3. Gyakorlat

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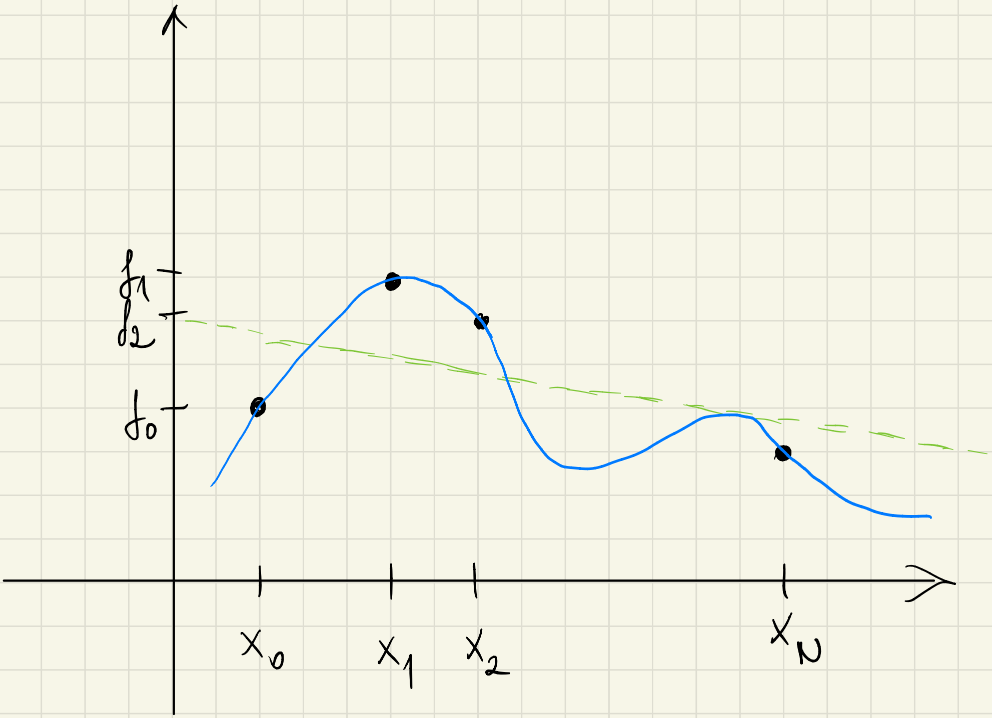
Feladatsor:

rs1.sze.hu/~szalayk

vagy

szelearning, 3. Téma, 3. Gyakorlat



f_1
f_2
f_0
x_0
x_1
x_2
x_N

① Határozzuk meg az alábbi adatokra illeszkedő Lagrange-interpolációs polinomot!

b) $x_0 = -1 < x_1 = 1 < x_2 = 2 < x_3 = 3$

$f_0 = -4 \quad f_1 = 2 \quad f_2 = 8 \quad f_3 = 24$

Mo: max $(4-1) = 3$-adfokú polinom

osztott differenciák módszere

$$\rightarrow L_3(x) = a_0 + a_1 \cdot (x - x_0) + a_2 \cdot (x - x_0)(x - x_1) +$$
$$+ a_3 \cdot (x - x_0)(x - x_1)(x - x_2)$$

a_0, a_1, a_2, a_3 ?

x_j	f_j			
-1	$\boxed{-4} = a_0$			
		$\frac{2-(-4)}{1-(-1)} = \boxed{3} = a_1$		
1	2		$\frac{6-3}{2-(-1)} = \boxed{1} = a_2$	
		$\frac{8-2}{2-1} = 6$		$\frac{5-1}{3-(-1)} = \boxed{1} = a_3$
2	8		$\frac{16-6}{3-1} = 5$	
		$\frac{24-8}{3-2} = 16$		
3	24			

$$\boxed{\begin{aligned} L_3(x) = -4 + 3\cdot(x-(-1)) + 1\cdot(x-(-1))\cdot(x-1) + \\ + a_3\cdot(x-(-1))(x-1)\cdot(x-2) \end{aligned}}$$

$$\Rightarrow L_3(x) = 2x - x^2 + x^3$$

c) $x_0 = 0$ $\quad x_1 = 2$ $\quad x_2 = 3$ $\quad x_3 = 4$

$f_0 = 0$ $\quad f_1 = 14$ $\quad f_2 = 51$ $\quad f_3 = 124$

Mo: (osztott diff.-k módszere)

x_j	f_j			
0	$\boxed{0} = a_0$	$\frac{14-0}{2-0} = \boxed{7} = a_1$	$\frac{37-7}{3-0} = \boxed{10} = a_2$	$\frac{18-10}{4-0} = \boxed{2} = a_3$
2	14	$\frac{51-14}{3-2} = 37$	$\frac{73-37}{4-2} = 18$	
3	51	$\frac{124-51}{4-3} = 73$		
4	124			

$$\boxed{\begin{aligned} L_3(x) &= 0 + 7\cdot(x-0) + 10\cdot(x-0)(x-2) + \\ &+ 2\cdot(x-0)(x-2)(x-3) \end{aligned}}$$

$$L_3(x) = 2x^3 - x$$

(2) Adott az $\boxed{f(x) = \cos\left(\frac{\pi}{2}\cdot x\right)}$ függvény.

Adottak az $x_0 = -2$; $x_1 = -1$; $x_2 = 0$; $x_3 = 1$; $x_4 = 2$ alappontok és a hozzájuk rendelt $\boxed{f_j = f(x_j)}$ $(j = 0,1,2,3,4)$ értékek.

a) Határozzuk meg az adatokra illeszkedő Lagrange-interpolációs polinomot.

b) Közelítsük $f(1{,}5)$ értékét az interpolációs polinom $x_0 = 1{,}5$ helyen vett értékével!

Mo: a) $\boxed{x_0 = -2}$ $\quad x_1 = -1$ $\quad x_2 = 0$ $\quad x_3 = 1$ $\quad \boxed{x_4 = 2}$

$f_0 = -1$ $\quad f_1 = 0$ $\quad f_2 = 1$ $\quad f_3 = 0$ $\quad f_4 = -1$

$$f_0 = f(x_0) = \cos\left(\frac{\pi}{2}\cdot(-2)\right) = -1$$

$$f_1 = f(x_1) = \cos\left(\frac{\pi}{2}\cdot(-1)\right) = 0$$

$$f_2 = f(x_2) = \cos\left(\frac{\pi}{2}\cdot 0\right) = 1$$

$\vdots$

x_j	y_j
-2	$\boxed{-1} = a_0$
-1	0
0	1
1	0
2	-1

$$\frac{0-(-1)}{-1-(-2)} = \boxed{1} = a_1$$

$$\frac{1-0}{0-(-1)} = 1$$

$$\frac{0-1}{1-0} = -1$$

$$\frac{-1-0}{2-1} = -1$$

$$\frac{1-1}{0-(-2)} = \boxed{0} = a_2$$

$$\frac{-1-1}{1-(-1)} = -1$$

$$\frac{-1-(-1)}{2-0} = 0$$

$$\frac{-1-0}{1-(-2)} = \boxed{-\frac{1}{3}} = a_3$$

$$\frac{0-(-1)}{2-(-1)} = \frac{1}{3}$$

$$\frac{\frac{1}{3}-\left(-\frac{1}{3}\right)}{2-(-2)} = \boxed{\frac{1}{6}} = a_4$$

$$\boxed{\begin{aligned} L_4(x) &= -1 + 1\cdot(x-(-2)) + 0\cdot(x+2)(x-(-1)) + \\ &+ \left(-\frac{1}{3}\right)\cdot(x+2)(x+1)\cdot(x-0) + \frac{1}{6}(x+2)(x+1)x\cdot(x-1) \end{aligned}}$$

$$\Rightarrow L_4(x) = \frac{1}{6}x^4 - \frac{7}{6}x^2 + 1$$

b)

$$f(1,5) \approx L_4(1,5) = \underline{\underline{-0,7813}}$$

pontos $f(1,5) = \cos\left(\frac{\pi}{2}\cdot 1,5\right) = -\frac{\sqrt{2}}{2} = \underline{\underline{-0,7071}}$

Hiba: $\left|f(1,5) - L_4(1,5)\right| = \boxed{\underline{0,0742}}$

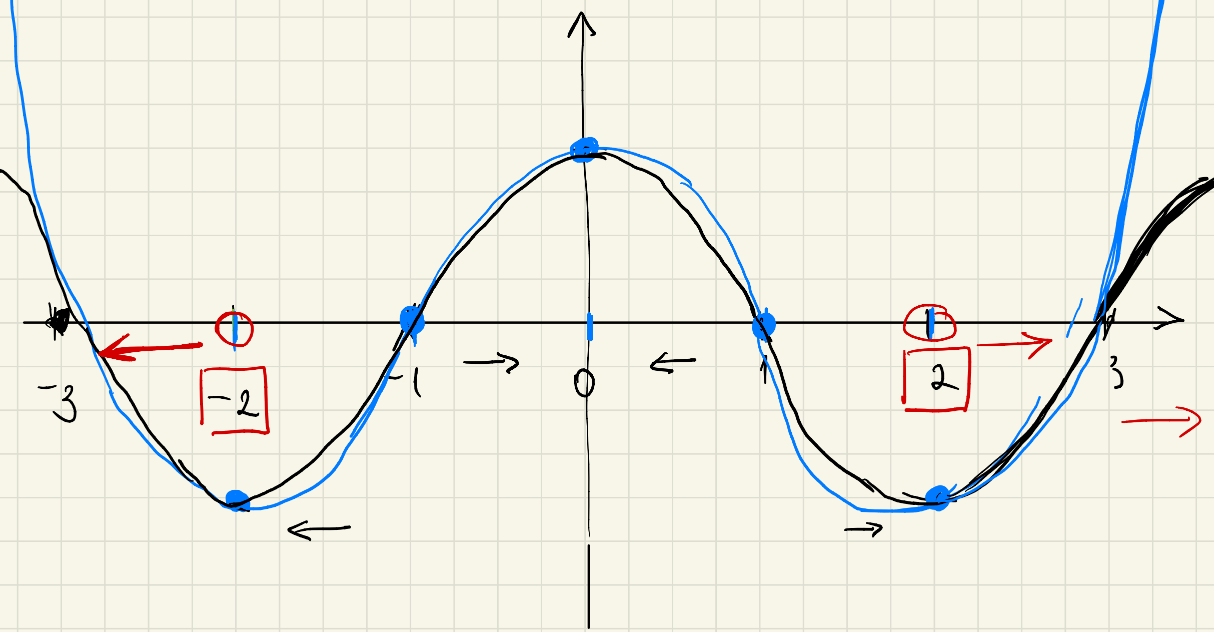
<u>Megj:</u>

$f(4) \approx L_4(4) = 25$

$f(4) = \cos\left(\frac{\pi}{2}\cdot 4\right) = 1$

$\lightning$ $4 \notin [-2,2]$

$f(x) = \cos\left(\frac{\pi}{2} \cdot x\right)$



3.) Határozzuk meg az alábbi adatokra illeszkedő Hermite-interpolációs polinomot!

a) $x_0 = 0$; $x_1 = 1$

$f_0 = 1$; $f_1 = 0$

$f'_0 = 2$; $f'_1 = 1$

Mo: max $(2+2)-1 = 3$-adfokú

$$\boxed{H_3(x) = A + B\cdot\frac{(x-x_0)}{h} + C\cdot\frac{(x-x_0)^2}{h^2} + D\cdot\frac{(x-x_0)^3}{h^3}}$$

$$\boxed{h = (x_1 - x_0)}$$

A, B, C, D ???

$$\boxed{\begin{aligned} A &= f_0 \\ A+B+C+D &= f_1 \\ B &= f'_0\cdot h \\ B+2C+3D &= f'_1\cdot h \end{aligned}}$$

$$h = 1 - 0 = 1$$

$$\left.\begin{array}{l} A = 1 \\ A + B + C + D = 0 \\ B = 2 \cdot 1 \\ B + 2C + 3D = 1 \cdot 1 \end{array}\right\} \quad \boxed{A=1} \quad \boxed{B=2}$$

$\Longrightarrow$ 12 gép

$$\left.\begin{array}{l} C + D = -1 - 2 = -3 \\ 2C + 3D = 1 - 2 = -1 \end{array}\right\} \cdot 2 \quad -$$

$$D = 5 \qquad C = -3 - 5 = -8$$

$$\boxed{H_3(x) =} \; 1 + 2 \cdot \frac{x-0}{1} - 8 \cdot \frac{(x-0)^2}{1^2} + 5 \cdot \frac{(x-0)^3}{1^3} =$$

$$\boxed{= 1 + 2x - 8x^2 + 5x^3}$$

4. Adott az $f(x) = \ln(x^2+2)$ függvény. Adottak az $x_0 = 0$; $x_1 = 0{,}5$ interpolációs alappontok és a hozzájuk rendelt $\boxed{f_0 = f(x_0)}$; $f_1 = f(x_1)$; $\boxed{f'_0 = f'(x_0)}$; $f'_1 = f'(x_1)$ értékek.

a) Határozzuk meg az adatokra illeszkedő Hermite-interpolációs polinomot!

b) Közelítsük $f(0{,}1)$ értékét az interpolációs polinom $x_0 = 0{,}1$ helyen vett helyettesítési értékével!

Mo: a)

$x_0 = 0$	$x_1 = 0{,}5$
$f_0 = \ln 2$	$f_1 = \ln 2{,}25$
$f'_0 = 0$	$f'_1 = \frac{4}{9}$

$f_0 = f(x_0) = f(0) = \ln(0^2+2) = \ln 2$

$$f_1 = f(x_1) = f(0{,}5) = \ln(0{,}5^2+2) = \ln 2{,}25$$

$$f'(x) = \frac{1}{x^2+2} \cdot 2x = \frac{2x}{x^2+2}$$

$$f'_0 = f'(x_0) = f'(0) = \frac{2\cdot 0}{0^2+2} = 0$$

$$f'_1 = f'(x_1) = f'(0{,}5) = \frac{2\cdot 0{,}5}{0{,}5^2+2} = \frac{1}{2{,}25} = \frac{4}{9}$$

$$\left.\begin{aligned} A &= \underline{\ln 2} \\ A+B+C+D &= \ln 2{,}25 \\ B &= 0\cdot 0{,}5 = \underline{0} \\ B+2C+3D &= \frac{4}{9}\cdot 0{,}5 = \frac{2}{9} \end{aligned}\right\} \quad \begin{aligned} h &= 0{,}5-0 \\ &\boxed{h = 0{,}5} \end{aligned}$$

$\Rightarrow$ 12. gép

$A = 0{,}6931$ $\qquad C = 0{,}1311$

$B = 0$ $\qquad D = -0{,}0133$

$$\boxed{H_3(x) = 0{,}6931 + 0\cdot\frac{x-0}{0{,}5} + 0{,}1311\cdot\frac{(x-0)^2}{0{,}5^2} - 0{,}0133\,\frac{(x-0)^3}{0{,}5^3}}$$

b) $f(0{,}1) \approx H_3(0{,}1) = \underline{0{,}6983}$

$f(0{,}1) = \ln(0{,}1^2+2) = \underline{\underline{0{,}6981}}$

Hiba: $|f(0{,}1) - H_3(0{,}1)| = |0{,}6981 - 0{,}6983| =$

$= \underline{\underline{0{,}0002}}$