dq Transformations

$$\text{abc} \longrightarrow \alpha\beta \longrightarrow \text{dq}$$

$$f_a + f_b + f_c = 0$$

$$\begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \times \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \qquad \begin{bmatrix} f_d \\ f_q \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \times \begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix}$$

$$\begin{bmatrix} f_d \\ f_q \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos(\phi) & \cos(\phi-\gamma) & \cos(\phi+\gamma) \\ -\sin(\phi) & -\sin(\phi-\gamma) & -\sin(\phi+\gamma) \end{bmatrix} \times \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

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$$f_a + f_b + f_c = 0$$

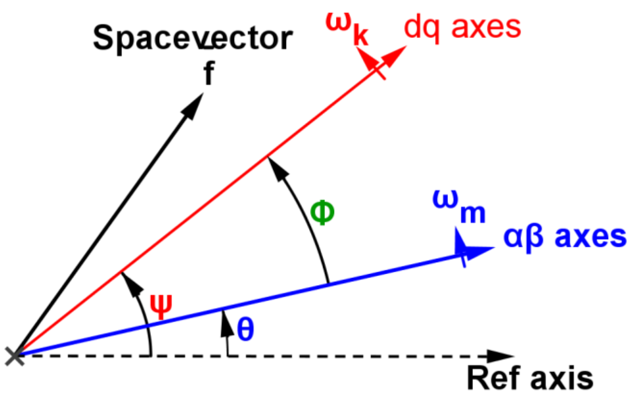
$$\begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \times \begin{bmatrix} f_d \\ f_q \end{bmatrix} \qquad \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \times \begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix}$$

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \cos(\phi-\gamma) & -\sin(\phi-\gamma) \\ \cos(\phi+\gamma) & -\sin(\phi+\gamma) \end{bmatrix} \times \begin{bmatrix} f_d \\ f_q \end{bmatrix}$$

$$\boldsymbol{\gamma = \frac{2\pi}{3}}$$

ϕ = angle between dq and αβ reference frames

A space vector $\overrightarrow{f_o}$ and its time rate of change $\overrightarrow{g_o}$ are attached to an αβ coordinate system rotating at the speed $\omega_m = \frac{d\theta}{dt}$. The transformation to a dq coordinate system rotating at the speed $\omega_k = \frac{d\psi}{dt}$ is performed using the rotating matrix $M(\phi)$ where $\phi = \psi - \theta$.



Specifically, in terms of **Space vectors** and **Rotating matrix,**

$$\overrightarrow{f_o} = \begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix} \qquad \overrightarrow{f_k} = \begin{bmatrix} f_d \\ f_q \end{bmatrix} \qquad M(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}$$

the transformation of variables takes the form

$$\boldsymbol{\overrightarrow{f_o} = M(\phi)\overrightarrow{f_k}} \qquad \text{or the reverse} \qquad \boldsymbol{\overrightarrow{f_k} = M^t(\phi)\overrightarrow{f_o}}$$

The time rate of change of the initial space vector $\overrightarrow{f_o}$ is

$$\overrightarrow{g_o} = \frac{d}{dt}\overrightarrow{f_o} = \frac{d}{dt}\left[M(\phi)\overrightarrow{f_k}\right] = M(\phi)\frac{d}{dt}\overrightarrow{f_k} + \frac{d}{dt}M(\phi) * \overrightarrow{f_k}$$

$$\overrightarrow{g_o} = M(\phi)\overrightarrow{g_k} = M(\phi)\frac{d}{dt}\overrightarrow{f_k} + (\omega_k - \omega_m)M(\phi)M\left(\frac{\pi}{2}\right)\overrightarrow{f_k}$$

so that, after cancelling $M(\phi)$, the time derivative in the k frame becomes

$$\boldsymbol{\overrightarrow{g_k} = \frac{d}{dt}\overrightarrow{f_k} + (\omega_k - \omega_m)M\left(\frac{\pi}{2}\right)\overrightarrow{f_k}}$$

Note that $\frac{d}{dt}M(\phi) = \frac{d\phi}{dt}\frac{d}{d\phi}M(\phi) = (\omega_k - \omega_m)\frac{d}{d\phi}M(\phi)$ and

$$\frac{d}{d\phi}M(\phi) = \begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= M(\phi)M\left(\frac{\pi}{2}\right)$$

A matter of scale in 3⟾2 and 2⟾3 phase transformations:

$$\begin{bmatrix} f_d \\ f_q \end{bmatrix} = \kappa \begin{bmatrix} \cos(\phi) & \cos(\phi-\gamma) & \cos(\phi+\gamma) \\ -\sin(\phi) & -\sin(\phi-\gamma) & -\sin(\phi+\gamma) \end{bmatrix} \times \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad \text{or}$$

$$f_{dq} = \kappa\, T\, f_{abc}$$

Pseudo-inverse conversion:

$$f_{abc} = k_i\, T^t\, f_{dq} \qquad \text{where } k_i = \frac{2}{3}\frac{1}{\kappa} \qquad \text{and} \qquad \frac{2}{3}\, T\, T^t = U_2$$

Quadratic form conversion:

$$f_a^2 + f_b^2 + f_c^2 = k_p\,(f_d^2 + f_q^2) \qquad \text{where} \qquad k_p = \frac{2}{3}\frac{1}{\kappa^2}$$

Magnitude conversion:

$$f_\alpha = k_m\, f_a \qquad \text{where} \qquad k_m = \frac{3}{2}\kappa$$

Common conventions:

	a	b	c	d
κ	$2/3$	$\sqrt{2/3}$	1	$\sqrt{2}/3$
k_i	1	$\sqrt{2/3}$	$2/3$	$\sqrt{2}$
k_p	$3/2$	1	$2/3$	3
k_m	1	$\sqrt{3/2}$	$3/2$	$1/\sqrt{2}$

a : Equal magnitude of 2- and 3-phase balanced sinusoidal signals

b : Equal 2- and 3-phase power (power invariant)

c : 2-phase amplitude equals 3/2 3-phase amplitude

d : 2-phase amplitude equals rms of 3-phase signal