt{7 + 4x}}{3x - 12}$ $\left[7 + 4x \geq 0; 3x - 12 \neq 0; D(f) = [-\frac{7}{4}; \infty[\setminus \{4\} \right]$
- (h) **B** $f(x) = \frac{\lg(11 - 4x)}{8x - 3}$ $\left[11 - 4x > 0; 8x - 3 \neq 0; D(f) =] - \infty; \frac{11}{4} \setminus \{\frac{3}{8}\} \right]$
- (i) **B** $f(x) = \frac{5}{4 - \sqrt{5x + 3}}$ $\left[5x + 3 \geq 0; 4 - \sqrt{5x + 3} \neq 0; D(f) = [-\frac{3}{5}; \infty[\setminus \{\frac{13}{5}\} \right]$

- (j) **B** $f(x) = \ln(10 - 2x)\sqrt{4x - 5}$ $[10 - 2x > 0; 4x - 5 \geq 0; D(f) = [\frac{5}{4}; 5[]]$
- (k) **B** $f(x) = \log_7(-5x + 7) + \sqrt{3x}$ $[-5x + 7 > 0; 3x \geq 0; D(f) = [0; \frac{7}{5}[]]$
- (l) **B** $f(x) = \frac{\log_6(5x + 8)}{\sqrt[5]{3x - 5}}$ $[5x + 8 \geq 0; 3x - 5 \neq 0; D(f) =] - \frac{8}{5}; \infty[\setminus\{\frac{5}{3}\}]$
- (m) **B** $f(x) = \frac{-3x + 8}{\sqrt[6]{x^2 - 6x - 7}}$
 $\left[\sqrt[6]{x^2 - 6x - 7} \neq 0; x^2 - 6x - 7 \geq 0; D(f) =] - \infty; -1 \cup]7; \infty[\right]$
- (n) **B** $f(x) = \frac{4x - 7}{\ln(3x - 6)}$ $[3x - 6 \geq 0; \ln(3x - 6) \neq 0; D(f) =]2; \infty[\setminus\{\frac{7}{3}\}]$

8. Határozza meg a valós számok legbővebb részhalmazát, melyen az adott függvény értelmezhető!

- (a) **V** $f(x) = \frac{x}{\sqrt[4]{x^2 - 9} - 4}$
 $[x^2 - 9 \geq 0; \sqrt[4]{x^2 - 9} - 4 \neq 0; D(f) =] - \infty; -3 \cup]3; \infty[\setminus\{-5; 5\}]$
- (b) **V** $f(x) = \sqrt{\frac{4x - 5}{-1 + 3x}}$ $\left[\frac{4x - 5}{1 - 3x} \geq 0; D(f) =]\frac{1}{3}; \frac{5}{4}[\right]$
- (c) **V** $f(x) = \sqrt{\log_2 x + 5}$
 $[\log_2 x + 5 \geq 0 (\log_2 x - szigorúan monoton növekvő); x > 0; D(f) = [2^{-5}; \infty] = [\frac{1}{32}; \infty]]$
- (d) **V** $f(x) = \sqrt[6]{\log_{0,4} x + 2}$
 $[\log_{0,4} x + 2 \geq 0 (\log_{0,4} x - szigorúan monoton csökkenő); x > 0;$
 $D(f) =]0; 0, 4^{-2}] =]0; 6, 25]]$
- (e) **V** $f(x) = \frac{2}{4 - \sqrt{x^2 + 7}}$ $[x^2 + 7 \geq 0; 4 - \sqrt{x^2 + 7} \neq 0; D(f) = R \setminus \{-3; +3\}]$
- (f) **V** $f(x) = \frac{-7}{5 - \sqrt{x^2 - 9}}$
 $[x^2 - 9 \geq 0; 5 - \sqrt{x^2 - 9} \neq 0; D(f) =] - \infty; -3] \cup [3; \infty[\setminus\{-\sqrt{34}; \sqrt{34}\}]$
- (g) **V** $f(x) = \frac{4}{\ln(5x) + 3}$
 $[5x > 0; \ln(5x) + 3 \neq 0; D(f) =]0; \infty[\setminus\{\frac{e^{-3}}{5}\} =]0; \infty[\setminus\{\frac{1}{5e^3}\}]$
- (h) **V** $f(x) = \sqrt{3^x - 81}$
 $[3^x - 81 \geq 0 (3^x - szigorúan monoton növekvő); D(f) = [4; \infty[]$
- (i) **V** $f(x) = \ln(x^2 - x)\sqrt{16 - 4x^2}$
 $[x^2 - x > 0; 16 - 4x^2 \geq 0; D(f) = [-2; 0 \cup]1; 2]]$
- (j) **V** $f(x) = \frac{x}{\ln(x^2 + x - 12)}$
 $[x^2 + x - 12 > 0; \ln(x^2 + x - 12) \neq 0; D(f) =] - \infty; -4 \cup]3; \infty[\setminus\{3, 14\}]$
- (k) **V** $f(x) = \sqrt{\frac{2x - 4}{7 + x}}$ $\left[\frac{2x - 4}{7 + x} \geq 0; D(f) =] - \infty; -7 \cup]2; \infty[\right]$

(k) **V** $f(x) = \frac{\ln(x^2 - 20)}{x - 6}$
 $[x^2 - 20 > 0; x - 6 \neq 0; D(f) =] - \infty; -\sqrt{20}[\cup]\sqrt{20}; \infty[\setminus\{6\}]$

9. Az alábbi $f : R \rightarrow R$ függvényeknek létezik inverze. Határozza meg az inverz függvény hozzárendelési utasítását!

- | | |
|---|---|
| (a) B $f(x) = 4x - 7$ | $[f^{-1}(x) = \frac{x+7}{4}]$ |
| (b) B $f(x) = \frac{x-5}{4-2x}$ | $[f^{-1}(x) = \frac{4x+5}{1+2x}]$ |
| (c) B $f(x) = \frac{3x-4}{x+2}$ | $[f^{-1}(x) = \frac{2x+4}{3-x}]$ |
| (d) B $f(x) = 3(x+4)^7 - 2$ | $[f^{-1}(x) = \sqrt[7]{\frac{x+2}{3}} - 4]$ |
| (e) B $f(x) = 6 - 5(x-1)^5$ | $[f^{-1}(x) = \sqrt[5]{\frac{x-6}{-5}} + 1]$ |
| (f) B $f(x) = 5\sqrt[3]{x-8} + 9$ | $[f^{-1}(x) = \left(\frac{x-9}{5}\right)^3 + 8]$ |
| (g) B $f(x) = -2\sqrt[4]{x+3} - 7$ | $[f^{-1}(x) = \left(\frac{x+7}{-2}\right)^5 - 3]$ |
| (h) B $f(x) = 3e^{2x-7} + 5$ | $[f^{-1}(x) = \frac{\ln(\frac{x-5}{3})+7}{2} = \frac{1}{2}\ln\left(\frac{x-5}{3}\right) + \frac{7}{2}]$ |
| (i) B $f(x) = 8^{x+2} - 4$ | $[f^{-1}(x) = \log_8(x+4) - 2]$ |
| (j) B $f(x) = 5^{2-\frac{x}{3}} - 1$ | $[f^{-1}(x) = -3(\log_2(x+1) - 2)]$ |
| (k) B $f(x) = \log_6(4x-8) + 3$ | $[f^{-1}(x) = \frac{6^{x-3}+8}{4}]$ |
| (l) B $f(x) = 5\log_2(6-4x) + 2$ | $[f^{-1}(x) = \frac{2^{\frac{x-2}{5}-6}}{-4} = -\frac{1}{4}2^{\frac{x-2}{5}} + \frac{3}{2}]$ |
| (m) B $f(x) = 4\ln(3+2x) - 7$ | $[f^{-1}(x) = \frac{e^{\frac{x+7}{4}}-3}{2} = \frac{1}{2}e^{\frac{x+7}{4}} - \frac{3}{2}]$ |

10. Határozza meg a következő $f : R \rightarrow R$ függvények legbővebb értelmezési tartományát és értékkészletét! Határozza meg az inverz függvényt és annak legbővebb értelmezési tartományát és értékkészletét!

- (a) **V** $f(x) = \ln(2x+5) - 8$
 $[D(f) =] - 2, 5; \infty[; R(f) = R; f^{-1}(x) = \frac{e^{x+8}-5}{2};$
 $D(f^{-1}) = R; R(f^{-1}) =]2, 5; \infty[]$
- (b) **V** $f(x) = 4\sqrt{6-2x} + 12$
 $[D(f) =] - \infty; 3]; R(f) = [12; \infty[; f^{-1}(x) = \frac{(\frac{x-12}{4})^2-6}{-2} = -\frac{1}{2}\left(\frac{x-12}{4}\right)^2 + 3;$
 $D(f^{-1}) = [12; \infty[; R(f^{-1}) =] - \infty; 3]]$
- (c) **V** $f(x) = e^{5x+3} - 4$
 $[D(f) = R; R(f) =] - 4; \infty[; f^{-1}(x) = \frac{\ln(x+4)-3}{5};$
 $D(f^{-1}) =] - 4; \infty[; R(f^{-1}) = R]$

(d) **V** $f(x) = \frac{3x - 15}{4 + x}$

$$[D(f) = R \setminus \{-4\}; R(f) = R \setminus \{3\}; f^{-1}(x) = \frac{-15 - 4x}{x - 3};$$

$$D(f^{-1}) = R \setminus \{3\}; R(f^{-1}) = R \setminus \{-4\}]$$

(e) **V** $f(x) = 3 - 2^{3x-5}$

$$[D(f) = R; R(f) =]-\infty; 3[; f^{-1}(x) = \frac{\log_2(3-x)+5}{3};$$

$$D(f^{-1}) =]-\infty; 3[; R(f^{-1}) = R]$$

(f) **V** $f(x) = 4(3x + 7)^5 + 6$

$$[D(f) = R; R(f) = R; f^{-1}(x) = \sqrt[5]{\frac{x-6}{4}} - 7;$$

$$D(f^{-1}) = R; R(f^{-1}) = R]$$

(g) **V** $f(x) = 4 \log_3(7x - 14) + 11$

$$[D(f) =]2; \infty[; R(f) = R; f^{-1}(x) = \frac{3^{\frac{x-11}{4}} + 14}{7};$$

$$D(f^{-1}) = R; R(f^{-1}) =]2; \infty[$$